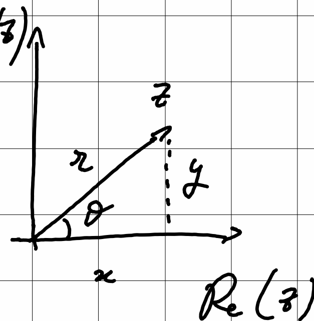
Numeri complessi

rappresentazione cartesiana



$z = x + jy$

$x = Re(z) \quad y = Im(z)$

$j^2 = -1 \quad \Rightarrow \quad \frac{1}{j} = -j$

rappresentazione trigonometrica

$z = r(\cos\theta \pm j \sin\theta)$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \Leftrightarrow \begin{cases} r = \sqrt{x^2+y^2} & \text{modulo} \\ \theta = \pm \arccos\frac{x}{r} & \text{argomento} \end{cases}$$

$\theta \in [-\pi, \pi]$

$$\theta = \begin{cases} \operatorname{arctg}\frac{y}{x} & \text{se } x > 0 \\ \operatorname{arctg}\frac{y}{x} + \pi & \text{se } x < 0 \text{ e } y \geq 0 \\ \operatorname{arctg}\frac{y}{x} - \pi & \text{se } x < 0 \text{ e } y < 0 \end{cases}$$

rappresentazione esponenziale

$$z = r e^{j\vartheta} = r(\cos\vartheta + j \sin\vartheta) \qquad \text{Eulero}$$

es: $j = \cos\frac{\pi}{2} + j \sin\frac{\pi}{2} = e^{j\frac{\pi}{2}}$ $\qquad \arg j = \frac{\pi}{2}$

$-j = e^{-j\frac{\pi}{2}}$ $\qquad \arg(-j) = -\frac{\pi}{2}$

complesso coniugato :

$$z = x + jy = r e^{j\vartheta}$$

$$z^* = x - jy = r e^{-j\vartheta}$$

stesso modulo e argomento opposto

prodotto :

$$A e^{j\alpha} \cdot B e^{j\beta} = A \cdot B \, e^{j(\alpha+\beta)}$$

$$z_1 \cdot z_2 = x_1 x_2 - y_1 y_2 + j(x_1 y_2 + x_2 y_1)$$

$$\mathrm{Re}(z_1 \cdot z_2) = \mathrm{Re}\, z_1 \, \mathrm{Re}\, z_2 - \mathrm{Im}(z_1)\, \mathrm{Im}(z_2)$$

rapporto:

$$\frac{z_1}{z_2} = \frac{r_1 e^{j\theta_1}}{r_2 e^{j\theta_2}} = \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)}$$

$$\arg \frac{z_1}{z_2} = \theta_1 - \theta_2 = \arg z_1 - \arg z_2$$

$$\frac{1}{z} = \frac{z^*}{z z^*} = \frac{z^*}{|z|^2} = \frac{x - jy}{x^2 + y^2} = \frac{e^{-j\theta}}{r}$$

modulo:

$$|x + jy|^2 = (x + jy)(x - jy) = x^2 + y^2$$

$$|e^{j\theta}| = 1 \implies |r e^{j\theta}|^2 = r^2$$