Fisica Teorica 2 / Advanced Quantum Field Theory

(the context)

Claudio Bonati

From "Fisica Teorica 1 / Quantum Field Theory"

Historical reason for introducing QFTs:

quantum mechanics, special relativity, and electromagnetism

So far you have seen

QFT in the canonical formalism

applied to the computation of

cross sections using the lowest perturbative order (tree level)

These computations are meaningful only for "weakly interacting" theories, useful typically for some aspects of high-energy physics (e.g., QED or weak interactions).

Modern formulation of QFTs (since $\simeq$ late '70s)

> QFT is the theoretical framework needed to accurately describe the "low-energy" physics of systems in which fluctuations are important.

In this generalized context one often speaks of quantum and statistical field theories, depending whether fluctuations are of quantum or of thermal origin.

If fluctuations are not important classical field theory is enough (fluid mechanics, elasticity, electromagnetism, general relativity), moreover in some cases QFT is well approximated by quantum mechanics.

The meaning of "low-energy" depends on what we are actually studying. It can goes from $\ll 10^{19}\ \mathrm{GeV}$ down to $\ll 10^{-3}\ \mathrm{eV}$.

Particles in the modern formulation of QFTs

Particles are the excitations of the underlying QFT
(identical particles)

Particles can be either of fundamental nature

leptons and quarks in the Standard Model (so far)

or describe effective excitations of the system we are considering

pions in chiral effective theory, phonons in a crystal,
electrons in a semiconductor, Cooper pairs in a superconductor

Both the cases can be described by a QFT as far as we limit ourselves to the appropriate energy range. It is possibile (likely) that today fundamental physics is tomorrow effective low energy physics.

How physical description can change with the scale

Lagrangian of the fundamental model:

$$L = \sum_i \left(\frac{1}{2} m \dot{x}_i^2 - \frac{\kappa}{2} (x_{i+1} - x_i - a)^2 \right)$$

Introducing $\phi_i = x_i - x_i^{\mathrm{(eq)}}$ and considering distances $\gg a$ (i.e., momenta $\ll 1/a$) we get the effective theory described by the Lagrangian density ($v_s^2 = \kappa/m$)

$$\mathcal{L} = \frac{1}{2} \Big(\partial_t \phi(t,x) \Big)^2 - \frac{v_s^2}{2} \Big(\partial_x \phi(t,x) \Big)^2 = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi$$

Original symmetry: shift ($x_i \to x_i + \Delta$; $\phi(t,x) \to \phi(t,x) + \Delta$)

Emergent symmetries: translation and Lorentz (with $c \to v_s$)

Classical FT: elasticity. Quantum FT: acoustic phonons.

High-energy physics / condensed matter physics

The traditional (?) subdivision in high-energy physics or low-energy/condensed matter physics is useless and harmful in theoretical physics.

The modern understanding of several key concepts of theoretical physics has benefited greatly from the contamination between high/low energy perspectives.

Spontaneous symmetry breaking

low-energy → high-energy → everything

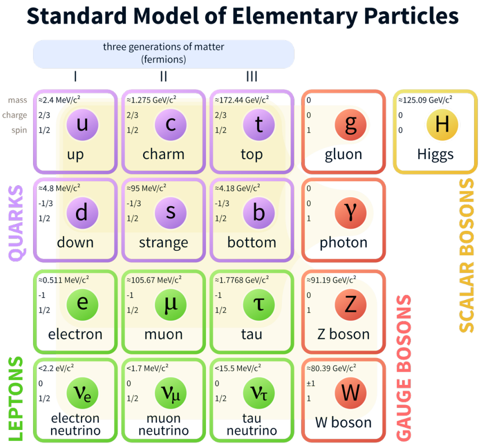
Higgs mechanism

high-energy → low-energy → high-energy → everything

Renormalization & renormalization group

high-energy → low-energy → everything

High-energy: Standard Model



Standard Model of Elementary Particles

three generations of matter (fermions)

		I	II	III		
QUARKS	mass charge spin	≈2.4 MeV/c² 2/3 1/2 u up	≈1.275 GeV/c² 2/3 1/2 c charm	≈172.44 GeV/c² 2/3 1/2 t top	0 0 1 g gluon	≈125.09 GeV/c² 0 0 H Higgs
		≈4.8 MeV/c² -1/3 1/2 d down	≈95 MeV/c² -1/3 1/2 s strange	≈4.18 GeV/c² -1/3 1/2 b bottom	0 0 1 γ photon	**SCALAR BOSONS**
LEPTONS		≈0.511 MeV/c² -1 1/2 e electron	≈105.67 MeV/c² -1 1/2 μ muon	≈1.7768 GeV/c² -1 1/2 τ tau	≈91.19 GeV/c² 0 1 Z Z boson	**GAUGE BOSONS**
		<2.2 eV/c² 0 1/2 ν_e electron neutrino	<1.7 MeV/c² 0 1/2 ν_μ muon neutrino	<15.5 MeV/c² 0 1/2 ν_τ tau neutrino	≈80.39 GeV/c² ±1 1 W W boson	

Caution: the masses of the light quarks can not be interpreted as masses in the standard sense of the word.

High-energy: Standard Model

So far the most economic and successful way to describe particle physics phenomenology.

Described by the QFT based on

- locality
- Poincarè invariance
- gauge symmetry $U(1) \times SU(2) \times SU(3)$
  (sort-of nonAbelian generalization of QED)
- renormalizability (more to be said later on)

Electroweak sector dates back to 1967, the strong sector was completed in 1973 (asymptotic freedom and third quark family).

Accurate results in the EW sector

Magnetic dipole moment and g-factor: $\boldsymbol{M} = g \frac{e}{2m} \boldsymbol{S}$

For elementary spin 1/2 particles: tree level $g = 2$,
one loop $a \equiv \frac{g-2}{2} = \frac{\alpha}{2\pi} \simeq 0.001\,161\,\ldots$ (Schwinger '48)

$a_e^{(\mathrm{exp})} = 0.001\,159\,652\,180\,59(13)$ and 5 loop QED used to fix α

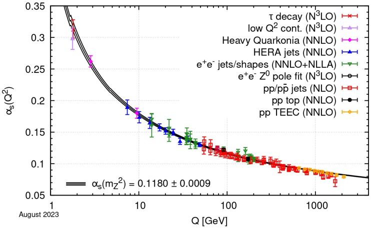
$$a_\mu^{(\mathrm{exp})} = 0.001\,165\,920\,72(15)$$
$$a_\mu^{(\mathrm{th})} = 0.001\,165\,920\,33(62)$$

$(a_\mu^{(\mathrm{exp})} - a_\mu^{(\mathrm{th})}) \times 10^{11} = 39 \pm 62_{\mathrm{th}} \pm 15_{\mathrm{exp}}$ Aliberti et al. 2505.21476

"New physics" bounds

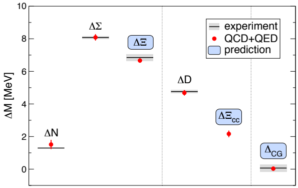
$$\frac{e}{2\Lambda} \bar{\psi} [\gamma_\mu, \gamma_\nu] \psi F^{\mu\nu} \quad \longrightarrow \quad \Lambda \gtrsim 10^8 \mathrm{GeV}$$

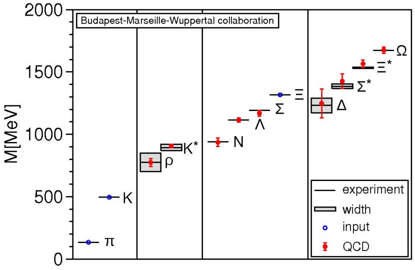
Accurate results in the strong sector



Perturbative running coupling constant $\alpha_s(Q^2)$ in $\overline{\mathrm{MS}}$ scheme at 5 loops compared with experiments

Hadron spectrum from numerical simulations of lattice QCD compared with experiments





Condensed matter physics

Condensed matter phenomena are typically characterized by the following properties:

- large number of degrees of freedom $\approx O(10^{23})$, problems has to be addressed using statistical methods (stat. mech.)
- the degrees of freedom describing the low-energy excitations are often very different from the microscopic fundamental ones (emergence)
- different systems may show common collective behaviors although they differ microscopically (universality)

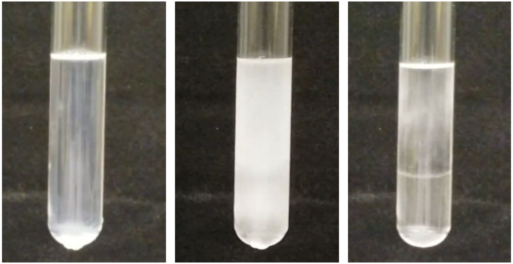
Examples: magnetism, theory of metals, superconductivity, ^{3}He and ^{4}He quantum liquids, quantum Hall effects, optical lattices,. . .

Depending on the physical setup thermal or quantum fluctuations can be the most important ones.

Continuous thermal phase transitions

Characterized by the divergence of the correlation length ξ of the critical modes: $\xi \sim (T - T_c)^{-\nu}$. Also $C_V \sim |T - T_c|^{-\alpha}$, $\chi \sim |T - T_c|^{-\gamma}$, $G(x) \sim |x|^{-1-\eta}$, ...

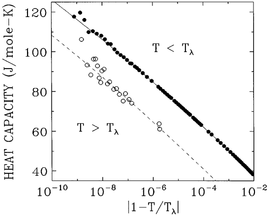
Critical opalescence happens when $\xi \gtrsim$ visible wavelength



Mixture of methanol and cyclohexane heated at 65-70°C
(by "Chemistry and Biochemistry Demo lab at OSU",
see https://www.youtube.com/watch?v=DIGdbmJvFUw)

Continuous thermal phase transitions: precision measures

C_{V} at the normal-superfluid transition in ^{4}He, up to a few nK from T_c.



order parameter: cond. wavefunction
complex field $\varphi(x)$ with U(1) symmetry
QFT: $\mathcal{L} = |\partial_\mu \varphi|^2 + r\,|\varphi|^2 + u\,|\varphi|^4$

Results for the 3d XY universality class

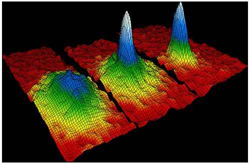
		ν	ref.
Experiment	^{4}He	0.66758(6)	Physica B **284-288** (2000), 49
	^{4}He	0.6709(1)	cond-mat/0310163
QFT	6,7-loop 3d-exp	0.6703(15)	cond-mat/9803240
	7-loop ϵ-exp	0.67076(38)	2005.12714
	conformal bootstrap	0.67175(10)	1912.03324
Stat mech	MC+HT	0.6717(1)	cond-mat/0605083
	MC	0.671718(23)	2507.19265

Phase transitions in the quantum regime

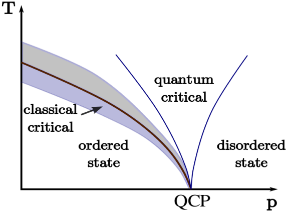
BEC in bosonic gases, when

$$\lambda_{\mathrm{deB}} = \left(\frac{h^2}{mT}\right)^{1/2} \approx d_{\mathrm{atoms}} = (N/V)^{-1/3}$$

Observed in weakly interacting gases of alkali atoms (rubidium, sodium, lithium, ...).
velocity distribution of rubidium atoms ⟶



At a quantum critical point scaling laws describe the interplay between quantum and thermal fluctuations.
Interesting phenomenology for quantum magnetism, high-T superconductors, quantum many particle systems, ...



Why are QFTs so effective?

The main ingredients behind the success of QFTs are

1. symmetries (exact and/or spontanously broken)
2. renormalization and renormalization group

Symmetries constraint the dynamics and the spectrum of a theory. In the presence of spontaneous symmetry breaking massless or light excitations can be present, whose interactions are greatly constrained by symmetry arguments.

Renormalization group in the Wilson formulation is a systematic way of disentangling the low-energy features of a theory from its high-energy details.

QFTs work when different scales exist, providing effective descriptions at low-energy, even when the microscopic details are unknown.

Modern point of view on renormalizability: non-renormalizable interactions are irrelevant at low-energy.

Reductionism or not?

P. A. M. Dirac (?) "... and the rest is chemistry!"

V. Weisskopf "Intensive research goes for the fundamental laws, extensive research goes for the explanation of phenomena in terms of known fundamental laws"

P. W. Anderson "The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of a simple extrapolation of the properties of a few particles. Instead, at each level of complexity entirely new properties appear, and the understanding of the new behaviors requires research which I think is as fundamental in its nature as any other"

It is rather ironic to note that QFTs have been initially developed for fundamental physics, but the modern approach to QFTs is that they are effective descriptions of something more fundamental.

Reductionism or not?

Q: Doesn't all this mean that quantum field theory, for all its successes, is an approximation that may have little to do with the underlying theory? And isn't renormalization a bad thing, since it implies that we can only probe the high energy theory through a small number of parameters?

Reductionism or not?

Q: Doesn't all this mean that quantum field theory, for all its successes, is an approximation that may have little to do with the underlying theory? And isn't renormalization a bad thing, since it implies that we can only probe the high energy theory through a small number of parameters?

A: Nobody ever promised you a rose garden.

J. Polchinksi "Effective Field Theory and the Fermi Surface"
hep-th/9210046

A (not so) short history of QFT & its friends

- P. A. M. Dirac "The quantum theory of emission and absorption of radiation" Proc. Roy. Soc. London A **114**, 243 (1927)
  E. Fermi "Sopra l'elettrodinamica quantistica" Atti Acc. Lincei **12**, 431 (1930)
- R. Peierls "On Ising's model of ferromagnetism" Proc. Cambridge Phil. Soc. **32**, 477 (1936)
- L. D. Landau "On the Theory of Phase Transitions" Zh. Eksp. Teor. Fiz. **7**, 19 (1937)
- L. Onsager "Crystal Statistics. I. A Two-Dimensional Model with an Order-Disorder Transition" Phys. Rev. **65**, 117 (1944)
- S. Tomonaga "On infinite field reactions in quantum field theory" Phys. Rev **74**, 224 (1948)
  J. Schwinger "On quantum electrodynamics and the magnetic moment of the electron" Phys. Rev. **73**, 416 (1948)
  R. P. Feynman "The theory of positrons" Phys. Rev. **76**, 749 (1949)
- F. J. Dyson "The radiation theories of Tomonaga, Schwinger, and Feynman" Phys. Rev. **75**, 486 (1949)
  F. J. Dyson "The S matrix in quantum electrodynamics" Phys. Rev. **75**, 1736 (1949)
- J. C. Ward "An Identity in Quantum Electrodynamics" Phys. Rev. **78**, 182 (1950)
  Y. Takahashi "On the generalized Ward identity" Nuovo Cim. **6**, 371 (1957)
- V. L. Ginzburg, L. D. Landau "On the Theory of superconductivity" Zh. Eksp. Teor. Fiz. **20**, 1064 (1950)
- R. P. Feynman "Atomic theory of the λ transition in Helium" Phys. Rev. **91**, 1291 (1953)
- E. C. G. Stueckelberg, A. Petermann "La renormalisation des constants dans la théorie de quanta" Helv. Phys. Acta **26**, 499 (1953)
  M. Gell-Mann, F. E. Low "Quantum Electrodynamics at Small Distances" Phys. Rev. **95**, 1300 (1954)
- C. N. Yang, R. L. Mills "Conservation of Isotopic Spin and Isotopic Gauge Invariance" Phys. Rev. **96**, 191 (1954)
- R. Kubo "Statistical-Mechanical Theory of Irreversible Processes. I." J. Phys. Soc. Jpn. **12**, 570 (1957)
  J. Schwinger "On the Euclidean structure of relativistic field theory" Proc. N. A. Sci. U.S. **44**, 956 (1958)
  P. C. Martin, J. Schwinger "Theory of many particle systems. 1." Phys. Rev. **115**, 1342 (1959)
- J. Bardeen, L. N. Cooper, J. R. Schrieffer "Theory of Superconductivity". Phys. Rev. **108**, 1175 (1957)

A (not so) short history of QFT & its friends

- P. Anderson "Coherent excited states in the theory of superconductivity; gauge invariance and the Meissner effect" Phys. Rev. **110**, 827 (1958)
  Y. Nambu "Quasi-particles and gauge invariance in the theory of superconductivity" Phys. Rev. **117**, 648 (1960)
- Y. Nambu "Axial vector current conservation in weak interactions" Phys. Rev. Lett. **4**, 380 (1960)
  M. Gell-Mann, M. Lévy "The axial vector current in beta decay", Nuovo Cim. **16**, 705 (1960)
  Y. Nambu, G. Jona-Lasinio "Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I" Phys. Rev. **122**, 345 (1961)
- S. Weinberg "High-Energy Behavior in Quantum Field Theory" Phys. Rev. **118**, 838 (1960)
- S. L. Glashow "Partial-symmetries of weak interactions" Nucl. Phys. B **22**, 579 (1961)
- J. Goldstone "Field Theories with Superconductor Solutions" Nuovo Cim. **19**, 154 (1961)
  J. Goldstone, A. Salam, S. Weinberg "Broken Symmetries" Phys. Rev. **127**, 965 (1962)
- J. Schwinger "Gauge invariance and mass" Phys. Rev. **125**, 397 (1962)
  P. Anderson "Plasmons, gauge invariance and mass" Phys. Rev. **130**, 439 (1963)
- N. Cabibbo "Unitary Symmetry and Leptonic Decays" Phys. Rev. Lett. **10**, 531 (1963)
- M. Gell-Mann "A Schematic Model of Baryons and Mesons" Phys. Lett. **8**, 214 (1964)
  O. W. Greenberg "Spin and Unitary-Spin Independence in a Paraquark Model of Baryons and Mesons" Phys. Rev. Lett. **13**, 598 (1964)
  M. Y. Han, Y. Nambu "Three-Triplet Model with Double SU(3) Symmetry" Phys. Rev. **139**, B1006 (1965)
- F. Englert, R. Brout "Broken symmetry and the mass of gauge vector mesons" Phys. Rev. Lett. **13**, 321 (1964)
  P. W. Higgs "Broken symmetries and the masses of gauge bosons" Phys. Rev. Lett. **13**, 508 (1964)
  G. S. Guralnik, C. R. Hagen, T. W. B. Kibble "Global conservation laws and massless particles" Phys. Rev. Lett. **13**, 585 (1964)
- L. Kadanoff "Scaling laws for Ising models near T_c" Physics **2**, 263 (1966)
- S. Weinberg "A Model of Leptons" Phys. Rev. Lett. **19**, 1264 (1967)
  A. Salam "Weak and Electromagnetic Interactions" Conf. Proc. C 680519, 367 (1968)
- L. D. Faddeev, V. N. Popov "Feynman diagrams for the Yang-Mills field" Phys. Lett. B. **25**, 29 (1967)

A (not so) short history of QFT & its friends

- H. E. Stanley "Spherical Model as the Limit of Infinite Spin Dimensionality" Phys. Rev. **176**, 718 (1968)
- S. L. Adler "Axial-Vector Vertex in Spinor Electrodynamics" Phys. Rev. **177**, 2426 (1969)
  J. S. Bell, R. Jackiw "A PCAC puzzle: $\pi^0 \to \gamma\gamma$ in the σ-model" Nuovo Cim. A **60**, 47 (1969)
- A. M. Polyakov "Conformal symmetry of critical fluctuations" Pisma Zh. Eksp. Teor. Fiz. **12**, 538 (1970), JETP Lett. **12**, 381 (1970)
- C. G. Callan "Broken Scale Invariance in Scalar Field Theory" Phys. Rev. D **2**, 1541 (1970)
  K. Symanzik "Small distance behaviour in field theory and power counting" Comm. Math. Phys. **18**, 227 (1970)
  S. Coleman, R. Jackiw "Why dilatation generators do not generate dilatations?" Annals Phys. **67**, 552 (1971)
  K. G. Wilson "Renormalization Group and Strong Interactions" Phys. Rev. D **3**, 1818 (1971).
- K. G. Wilson "Renormalization Group and Critical Phenomena. I. Renormalization Group and the Kadanoff Scaling Picture" Phys. Rev. B **4**, 3174 (1971)
- F. Wegner "Duality in Generalized Ising Models and Phase Transitions without Local Order Parameter" J. Math. Phys. **12**, 2259 (1971)
- J. C. Taylor "Ward identities and charge renormalization of the Yang-Mills field" Nucl. Phys. B **33**, 436 (1971)
  A. A. Slavnov "Ward identities in gauge theories" Th. Math. Phys. **10**, 99 (1972)
- G. 't Hooft, M. Veltman "Regularization and renormalization of gauge fields" Nucl. Phys. B **44**, 189 (1972)
  B. W. Lee, J. Zinn-Justin "Spontaneously Broken Gauge Symmetries. I. Preliminaries" Phys. Rev. D **5**, 3121 (1972)
- K. G. Wilson, M. E. Fisher "Critical Exponents in 3.99 Dimensions" Phys. Rev. Lett. **28**, 240 (1972)
- D. J. Gross, F. Wilczek "Ultraviolet behavior of non-abelian gauge theories" Phys. Rev. Lett. **30**, 1343 (1973)
  H.D. Politzer "Reliable perturbative results for strong interactions" Phys. Rev. Lett. **30**, 1346 (1973)
- M. Kobayashi, T. Maskawa "CP-violation in the renormalizable theory of weak interaction" Prog. Theor. Phys. **49**, 652 (1973)
- K. G. Wilson "Quantum Field-Theory Models in Less Than 4 Dimensions" Phys. Rev. D **7**, 2911 (1973)

A (not so) short history of QFT & its friends

- E. Brezin, J. C. Le Guillou, J. Zinn-Justin "Wilson's Theory of Critical Phenomena and Callan-Symanzik Equations in $4 - \varepsilon$ Dimensions" Phys. Rev. D **8**, 434 (1973)
  G. Parisi "Field-theoretic approach to second-order phase transitions in two- and three-dimensional systems" Lectures at Cargese Summer School 1973 [J. Stat. Phys. **23**, 49 (1980)]
- K. Osterwalder, R. Schrader "Axioms for Euclidean Green's functions" Comm. Math. Phys. **31**, 83 (1973)
  B. Simon, R. Griffiths "The $(\varphi^4)_2$ field theory as a classical Ising model" Commun. Math. Phys. **33**, 145 (1973)
  F. Guerra, L. Rosen, B. Simon "The $P(\phi)_2$ Eulidean quantum field theory as classical statistical mechanics. Part I" Annals of Mathematics **101**, 111 (1975)
- K. G. Wilson, J. B. Kogut "The renormalization group and the ε expansion" Phys. Rep. **12**, 75 (1974)
  K. G. Wilson "Confinement of quarks" Phys. Rev. D **10**, 2445 (1974)
- G. 't Hooft "Magnetic monopoles in unified gauge theories" Nucl. Phys. B. **79**, 276 (1974)
  A. M. Polyakov "Particle Spectrum in the Quantum Field Theory" JETP Lett. **20**, 194 (1974)
  A. A. Belavin, A. M. Polyakov, A. S. Schwartz, Y. S. Tyupkin "Pseudoparticle Solutions of the Yang-Mills Equations" Phys. Lett. B **59**, 85 (1975)
  T. T. Wu, C. N. Yang "Concept of nonintegrable phase factors and global formulation of gauge fields" Phys. Rev. D **12**, 3845 (1975)
- C. Becchi, A. Rouet, R. Stora "Renormalization of the Abelian Higgs-Kibble model" Comm. Math. Phys. **42**, 127 (1975)
- G. A. Baker, B. G. Nickel, M. S. Green, D. I. Meiron "Ising Model Critical Indices in Three-Dimensions from the Callan-Symanzik Equation" Phys. Rev. Lett. **36**, 1351 (1976)
  J. C. Le Guillou, J. Zinn-Justin "Critical Exponents for the N Vector Model in Three-Dimensions from Field Theory" Phys. Rev. Lett. **39**, 95 (1977)
- S. Weinberg "Phenomenological Lagrangians" Physica A **96** 327 (1979)
- S. Weinberg "Ultraviolet divergences in quantum theories of gravitation" in S. W. Hawking; W. Israel (eds.) "General Relativity: An Einstein centenary survey" (1979)
- M. Creutz "Monte Carlo study of quantized SU(2) gauge theory" Phys. Rev. D **21**, 2308 (1980)
  M. Creutz "Monte Carlo study of renormalization in lattice gauge theory" Phys. Rev. D **23**, 1815 (1981)