Limiti

$$\lim_{n\to+\infty} n^a = \begin{cases} +\infty & \text{se } a>0 \\ 0 & \text{se } a<0 \end{cases}$$

$$\lim_{n\to+\infty} a^n = \begin{cases} +\infty & \text{se } a>1 \\ 0 & \text{se } a\in(0,1) \end{cases}$$

Dim. $\quad a^n = (1+a-1)^n \geqslant 1+(a-1)n$

$1+(a-1)n \longrightarrow +\infty \quad$ se $a>1$

Se $a\in(0,1)$ lo scrivo come $a=\dfrac{1}{b}$

$$a^n = \frac{1}{b^n} \longrightarrow \frac{1}{+\infty} = 0$$

$$\lim_{n\to+\infty} \frac{n^2+3n+5}{2n^2-n} = \lim_{n\to+\infty} \frac{n^2\left(1+\frac{3}{n}+\frac{5}{n^2}\right)}{n^2\left(2-\frac{1}{n}\right)} = \frac{1}{2}$$

$$\lim_{n\to+\infty} \sqrt[n]{2} = 1$$

<u>Dim.</u> $(1+x)^n \geq 1+nx$ uso $x=\frac{1}{n}$

$$\left(1+\frac{1}{n}\right)^n \geq 2 \implies 1+\frac{1}{n} \geq \sqrt[n]{2}$$

$$\implies \sqrt[n]{1} = 1 \leq \sqrt[n]{2} \leq 1+\frac{1}{n}$$

— o —

La successione $e_n = \left(1+\frac{1}{n}\right)^n$ è limitata e crescente

$$2 \leq e_n \leq 3 \quad \forall n \geq 1$$

quindi ha un limite reale che si indica con "e"

$$\lim_{n\to+\infty} \left(1+\frac{1}{n}\right)^n = e = 2{,}718\ldots$$

<u>Dim.</u> $\left(1+\frac{1}{n}\right)^n \geq 2$ dalla disuguaglianza di Bernoulli

$$\left(1+\frac{1}{n}\right)^n = 1 + n\frac{1}{n} + \frac{n(n-1)}{2!}\frac{1}{n^2} + \frac{n(n-1)(n-2)}{3!}\frac{1}{n^3} + \ldots$$

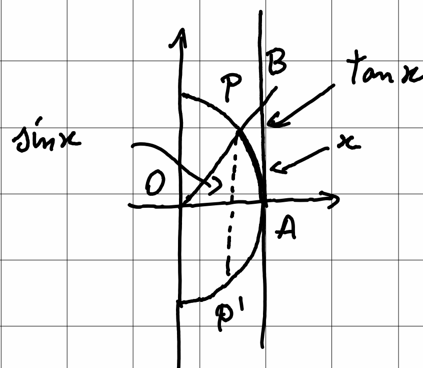
$$\leq 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \ldots + \frac{1}{n!} \leq 1+1+\frac{1}{2}+\frac{1}{4}+\ldots+\frac{1}{2^{n-1}}$$

$$\leq 1+2 = 3$$ dal binomio di Newton

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

<u>Dim.</u> $\sin x \le x \le \tan x \quad \forall x \in \left(0, \frac{\pi}{2}\right)$

infatti



$\overline{PP'} = 2 \sin x$

$\widehat{PP'} = 2x$

$\overline{PP'} \le \widehat{PP'} \Rightarrow \sin x \le x$

Area Triangolo $OAB = \frac{1}{2} \cdot 1 \cdot \tan x = \frac{\tan x}{2}$

Area settore $OAP = \frac{x \cdot \pi}{2\pi} = \frac{x}{2}$

Area settore $\le$ Area Triangolo $\Rightarrow x \le \tan x$

$\sin x \le x \le \tan x \quad \forall x \in \left(0, \frac{\pi}{2}\right)$

posso dividere per $\sin x > 0$

$$1 \le \frac{x}{\sin x} \le \frac{1}{\cos x} \Rightarrow \lim_{x \to 0^+} \frac{x}{\sin x} = 1$$

Per il teorema algebrico quindi

$$\lim_{x \to 0^+} \frac{\sin x}{x} = 1$$

e poiché $\frac{\sin x}{x}$ é una funzione pari anche

$$\lim_{x \to 0^-} \frac{\sin x}{x} = 1 \quad \text{e quindi} \quad \lim_{x \to 0} \frac{\sin x}{x} = 1$$

— o —

$$\lim_{x \to +\infty} \frac{\sin x}{x} = 0$$

<u>Dim.</u> $-1 \leq \sin x \leq 1$ e dividendo per $x > 0$

$$\Rightarrow \quad -\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$$

$$\lim_{x \to -\infty} \frac{\sin x}{x} = 0$$

<u>Dim.</u> $-1 \leq \sin x \leq 1$ e dividendo per $x < 0$

$$\Rightarrow \quad -\frac{1}{x} \geq \frac{\sin x}{x} \geq \frac{1}{x}$$