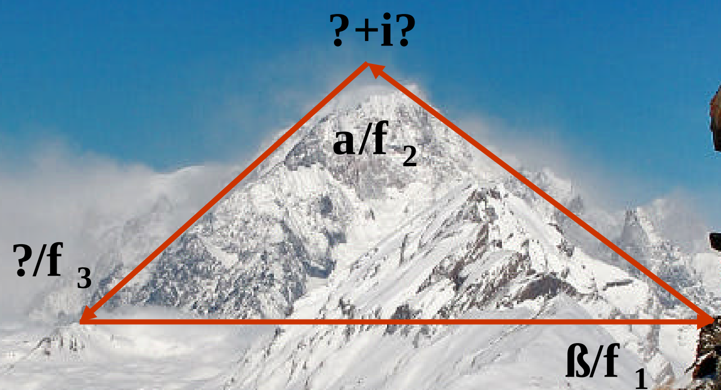
?/f_3 determination at B-factories



A. Poluektov
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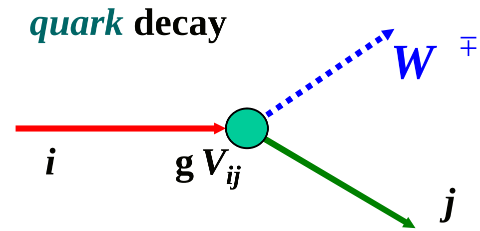
Novosibirsk, Russia

- CKM matrix and unitarity triangle
- GLW method
- ADS method
- Dalitz analysis method !
- sin(2ß+?) !

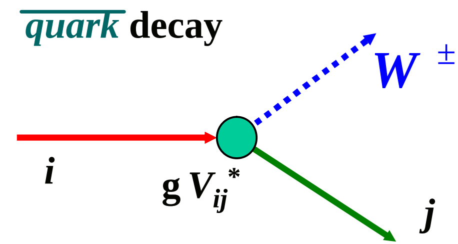
Weak decays and ??? matrix



Coupling constant g

Cabibbo-Kobayashi-Maskawa mixing matrix V_{ij}

Unitarity: $V_{ij}^{*}V_{jk}=d_{ik}$



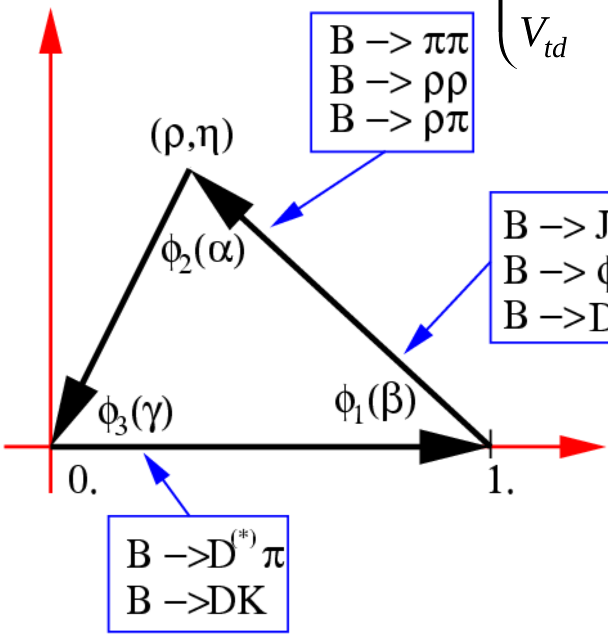
V_{ij} parameterization (Wolfenstein):

$$V=\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1-l^2/2 & l & Al^3(r-ih) \\ -l & 1-l^2/2 & Al^2 \\ Al^3(1-r-ih) & -Al^2 & 1 \end{pmatrix}$$

$\lambda = 0.2235 \pm 0.0033$ $A = 0.81 \pm 0.08$ $|\rho - i\eta| = 0.36 \pm 0.09$ $|1-\rho-i\eta| = 0.79 \pm 0.19$

Unitarity triangle

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1-l^2/2 & l & Al^3(r-ih) \\ -l & 1-l^2/2 & Al^2 \\ Al^3(1-r-ih) & -Al^2 & 1 \end{pmatrix}$$



Using unitarity requirement:

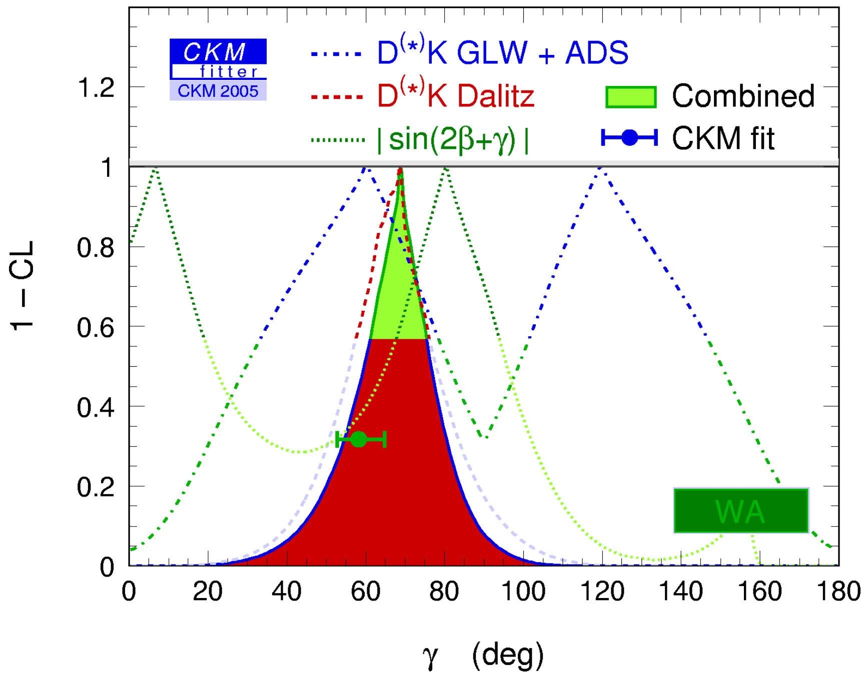
$$\{i = 1, k = 3\} : V^*_{ub}V_{ud} + V^*_{cb}V_{cd} + V^*_{tb}V_{td} = 0$$

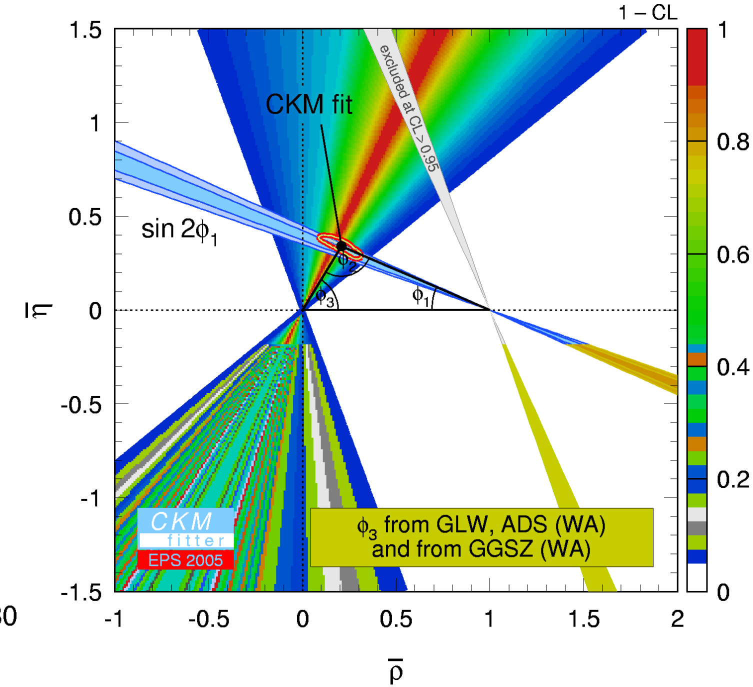
$$\Rightarrow \frac{V^*_{ub}V_{ud}}{V^*_{cb}V_{cd}} + 1 + \frac{V^*_{tb}V_{td}}{V^*_{cb}V_{cd}} = 0$$

$\sin 2f_{1}(ß)$ is measured with a good accuracy at B-factories.

Measurement of all the angles needed to test SM.

Constraints of the Unitarity Triangle

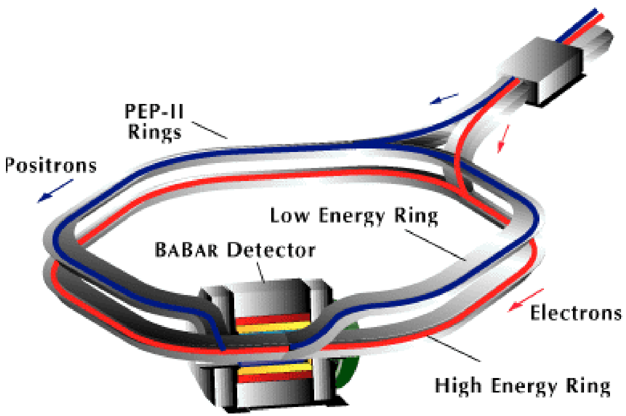


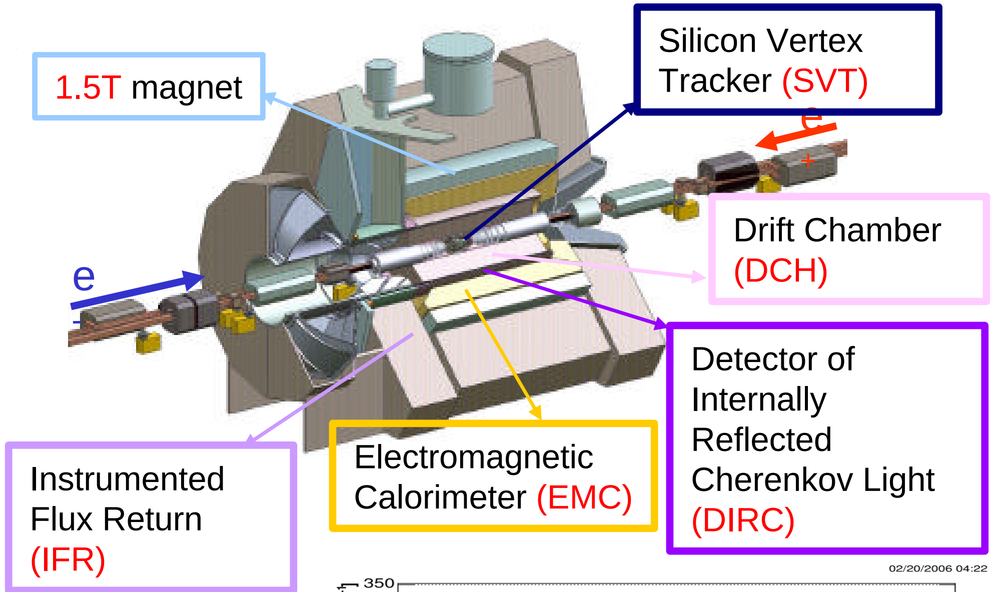


World average as of summer 2005
from GLW, ADS, Dalitz and $\sin(2f_1+f_3)$:

$?/f_3=70^{+12}_{-14}{}^\circ$

PEP-II and BaBar

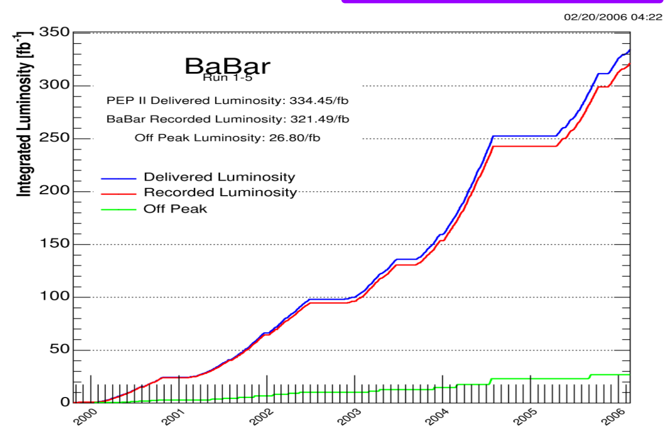




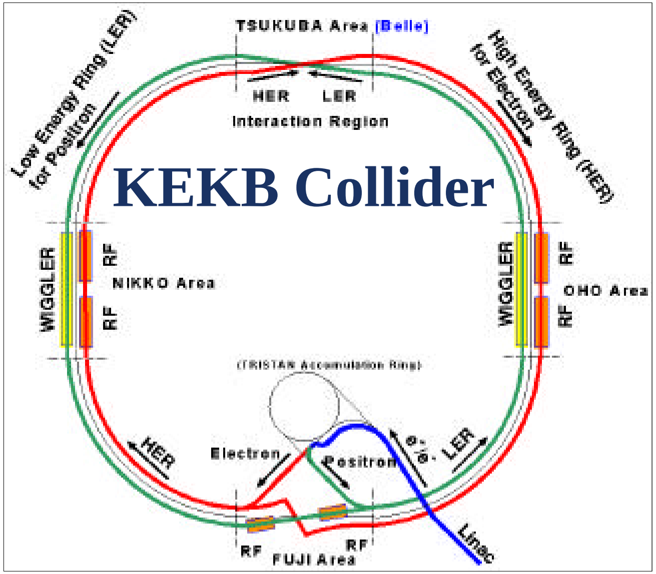
3.1 GeV e^+ & 9 GeV e^- beams

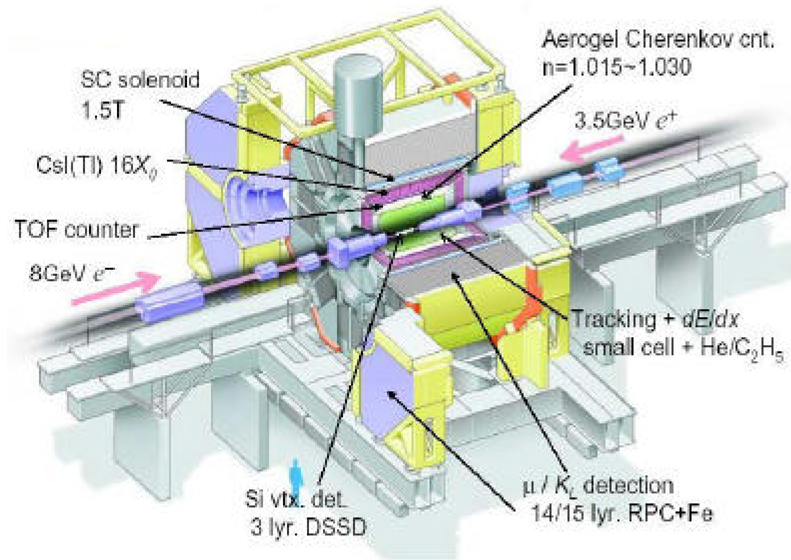
L= 1.00 x 10^{34} cm^{-2}s^{-1} (Oct 9, 2005)

$\int L\,dt$ = 335 fb^{-1} @ ? (4S)+off (~10%)

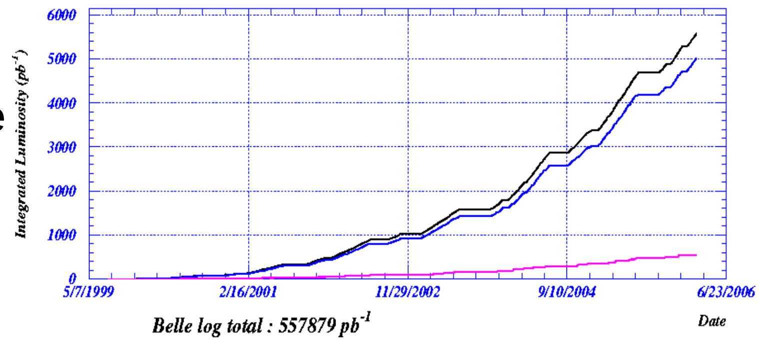


KEKB and Belle





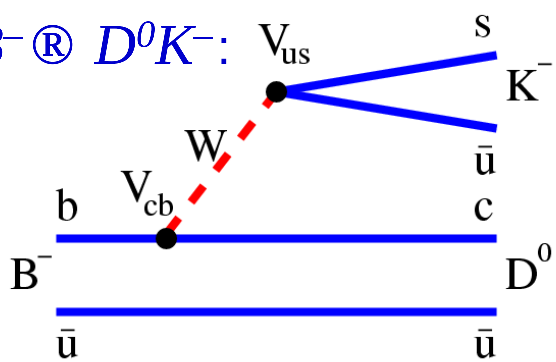
- 3.5 GeV e^+ & 8 GeV e^- beams
- 3 km circumference, 11 mrad crossing angle
- L= 1.63 x 10^{34} $cm^{-2}s^{-1}$ (world record)
- $\int L\,dt$ = 550 fb^{-1} @ ?(4S)+off(~10%)



B^+ ® D^0K^+ decay

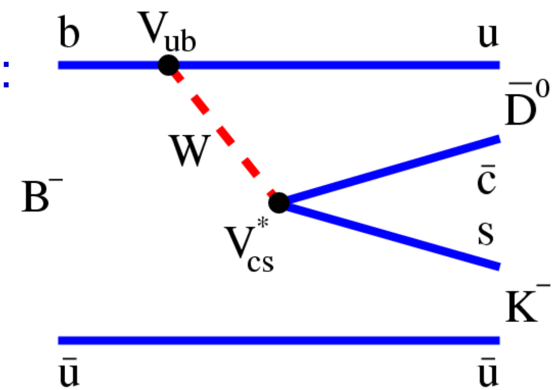
Need to use the decay where V_{ub} contribution interferes with another weak vertex.

B^- ® D^0K^-:



$$\mathbf{A}_1 \sim V_{cb}V_{us}^* \sim A\,l^3$$

B^- ® $\bar{D}^0K^-$:



$$\mathbf{A}_2 \sim V_{ub}V_{cs}^* \sim A?^3(? - ih) \sim e^{-ig}$$

If D^0 and $\bar{D}^0$ decay into the same final state, $\left|\tilde{D}^0\right\rangle = \left|D^0\right\rangle + re^{iq}\left|\bar{D}^0\right\rangle$

Relative phase: $q = -g + d$ (B^- ® DK^-), $q = +g + d$ (B^+ ® DK^+)

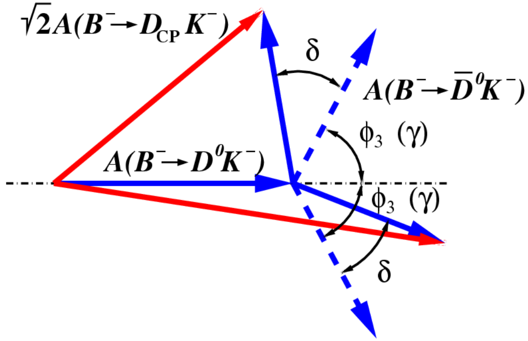
includes weak ($?/f_3$) and strong (d) phase.

Amplitude ratio:

$$r_B = \left|\mathbf{A}(B^- \to \bar{D}^0K^-)/\mathbf{A}(B^- \to D^0K^-)\right| \approx \frac{\left|V_{ub}^*V_{cs}\right|}{\left|V_{cb}^*V_{us}\right|} \times [\text{color supp}] \approx 0.1 \div 0.2$$

GLW method

M. Gronau and D. London, PLB **253**, 483 (1991);
M. Gronau and D. Wyler, PLB **265**, 172 (1991)



?? eigenstate of D-meson is used (D_{CP}).

CP-even : $D_1 \to K^+K^-, p^+p^-$

CP-odd : $D_2 \to K_S p^0, K_S ?, K_S f, K_S ?...$

??-asymmetry:

$$?_{1,2} = \frac{Br(B^- \to D_{1,2}K^-) - Br(B^+ \to D_{1,2}K^+)}{Br(B^- \to D_{1,2}K^-) + Br(B^+ \to D_{1,2}K^+)} = \frac{2r_B \sin d' \sin g}{1 + r_B^2 + 2r_B \cos d' \cos g}$$

$$d' = \begin{cases} d & \text{for } D_1 \\ d + p & \text{for } D_2 \end{cases} \quad \Rightarrow \quad A_{1,2} \text{ of different signs}$$

Additional constraint:

$$R_{1,2} = \frac{Br(B \to D_{1,2}K) / Br(B \to D_{1,2}p)}{Br(B \to D^0K) / Br(B \to D^0p)} = 1 + r_B^2 + 2r_B \cos d' \cos g$$

4 equations (3 independent: $A_1R_1 = -A_2R_2$), 3 unknowns (r_B, d, g)

GLW method (BaBar)

BaBar results (211 fb^{-1})

hep-ex/0512067,
hep-ex/0507002, PRD **72**, 071103

$B^{\pm}$ ® $DK^{\pm}$ decay:

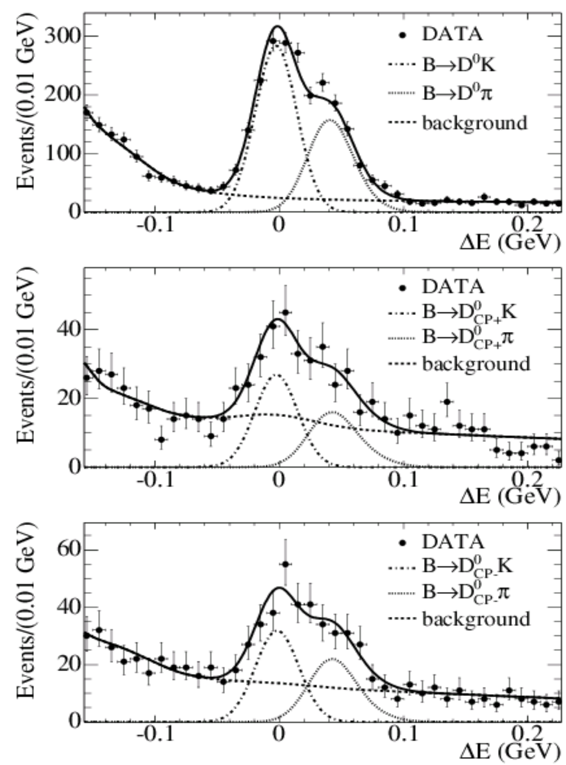
	R	*A*
B ® $D_{CP+}K$	0.90±0.12±0.04	0.35±0.13±0.04
B ® $D_{CP-}K$	0.86±0.10±0.05	-0.06±0.13±0.03

r_B^2=-0.12±0.08(stat)±0.03(syst)

$B^{\pm}$ ® $DK^{*\pm}$, $K^{*\pm}$ ® $K_S p^{\pm}$ decay:

	R	*A*
B ® $D_{CP+}K^*$	1.96±0.40±0.11	-0.08±0.19±0.08
B ® $D_{CP-}K^*$	0.65±0.26±0.08	-0.26±0.40±0.12

r_B^2=0.30±0.25



See talk by Marcello Rotondo
"V_{ub} and ? at BaBar"

GLW method (Belle)

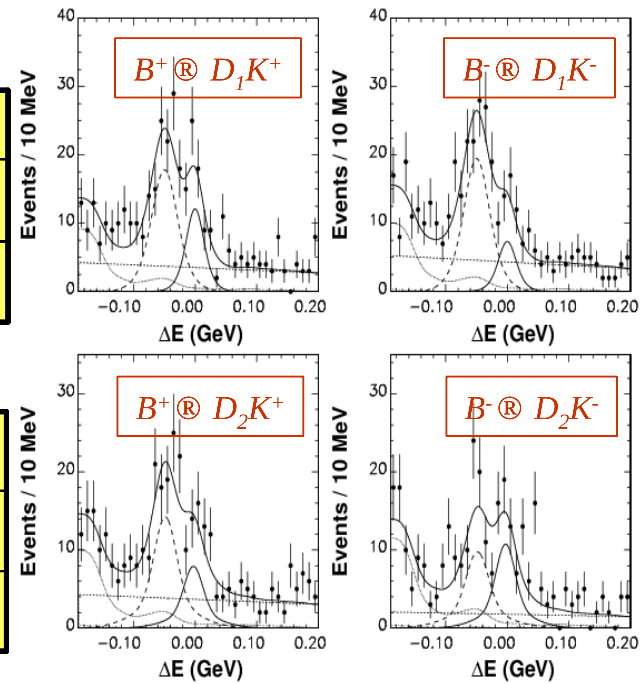
Belle results (253 fb^{-1})   hep-ex/0601032

$B^{\pm}$ ® $DK^{\pm}$ decay:

	R	*A*
B® D_1K	1.13 ±0.16±0.05	0.06±0.14±0.05
B® D_2K	1.17±0.14±0.14	-0.12±0.14±0.05

$B^{\pm}$ ® $D^*K^{\pm}$, D^*®Dp^0 decay:

	R	*A*
B® D^*_1K	1.41±0.25±0.06	-0.20±0.22±0.04
B® D^*_2K	1.15±0.31±0.12	0.13±0.30±0.08



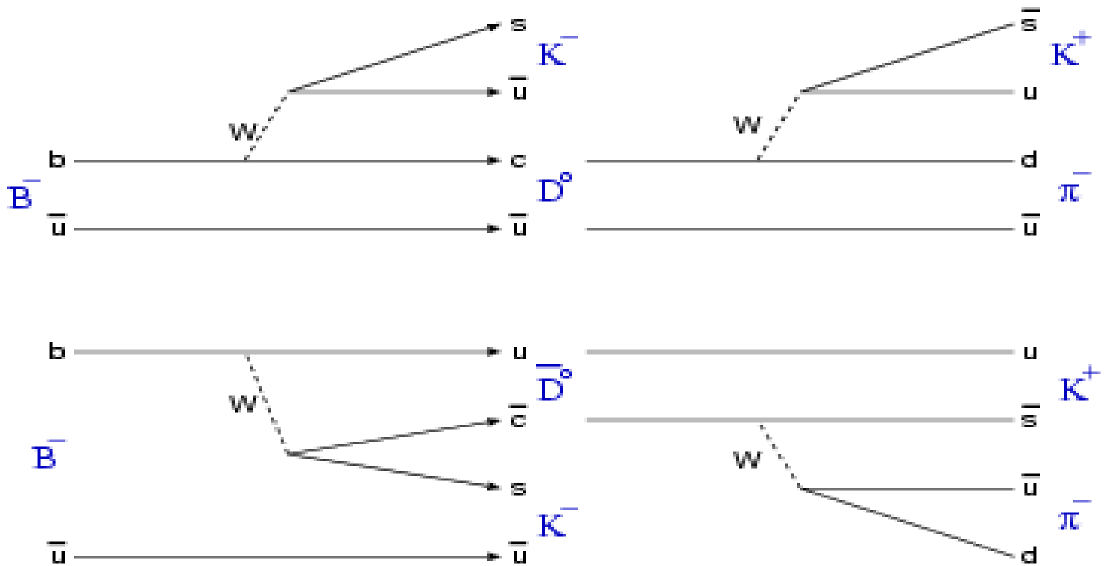
GLW analyses alone do not constrain ?/f $_3$ significantly yet, but

- can be combined with other measurements
- provide information on r_B

ADS method

D. Atwood, I. Dunietz and A. Soni, PRL **78**, 3357 (1997);
PRD **63**, 036005 (2001)

Enhancement of ??-violation due to use of Cabibbo-suppressed *D* decays



B^-®D^0K^- - color allowed

D^0®K^+p^- - doubly Cabibbo-suppressed

B^-®$\overline{D}^0K^-$ - color suppressed

$\overline{D}^0$®K^+p^- - Cabibbo-allowed

$\Longrightarrow$ Interfering amplitudes are comparable

ADS method (Belle)

Belle results (357 fb^{-1}) hep-ex/0508048

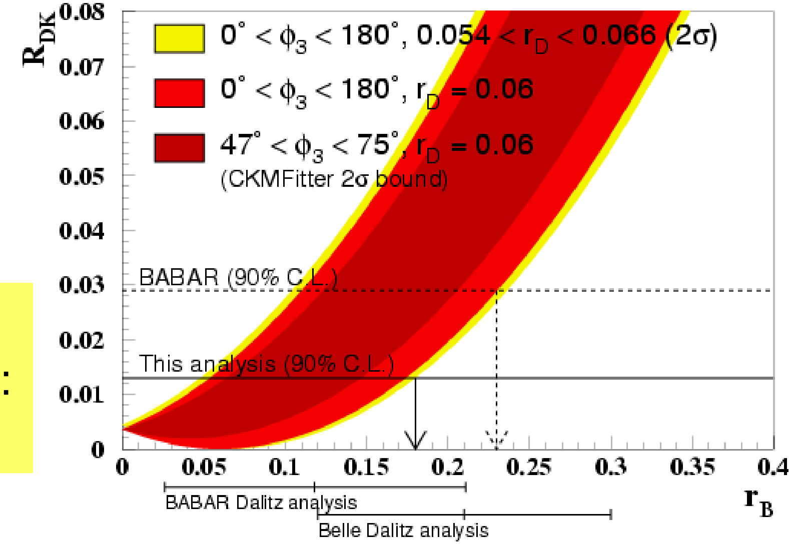
$$R_{DK} = \frac{Br(B \to D_{\text{supp}} K)}{Br(B \to D_{\text{fav}} K)} = r_B^2 + r_D^2 + 2 r_B r_D \cos j_3 \cos d$$

$$r_D = \left| \mathbf{A}(D^0 \to K^+ p^-) / \mathbf{A}(D^0 \to K^- p^+) \right|$$

Suppressed channel not visible yet:

$$R_{DK} = (0.0^{+8.4}_{-7.9} \pm 1.0) \times 10^{-3}$$

Using r_D=0.060±0.003,
for maximum mixing (f_3=0, d=180°):
r_B<0.18 (90% CL)



ADS method (BaBar)

BaBar results (211 fb^{-1}) hep-ex/0504047, PRD **72**, 032004

Suppressed channel not visible either:

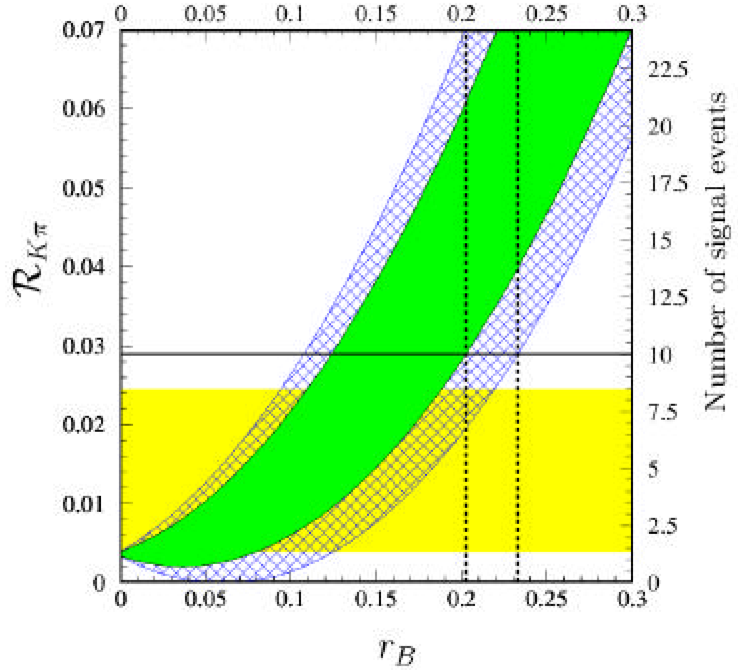
$$R_{DK} = 13^{+11}_{-9} \times 10^{-3}$$

$$R_{D^*[D\pi^0]K} = -2^{+10}_{-6} \times 10^{-3}$$

$$R_{D^*[D\gamma]K} = 11^{+18}_{-13} \times 10^{-3}$$

$r_B < 0.23$ (90% CL) for $B \to DK$

$r_B < 0.16$ for $B \to D^*K$



Like in GLW analyses, no significant constraint on $?/\phi_3$, but upper limit on r_B

Dalitz analysis method

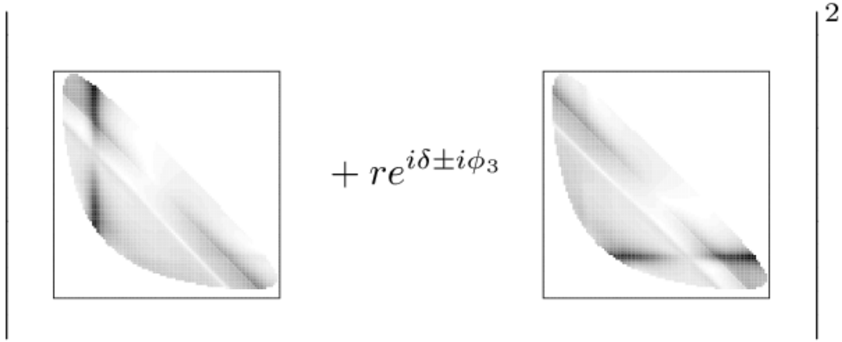
A. Giri, Yu. Grossman, A. Soffer, J. Zupan, PRD **68**, 054018 (2003)
A. Bondar, Proc. of Belle Dalitz analysis meeting, 24-26 Sep 2002.

$$\left|\tilde{D}^0\right\rangle = \left|D^0\right\rangle + re^{iq}\left|\overline{D}^0\right\rangle$$

Using 3-body final state, identical for D^0 and $\bar{D}^0$: $K_s p^+ p^-$.

Dalitz distribution density: $ds\,(m^2_{K_s p^+}, m^2_{K_s p^-}) \propto |\mathbf{A}|^2\, dm^2_{K_s p^+}\, dm^2_{K_s p^-}$

$$|\mathbf{A}(m^2_{K_s p^+}, m^2_{K_s p^-})|^2 =$$



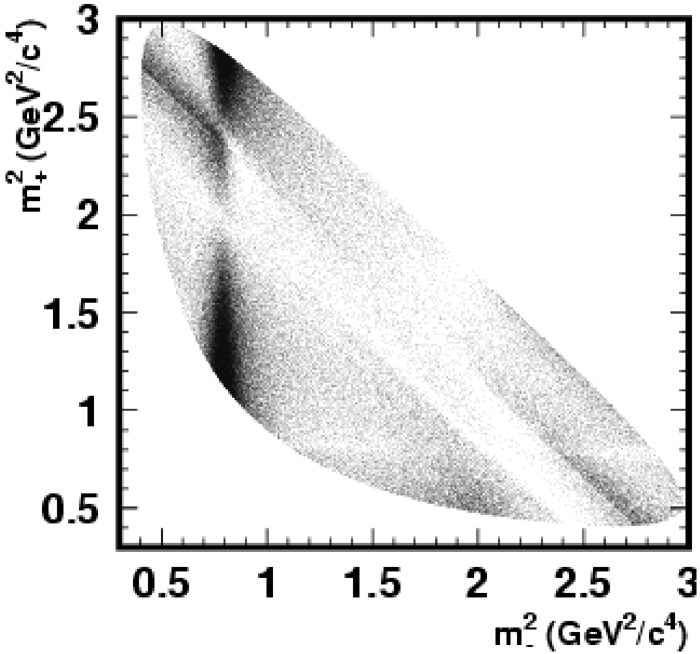
(assuming ??-conservation in D^0 decays)

If $f(m^2_{K_s p^+}, m^2_{K_s p^-})$ is known, parameters (r_B, d, g) are obtained from the fit to Dalitz distributions of D ® $K_s p^+ p^-$ from $B^\pm$ ® $DK^\pm$ decays

Dalitz analysis: D^0 ® $K_s p^+ p^-$ decay

Statistical sensitivity of the method depends on the properties of the 3-body decay involved.

(For $|M|^2=Const$ there is no sensitivity to the phase $?$)
Large variations of D^0 decay strong phase are essential

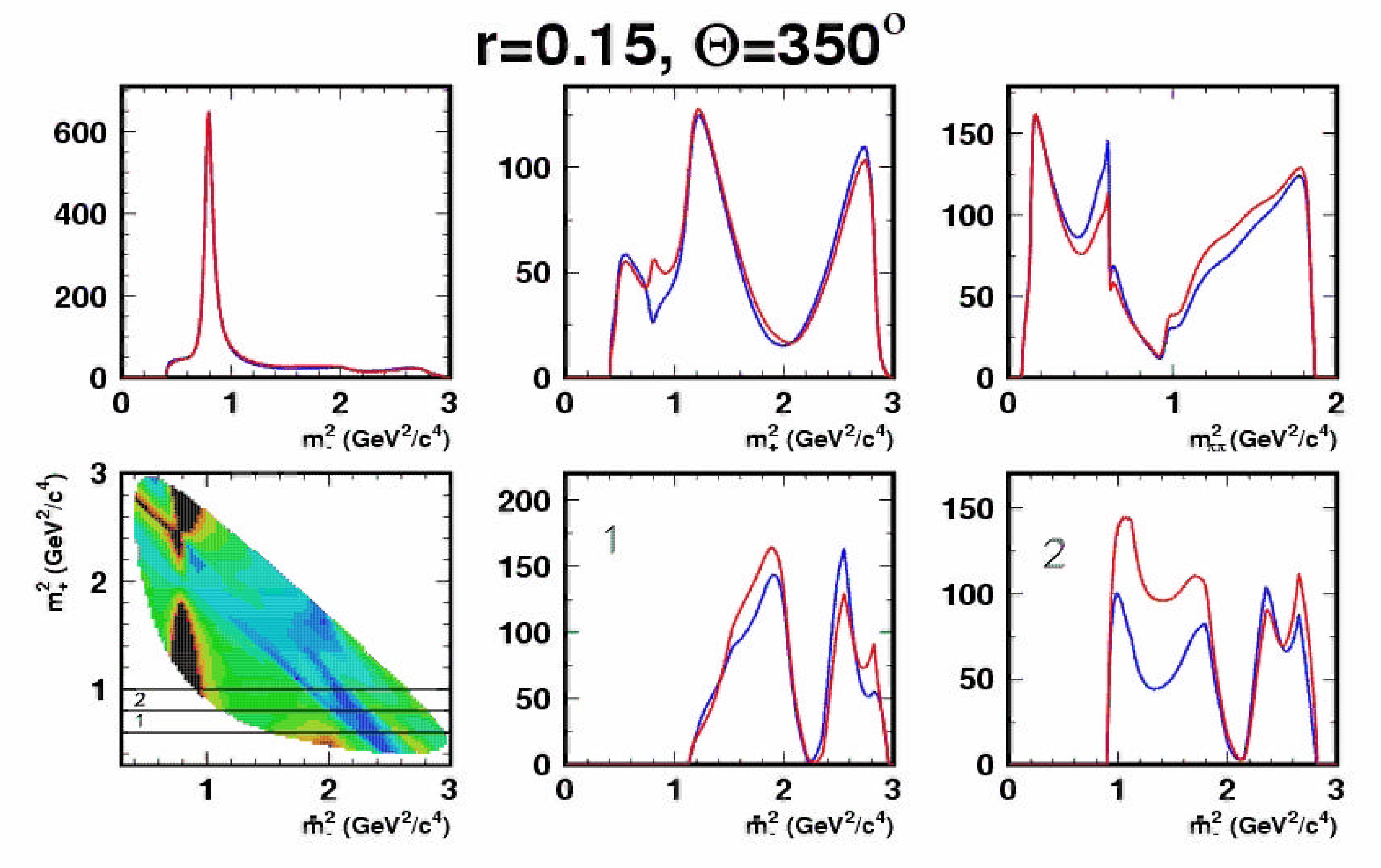


Use the model-dependent fit to experimental data from flavor-tagged D^* ®D^0p sample.

Model is described by the set of two-body amplitudes
+ flat nonresonant term.

As a result, model uncertainty in the ?/f $_3$ measurement.

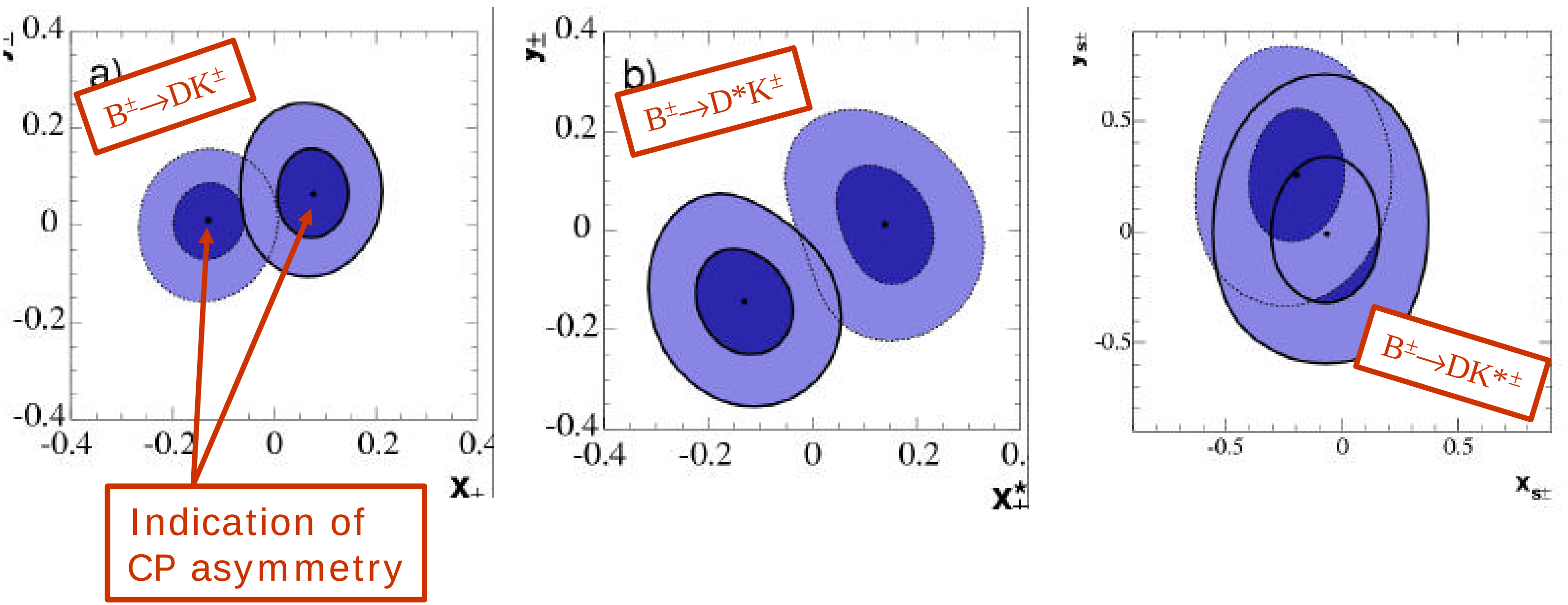
Dalitz analysis: sensitivity to the phase



Dalitz analysis (BaBar)

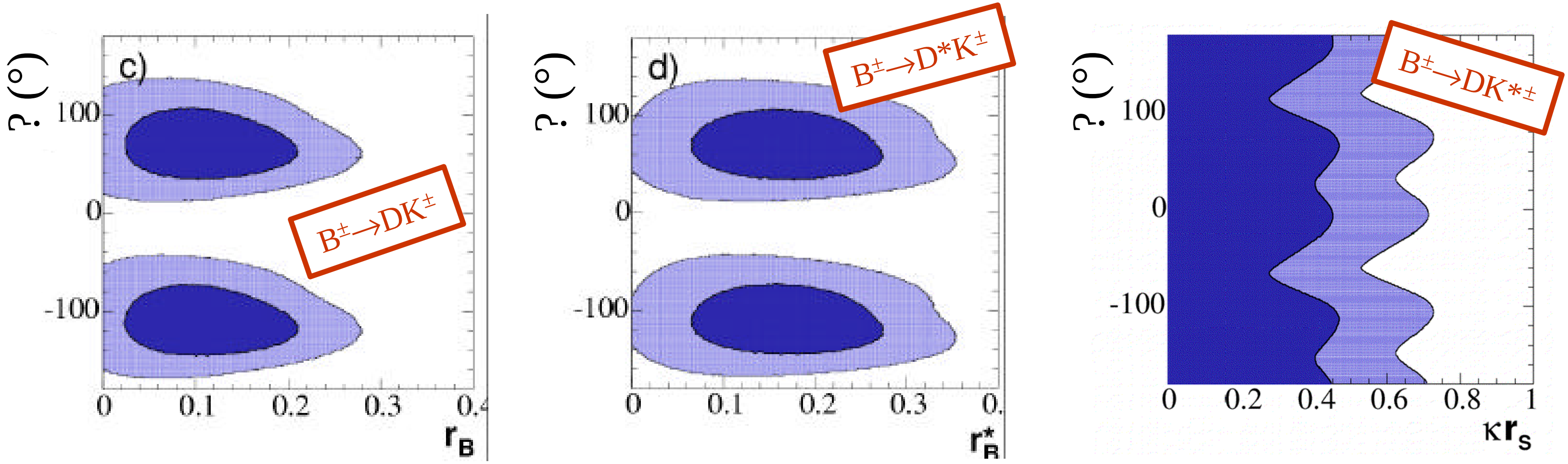
BaBar results (205 fb^{-1})

hep-ex/0504039, PRL **95**, 121802
hep-ex/0507101



Fit parameters are $x_\pm = r\cos(\pm?+d)$ and $y_\pm = r\sin(\pm?+d)$
(better behaved statistically than r, d, g)
r, d, g are obtained from frequentist statistical treatment based on PDFs from toy MC simulation.

Dalitz analysis (BaBar)



Model uncertainty from *pp* s-wave estimated with K-matrix formalism: 3°.

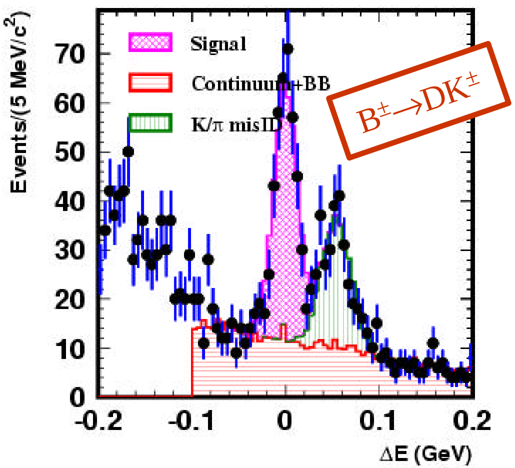
Nonresonant contribution in $B^{\pm}$ ® $DK^{*\pm}$ is treated by introducing additional free parameter 0<?<1 accounting for $B^{\pm}$ ® $DK_S p^{\pm}$ contribution.

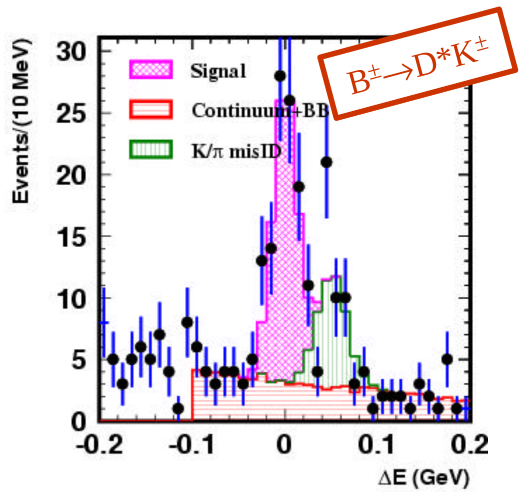
Combined for 3 modes: ?=67°±28°±13° (syst)±11° (model)

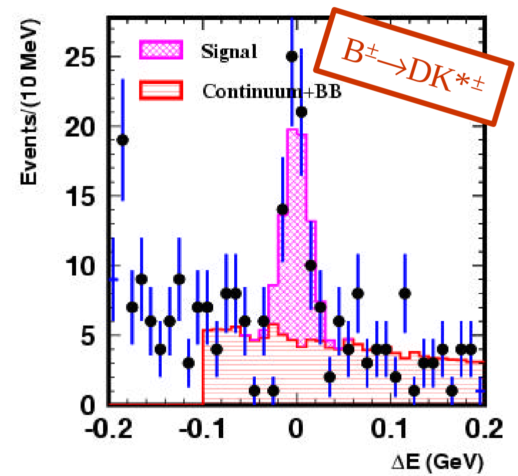
Dalitz analysis (Belle)

Belle result (357 fb^{-1})

New! Preliminary!



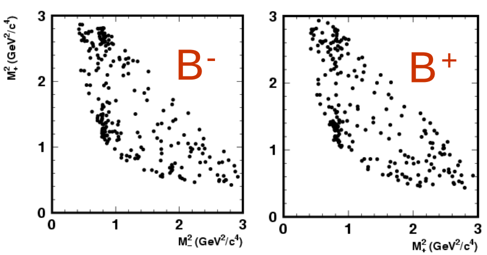


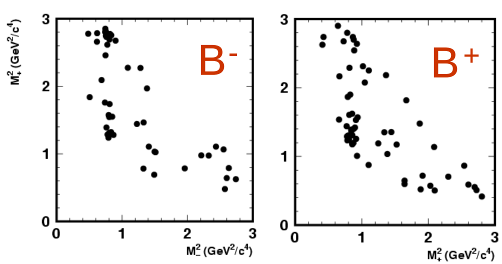


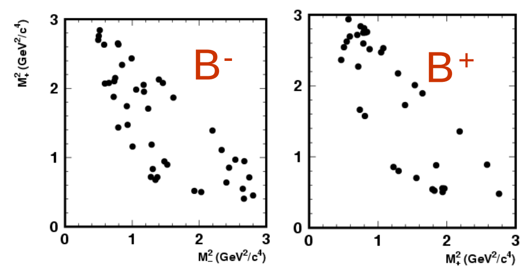
331±17 events

81±8 events

54±8 events



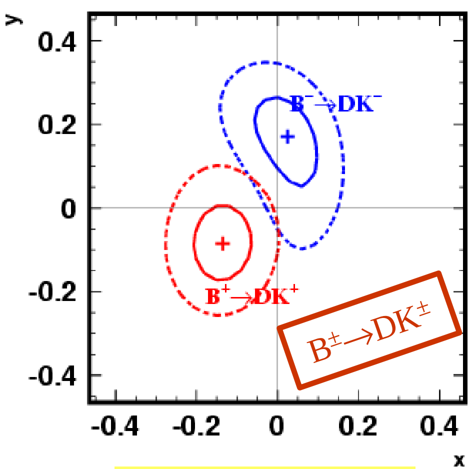




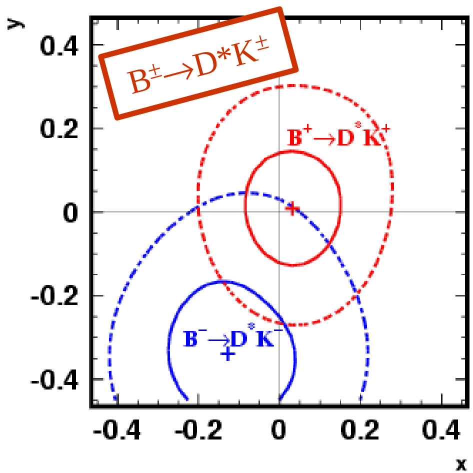
Dalitz analysis (Belle)

Fit parameters are $x_{\pm} = r\cos(\pm f_3 + d)$ and $y_{\pm} = r\sin(\pm f_3 + d)$
(better behaved statistically than r, d, j_3)
r, d, j_3 are obtained from frequentist statistical treatment based on PDFs from toy MC simulation.

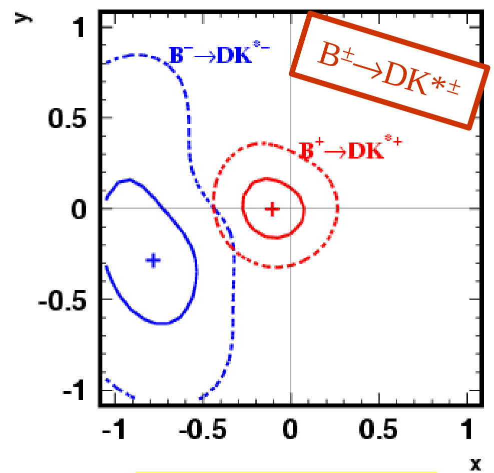
Similar to BaBar, easier to combine results



$x_- = \mathbf{0.025}^{+0.072}_{-0.080}$
$y_- = \mathbf{0.170}^{+0.093}_{-0.117}$
$x_+ = -\mathbf{0.135}^{+0.069}_{-0.070}$
$y_+ = -\mathbf{0.085}^{+0.090}_{-0.086}$

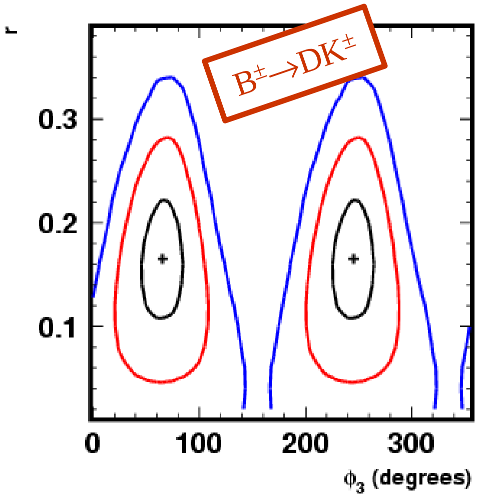


$x_- = -\mathbf{0.128}^{+0.167}_{-0.146}$
$y_- = -\mathbf{0.339}^{+0.172}_{-0.158}$
$x_+ = \mathbf{0.032}^{+0.120}_{-0.116}$
$y_+ = \mathbf{0.008}^{+0.137}_{-0.136}$

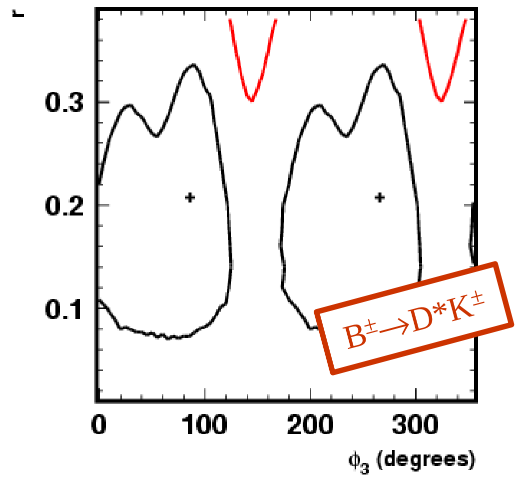


$x_- = -\mathbf{0.784}^{+0.249}_{-0.295}$
$y_- = -\mathbf{0.281}^{+0.440}_{-0.335}$
$x_+ = -\mathbf{0.105}^{+0.177}_{-0.167}$
$y_+ = -\mathbf{0.004}^{+0.164}_{-0.156}$

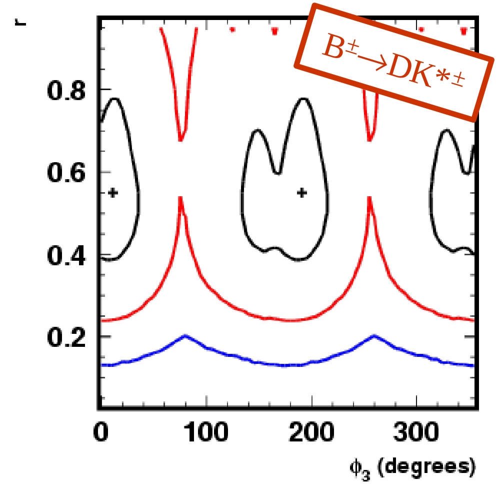
Dalitz analysis (Belle)



$f_3 = 66^{+19}_{-20}\,°$(stat)



$f_3 = 86^{+37}_{-93}\,°$(stat)



$f_3 = 11^{+23}_{-57}\,°$(stat)

Combined for 3 modes: $f_3 = 53°\,^{+15}_{-18} \pm 3°$ (syst) $\pm 9°$ (model)

$8° < f_3 < 111°$ (2s interval)

$r_{DK} = 0.159^{+0.054}_{-0.050} \pm 0.012$(syst)$\pm 0.049$(model)

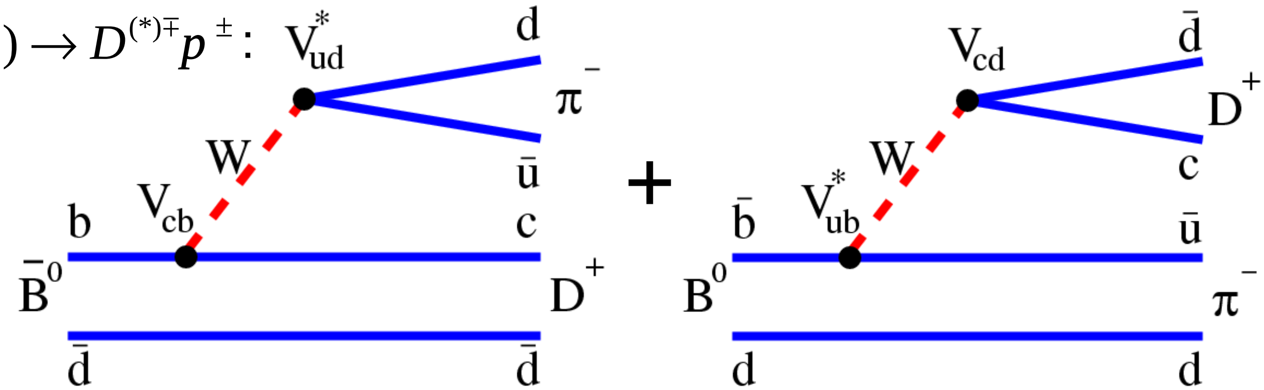
CPV significance: 78%

$r_{D^*K} = 0.175^{+0.108}_{-0.099} \pm 0.013$(syst)$\pm 0.049$(model)

$r_{DK^*} = 0.564^{+0.216}_{-0.155} \pm 0.041$(syst)$\pm 0.084$(model)

$\sin(2f_1+f_3)$ from B^0 ® D^*p decay

Decay $B^0(\overline{B}^0) \to D^{(*)\mp} p^{\pm}$:



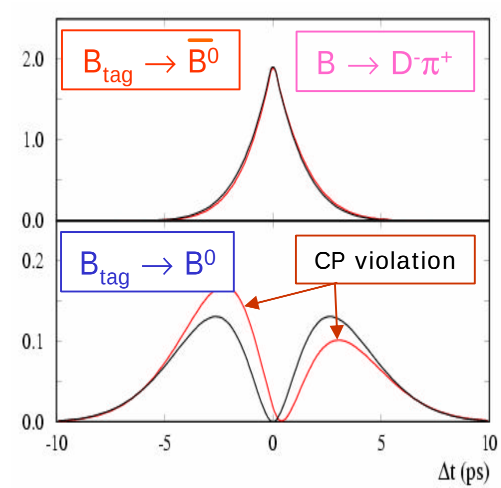
Use B flavor tag, measure time-dependent decay rates:

$$P(B^0 \to D^{(*)\pm} p^{\mp}) = \frac{1}{8t_B} e^{-|\Delta t|/t_B} \left[1 \mp C\cos(\Delta m \Delta t) - S^{\pm}\sin(\Delta m \Delta t)\right]$$

$$P(\overline{B}^0 \to D^{(*)\pm} p^{\mp}) = \frac{1}{8t_B} e^{-|\Delta t|/t_B} \left[1 \pm C\cos(\Delta m \Delta t) + S^{\pm}\sin(\Delta m \Delta t)\right]$$

where $$S^{\pm} = \frac{2R}{1+R^2}(-1)^L \sin(2j_1 + j_3 \pm d),$$

$$C = \frac{1-R^2}{1+R^2} \approx 1 \qquad R \approx 0.02$$



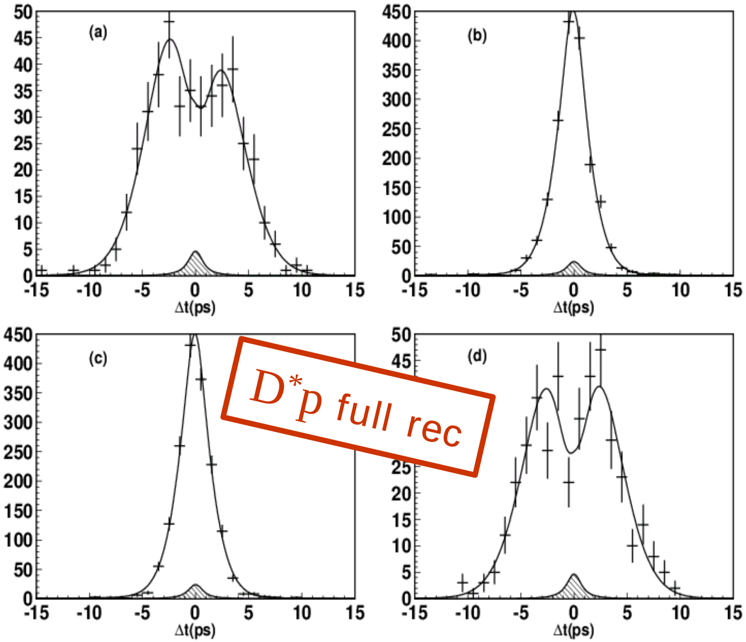
$\sin(2f_1+f_3)$ (Belle)

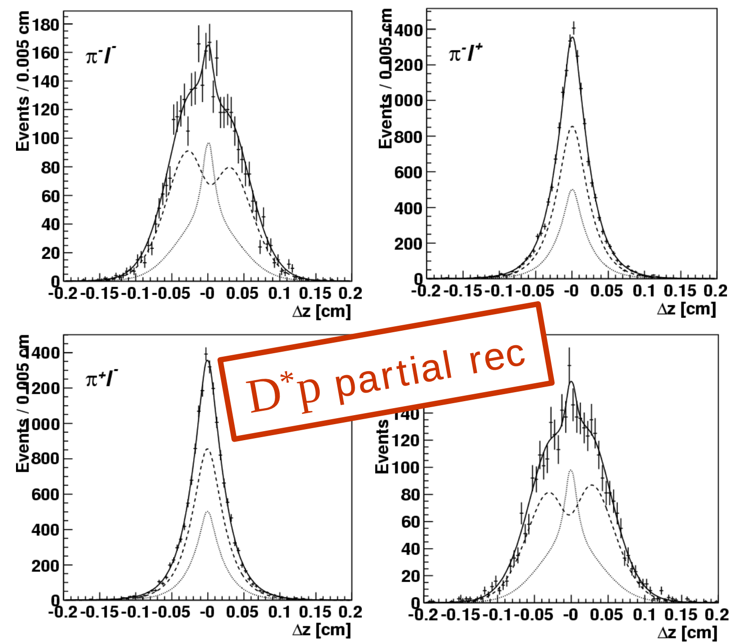
Belle result (357 fb^{-1})

New! Preliminary!

$B^0(\overline{B}^0) \to D^{(*)\mp}p^{\pm}$ - full reconstruction

$B^0(\overline{B}^0) \to D^{*\mp}p^{\pm}$ with $D^{*\mp} \to D^0 p^{\pm}$ - partial reconstruction (reconstruct only pions)





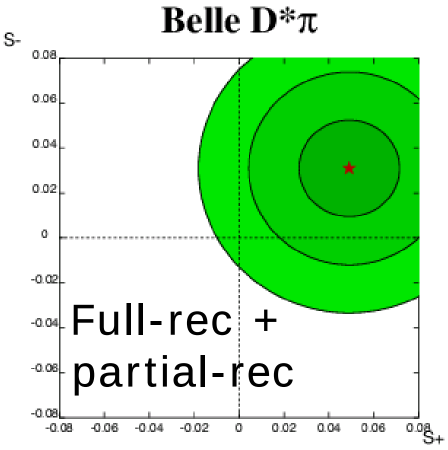
$\sin(2f_1+f_3)$ (Belle)

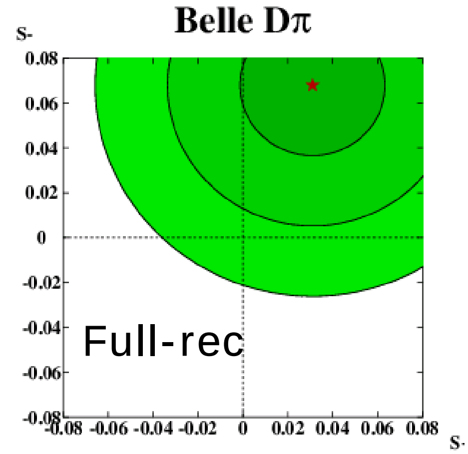
$S^+(D^*p)=0.049\pm0.020\pm0.011$

$S^-(D^*p)=0.031\pm0.019\pm0.011$

$S^+(Dp)=0.031\pm0.030\pm0.012$

$S^-(Dp)=0.068\pm0.029\pm0.012$





CP violation significance: 2.5s

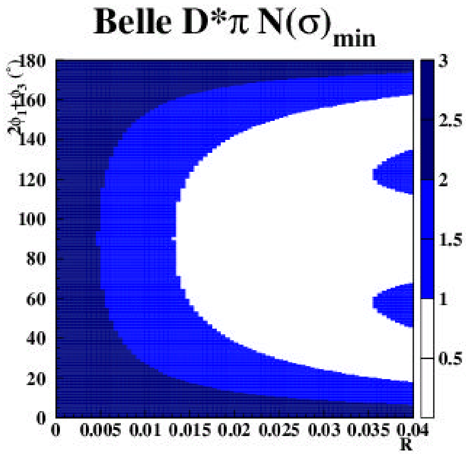
If $R\sim0.02$:

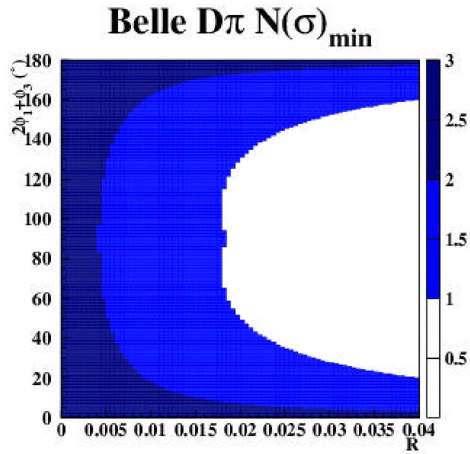
$B^0(\overline{B}^0)\rightarrow D^{*\mp}p^{\pm}$

$|\sin(2f_1+f_3)|>0.46$ (0.13)

$B^0(\overline{B}^0)\rightarrow D^{\mp}p^{\pm}$

$|\sin(2f_1+f_3)|>0.48$ (0.07)
at 68% (95%) CL





sin(2ß+?) (BaBar)

BaBar result (211 fb^{-1})

hep-ex/0602049, EPS-2005

$S^{\pm} = a_f \pm c_f$, where

$$a_f = 2r_f \sin(2b + g)\cos d_f$$

$$c_{f,lep} = 2r_f \cos(2b + g)\sin d_f$$

Full reconstruction:

$a^{Dp} = -0.013 \pm 0.022(\text{stat}) \pm 0.007(\text{syst})$

$a^{D^*p} = -0.043 \pm 0.023(\text{stat}) \pm 0.010(\text{syst})$

$a^{Dr} = -0.024 \pm 0.031(\text{stat}) \pm 0.010(\text{syst})$

$c_{lep}^{Dp} = -0.043 \pm 0.042(\text{stat}) \pm 0.011(\text{syst})$

$c_{lep}^{D^*p} = 0.047 \pm 0.042(\text{stat}) \pm 0.015(\text{syst})$

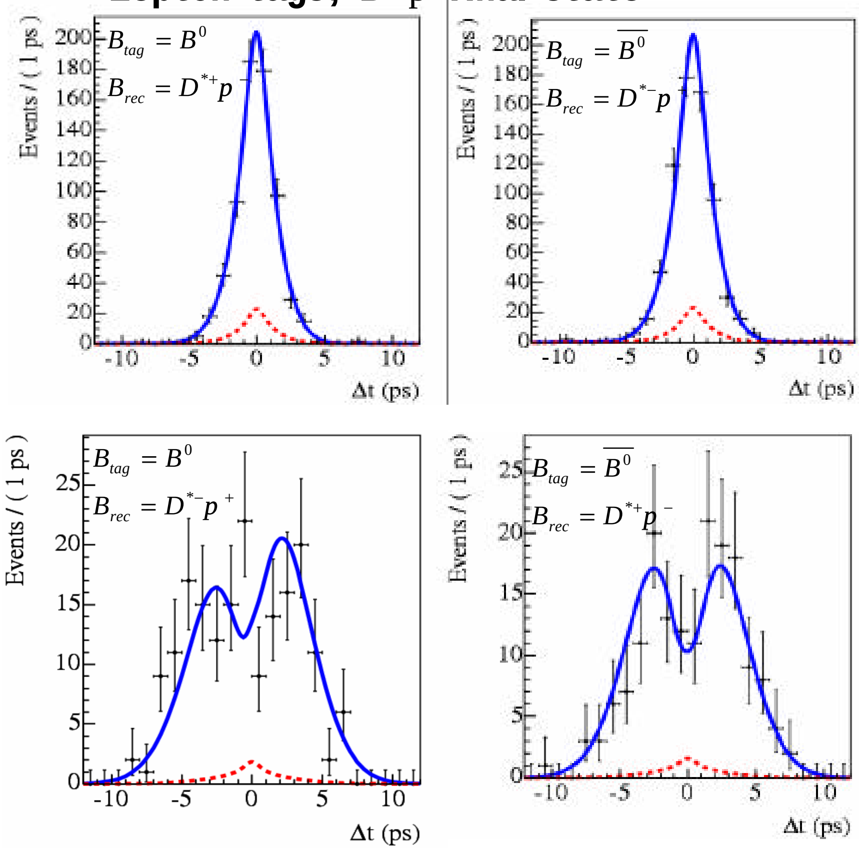
$c_{lep}^{Dr} = -0.098 \pm 0.055(\text{stat}) \pm 0.019(\text{syst})$

Partial reconstruction:

$a^{D^*p} = -0.034 \pm 0.014(\text{stat}) \pm 0.009(\text{syst})$

$c_{lep}^{D^*p} = -0.025 \pm 0.020(\text{stat}) \pm 0.013(\text{syst})$

Lepton tags, D*p final state



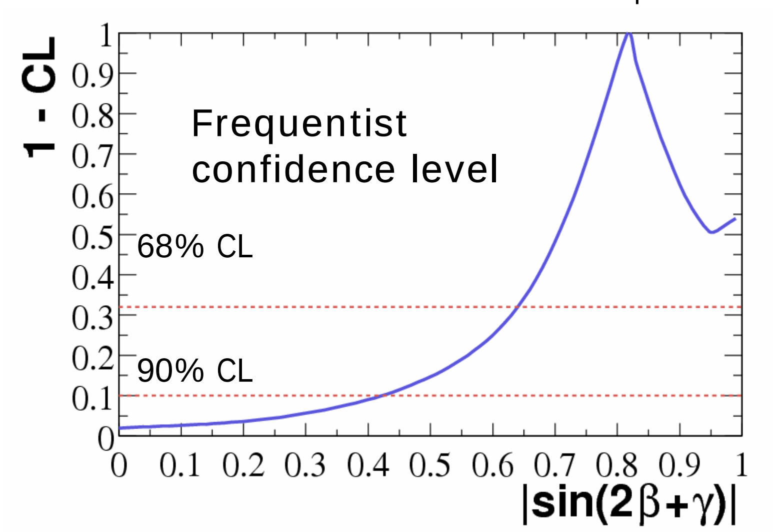
sin(2ß+?) (BaBar)

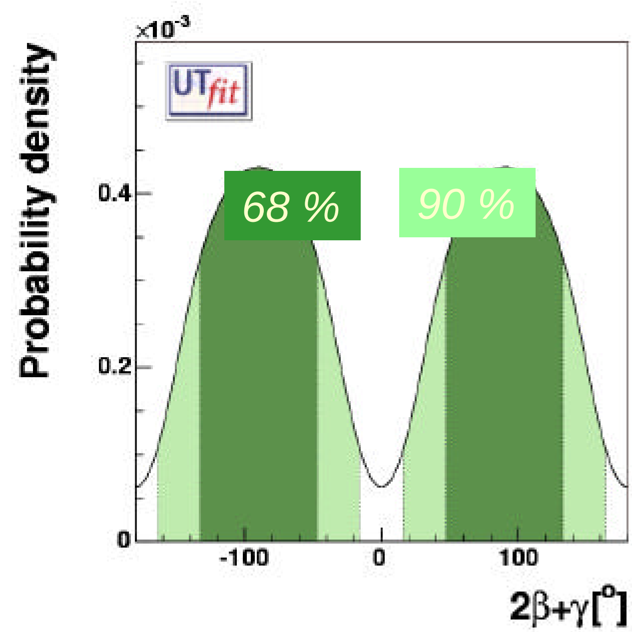
Combine partial and full reconstruction results for the a and c_{lep} parameters and use the r_f value from SU(3) symmetry

$|\sin(2\beta+\gamma)| > 0.64$ @ 68 % C.L.
$|\sin(2\beta+\gamma)| > 0.42$ @ 90 % C.L.

$|2\beta+\gamma| = 90^o \pm 43^o$

30% theoretical error on r_f





See talk by Marcello Rotondo later today for more details

Summary

The angle ?/f $_3$ remains the most difficult angle of the Unitarity Triangle to measure, although B-factories are working hard.

Many new analyses appeared since summer conferences:

- GLW method (Belle and BaBar)
- Dalitz analysis (Belle)
- $\sin(2f_1+f_3)$ (Belle)
- See talk by Marcello Rotondo for more hot BaBar results

Good perspectives with higher statistics since the theoretical uncertainties are very low.

Backup

ADS method

$$Br(B^{\pm} \rightarrow D_{\text{supp}} K^{\pm}) = [r_B^2 + r_D^2 + 2 r_B r_D \cos(\pm\phi_3 + \delta)] |\mathbf{A}_B|^2 |\mathbf{A}_D|^2$$

$$r_B = \left| \mathbf{A}(B^- \rightarrow \overline{D}^0 K^-) / \mathbf{A}(B^- \rightarrow D^0 K^-) \right|$$

$$r_D = \left| \mathbf{A}(D^0 \rightarrow K^+ \pi^-) / \mathbf{A}(D^0 \rightarrow K^- \pi^+) \right|$$

r_D, $|\mathbf{A}_D|^2$ – determined from D decay analysis

$|\mathbf{A}_B|^2$ – from $B^{\pm} \rightarrow D_{\text{flavor}} K^{\pm}$

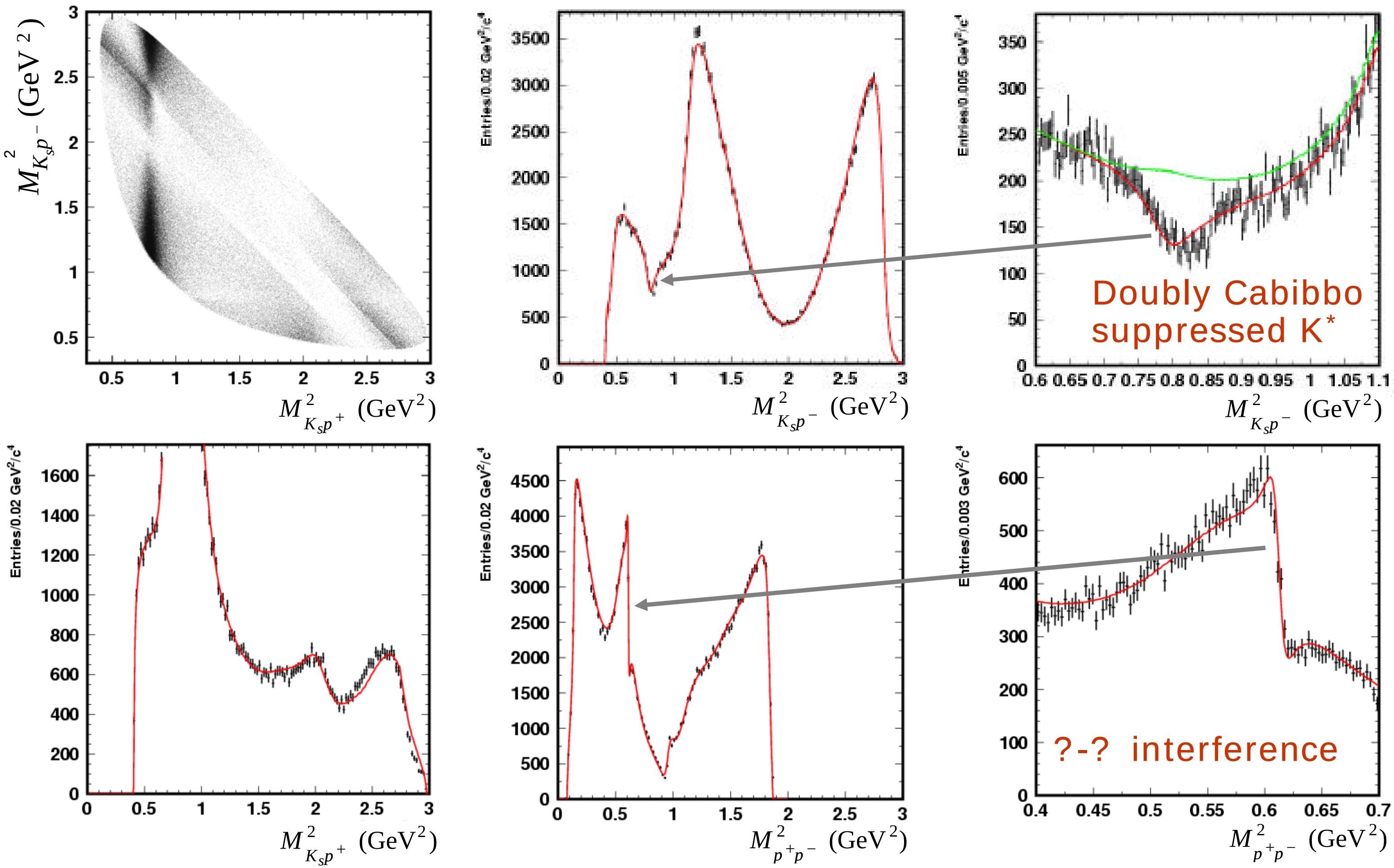
Using single D decay channel:

2 equations, 3 unknowns (r, δ, ϕ_3)

With one more channel added:

4 equations, 4 unknowns $(r, \delta_1, \delta_2, \phi_3)$

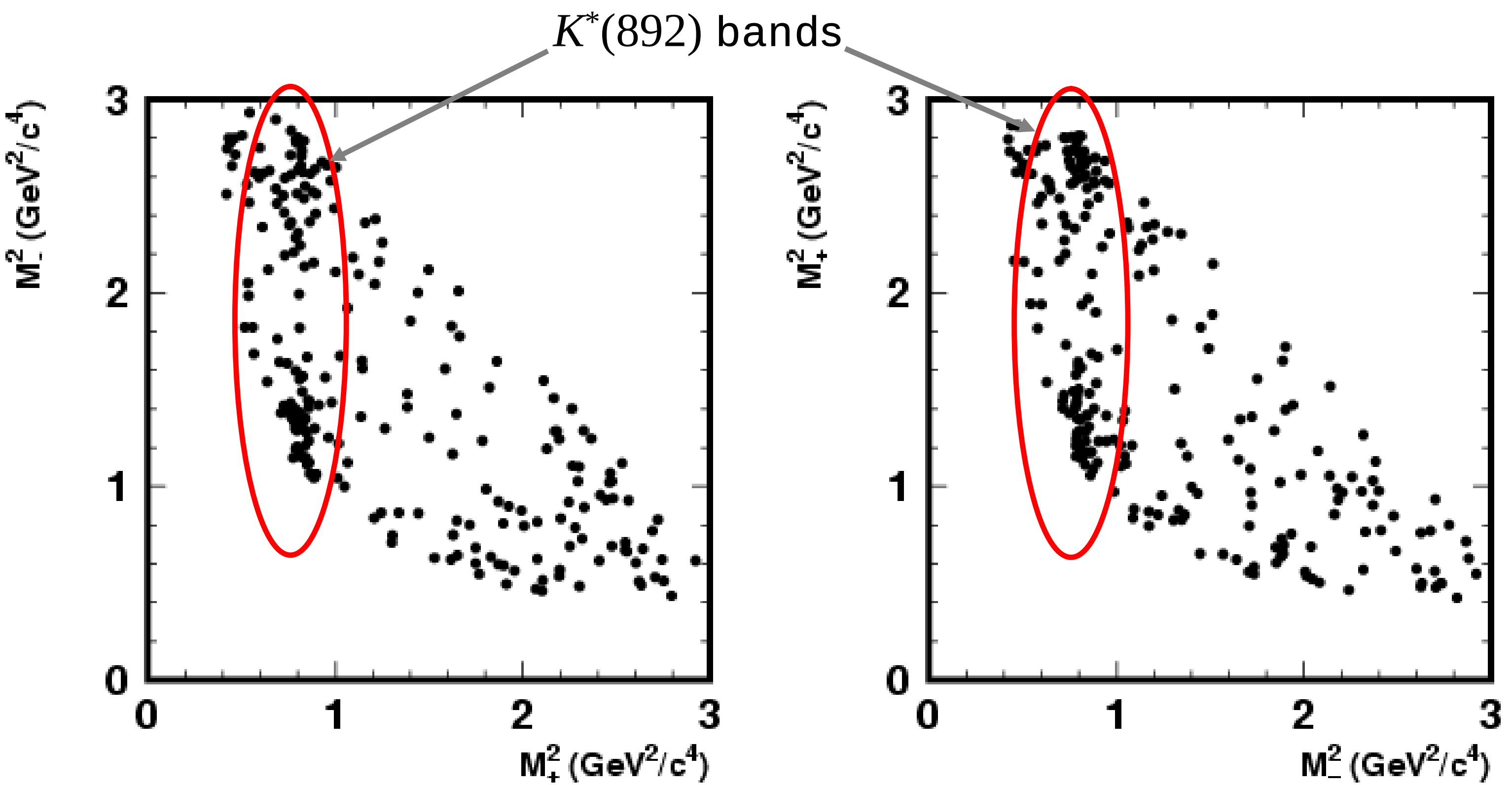
D^0 ® $K_s p^+ p^-$ decay model



D^0 ® $K_s p^+ p^-$ decay model

Intermediate state	Amplitude	Phase, °	Fit fraction
K_S s $_1$ (M=520±15 MeV, G=466±31 MeV)	1.43±0.07	212±4	9.8%
K_S ?(770)	1 (fixed)	0 (fixed)	21.6%
K_S ?	0.0314±0.0008	110.8±1.6	0.4%
K_S f_0(980)	0.365±0.006	201.9±1.9	4.9%
K_S s $_2$ (M=1059±6 MeV, G=59±10 MeV)	0.23±0.02	237±11	0.6%
K_S f_2(1270)	1.32±0.04	348±2	1.5%
K_S f_0(1370)	1.44±0.10	82±6	1.1%
K_S ?(1450)	0.66±0.07	9±8	0.4%
$K^*(892)^+p^-$	1.644±0.010	132.1±0.5	61.2%
$K^*(892)^-p^+$	0.144±0.004	320.3±1.5	0.55%
$K^*(1410)^+p^-$	0.61±0.06	113±4	0.05%
$K^*(1410)^-p^+$	0.45±0.04	254±5	0.14%
$K^*_0(1430)^+p^-$	2.15±0.04	353.6±1.2	7.4%
$K^*_0(1430)^-p^+$	0.47±0.04	88±4	0.43%
$K^*_2(1430)^+p^-$	0.88±0.03	318.7±1.9	2.2%
$K^*_2(1430)^-p^+$	0.25±0.02	265±6	0.09%
$K^*(1680)^+p^-$	1.39±0.27	103±12	0.36%
$K^*(1680)^-p^+$	1.2±0.2	118±11	0.11%
Nonresonant	3.0±0.3	164±5	9.7%

$B^{\pm}$ ® $DK^{\pm}$, D ® $K_S p^+ p^-$ Dalitz plots (Belle)



D^0 from B^+ ® D^0K^+

D^0 from B^- ® D^0K^-
(p^+ and p^- interchanged)

Model-independent approach

D^0 decay amplitude: $f = |f(m_+^2, m_-^2)|\, e^{if(m_+^2, m_-^2)}$

D^0-$\bar{D}^0$ interference from B^+ ® D^0K^+:

$$A_{\tilde{D}^0} = |f(m_+^2, m_-^2)|\, e^{if(m_+^2, m_-^2)} + re^{iq}\, |f(m_-^2, m_+^2)|\, e^{if(m_-^2, m_+^2)}$$

$$= |f(m_+^2, m_-^2)| + re^{iq}\, |f(m_-^2, m_+^2)|\, e^{i[f(m_+^2, m_-^2) - f(m_-^2, m_+^2)]}$$

$|f|$ is measured directly, $f(m_+^2, m_-^2) - f(m_-^2, m_+^2)$ is model-dependent

If CP-tagged D^0 are available (e.g. from ?'' ® $D^0\bar{D}^0$, where tag-side D^0 decays into CP-eigenstate) phase difference can be measured:

$$A_{CP} = \frac{|f(m_+^2, m_-^2)|\, e^{if(m_+^2, m_-^2)} \pm |f(m_-^2, m_+^2)|\, e^{if(m_-^2, m_+^2)}}{\sqrt{2}}$$

$$= \frac{|f(m_+^2, m_-^2)| \pm |f(m_-^2, m_+^2)|\, e^{i[f(m_-^2, m_+^2) - f(m_+^2, m_-^2)]}}{\sqrt{2}}$$

Model-independent approach

hep-ph/0510246

50 ab^{-1} at SuperB factory should be enough for model-independent ?/f $_3$ measurement with accuracy below 2°

~10 fb^{-1} at ?(3770) needed to accompany this measurement.