

Final projects for “Metodi Numerici per la Fisica”

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Module 1. “Introduction to Markov Chain Monte-Carlo with applications to statistical mechanics”

1. 2d Ising model: determination of critical properties (Metropolis single site update)
2. 2d Ising model on non-square lattices: determination of critical properties
3. 2d Ising model: determination of critical properties (cluster update)
4. 2d clock model: check that for $q = 4$ the transition is in the same universality class of the 2d Ising model
5. 3d clock model: check that for $q \geq 5$ the transition is in the same universality class of the 3d XY model
6. 2d Blume-Capel model: check that in some points of the phase diagram a transition in the 2d Ising universality class exists (see Ref. [1])
7. 3d XY model: determination of critical properties (Metropolis single site update and microcanonical update)
8. 3d XY model: determination of critical properties (cluster update)

Module 3. “Application of Monte-Carlo methods to the study of path-integral in quantum mechanics”

1. Thermodynamics and spectrum of the harmonic oscillator
2. Thermodynamics and spectrum of the harmonic oscillator with an anharmonic gx^4 correction
3. Study of the topological properties of the quantum mechanics of a particle on a circumference (local algorithms)
4. Study of the topological properties of the quantum mechanics of a particle on a circumference (parallel tempering and multicanonical update)
5. Study of the thermodynamics of a couple of identical particles (bosons and fermions) in an harmonic oscillator potential

Module 6. “Simulation of path-integral for quantum field theories”

1. Thermodynamics and spectrum of the free scalar field
2. Implementation of the HMC algorithm for the 2d $U(1)$ gauge model and validation of the algorithm against a local update
3. Study of the static potential and topological properties of the 2d $U(1)$ gauge model
4. Study of the static potential/string tension in the confined and deconfined phases of the 3d Z_2 gauge model (see Ref. [2] for useful strong and weak coupling expansions)
5. Study of the thermodynamics of $SU(2)$ gauge theory in 2+1 dimensions (see [3] for the relation between the bare coupling and the lattice spacing and [4] for the critical couplings)

References

- [1] J. Zierenberg, N. G. Fytas, M. Weigel, W. Janke, A. Malakis “Scaling and universality in the phase diagram of the 2D Blume-Capel model” Eur. Phys. J. Special Topics **226**, 789 (2017) [arXiv:1612.02138]
- [2] J. P. Kowall and H. Neuberger “Finite size Wilson loops in $Z(2)$ ” Phys. Lett. B **106**, 197 (1981)
- [3] M. J. Teper, “ $SU(N)$ gauge theories in (2+1)-dimensions” Phys. Rev. D **59**, 014512 (1999) [arXiv:9804008 [hep-lat]]
- [4] S. Edwards and L. von Smekal, “ $SU(2)$ lattice gauge theory in 2+1 dimensions: Critical couplings from twisted boundary conditions and universality,” Phys. Lett. B **681**, 484 (2009) [arXiv:0908.4030 [hep-lat]]