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# *Neutrinoless double $\beta$ -decay*

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Model independent evidence of neutrino masses  
and mixing

## I. S-K ATMOSPHERIC NEUTRINO EVIDENCE

Zenith angle dependence

If there are no oscillations

$$N(\cos \theta_z) = N(-\cos \theta_z)$$

Deficit of the up-going high energy  $\nu_\mu$  (Fig.)

Clear demonstration of the dependence of the  
number of  $\nu_\mu$  on distance

Perfectly described by  $\nu_\mu \rightarrow \nu_\tau$  oscillations

Best-fit values of the oscillation parameters

$$\Delta m_{\text{atm}}^2 = 2 \cdot 10^{-3} \text{eV}^2; \quad \sin^2 2\theta_{\text{atm}} = 1.0$$

$$(\chi_{\text{min}}^2 = 170.8/170 \text{ d.o.f.})$$

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## II. SNO SOLAR NEUTRINO EVIDENCE

Neutrinos from the decay  ${}^8\text{B} \rightarrow {}^8\text{Be} + e^+ + \nu_e$   
are detected via the observation

$$\nu_e + d \rightarrow e^- + p + p$$

$$\nu_l + d \rightarrow \nu_l + n + p \quad (l = e, \mu, \tau)$$

The total flux of the detected  $\nu_e$  (CC)

$$\Phi_{\nu_e}^{\text{SNO}} = (1.59^{+0.09}_{-0.07}(\text{stat.})^{+0.06}_{-0.08}(\text{syst.})) \cdot 10^6 \text{ cm}^{-2}\text{s}^{-1}$$

The total flux of the detected  $\nu_e, \nu_\mu, \nu_\tau$  (NC)

$$\sum_{l=e,\mu,\tau} \Phi_{\nu_l}^{\text{SNO}} = (5.21 \pm 0.27 \pm 0.38) \cdot 10^6 \text{ cm}^{-2}\text{s}^{-1}$$

Clear demonstration of the transitions of the  
solar  $\nu_e$  into  $\nu_\mu$  and  $\nu_\tau$

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### III. KamLAND REACTOR ANTINEUTRINO EVIDENCE

The reactor  $\bar{\nu}_e$  are detected via observation of  $e^+$  and  $n$  in  $\bar{\nu}_e + p \rightarrow e^+ + n$

Average distance is 180 km

$$\frac{N_{obs}}{N_{exp}} = 0.611 \pm 0.085 \pm 0.041$$

clear demonstration of the disappearance of the reactor  $\bar{\nu}_e$  (Fig)

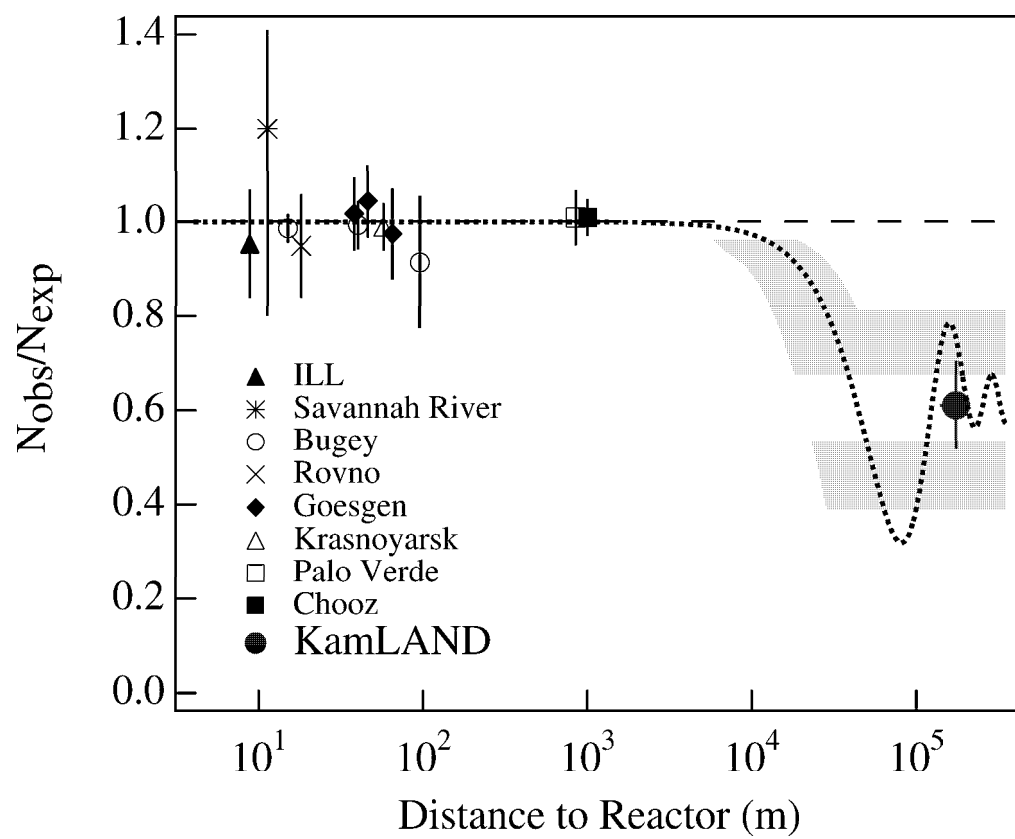
All solar neutrino data are described by the two-neutrino  $\nu_e$  survival probability in matter

KamLAND data are described by the two-neutrino  $\bar{\nu}_e$  survival probability in vacuum.

The values of the oscillation parameters found from analysis of the solar and KamLAND data are compatible

From joint analysis of the solar and KamLAND data (assuming CPT) best fit values of the parameters

$$\Delta m_{sol}^2 = 7.1 \cdot 10^{-5} \text{eV}^2; \tan^2 \theta_{sol} = 0.41$$



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Atmospheric neutrino evidence of neutrino  
oscillations was confirmed by the accelerator  
K2K experiment

Average  $\nu_\mu$  energy 1.3 GeV. Distance 250 km.

56  $\nu_\mu$  observed events;  $80.1^{+6.2}_{-5.4}$  expected.

best-fit values

$$\sin^2 2\theta_{K2K} = 1 : \Delta m_{K2K}^2 = 2.8 \cdot 10^{-3} \text{ eV}^2$$

agreement with  $\sin^2 2\theta_{\text{atm}}, \Delta m_{\text{atm}}^2$

Negative result of the CHOOZ experiment is  
very important

$\bar{\nu}_e$  detected via  $\bar{\nu}_e + p \rightarrow e^+ + n$

Distance about 1 km

$$\frac{N_{obs}}{N_{exp}} = 1.01 \pm 2.4\% \pm 2.7\%$$

From CHOOZ exclusion curve

$$\sin^2 2\theta_{\text{CHOOZ}} \leq 2 \cdot 10^{-1}$$

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## BASICS

### I. The Standard CC and NC Lagrangian

$$j_{\alpha}^{\text{CC}} = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL} \gamma_{\alpha} l_L; \quad j_{\alpha}^{\text{NC}} = \sum_{l=e,\mu,\tau} \bar{\nu}_{lL} \gamma_{\alpha} \nu_{lL}$$

### II. Three flavor neutrinos $\nu_e, \nu_{\mu}, \nu_{\tau}$

### *III. Neutrino mixing*

$$\nu_{lL} = \sum_i U_{li} \nu_{iL}$$

$U^{\dagger} U = 1$   $\nu_i$  is the field with the mass  $m_i$

Two basic facts

1.  $\Delta m_{\text{sol}}^2 \ll \Delta m_{\text{atm}}^2$
2.  $|U_{e3}|^2 \ll 1$

Leading transitions

$\nu_{\mu} \rightarrow \nu_{\tau}$  in the atmospheric range of L/E

Transitions of solar  $\nu_e$  in matter and reactor  $\bar{\nu}_e$   
in the KamLAND range of L/E are

$$\nu_e \rightarrow \nu_{\mu,\tau} \text{ and } \bar{\nu}_e \rightarrow \bar{\nu}_{\mu,\tau}$$

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After neutrino masses and mixing were established one of the most fundamental problem

*Are  $\nu_i$  Dirac or Majorana particles?*

The solution of this problem will have important impact on our understanding of the origin of neutrino masses

It is impossible to distinguish D and Mj in neutrino oscillations

$U^{Mj} = U^D S(\beta);$   $S(\beta)$  is a diagonal phase matrix

$$P(\nu_\alpha \rightarrow \nu_{\alpha'}) = \left| \sum_i U_{\alpha'i} e^{-i\Delta m_{i1}^2 \frac{L}{2E}} U_{\alpha i}^* \right|^2$$
$$P^{Mj}(\nu_\alpha \rightarrow \nu_{\alpha'}) = P^D(\nu_\alpha \rightarrow \nu_{\alpha'})$$

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### $0\nu\beta\beta$ - decay

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^-.$$

The most sensitive way to search for small  
Majorana masses

The half-life

$$\frac{1}{T_{1/2}^{0\nu}(A, Z)} = |m_{ee}|^2 |M^{0\nu}(A, Z)|^2 G^{0\nu}(E_0, Z)$$

The effective Majorana mass

$$m_{ee} = \sum_i U_{ei}^2 m_i$$

$M^{0\nu}(A, Z)$  nuclear matrix element,  $G^{0\nu}(E_0, Z)$   
phase-space factor

- Experimental data, future experiments
- Possible values of  $|m_{ee}|$
- The problem of nuclear matrix elements



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## *Experimental data*

$0\nu\beta\beta$ - decay possible for even-even nuclei for which usual  $\beta$ -decay is forbidden

$^{76}\text{Ge}(2.040\text{ MeV})$ ,  $^{130}\text{Te}(2.533\text{ MeV})$ ,  
 $^{136}\text{Xe}(2.479\text{ MeV})$ ,  $^{150}\text{Nd}(3.367\text{ MeV})$ , ....

The probability is proportional to  $E_0^5$

The results of many experiments are available

Some data (90 % CL bounds)

Hedelberg-Moscow  $^{76}\text{Ge}$  crystals, 86%  
enriched (11 kg)

$$T_{1/2}^{0\nu} \geq 1.9 \cdot 10^{25} \text{ years}; \quad |m_{ee}| \leq (0.3 - 1.2) \text{ eV}$$

MIBETA  $^{130}\text{Te}$ , cryogenic detector (6.8 kg)

$$T_{1/2}^{0\nu} \geq 2.1 \cdot 10^{23} \text{ years}; \quad |m_{ee}| \leq (1.1 - 2.6) \text{ eV}$$

ranges due to NME uncertainties

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*Future goal*  $|m_{ee}| \simeq \text{a few } 10^{-2} \text{ eV}$

Can be reached with  $\simeq 1$  ton detectors (now  $\simeq 10$  kg ), low background, good energy resolution, efficient signature for real events

15 proposals

CUORE Cryogenic detector, 800 kg Te O<sub>2</sub> crystals, resolution 5 keV

$$T_{1/2}^{0\nu} \simeq 9.5 \cdot 10^{26} \text{ years}; \quad |m_{ee}| \simeq (2 - 5.2) 10^{-2} \text{ eV}$$

EXO 60-80 % enreached  $^{136}\text{Xe}$  ,  $\simeq 10$  tons;  
additional signature:  $\text{Ba}^+$  atoms from the decay  $^{136}\text{Xe} \rightarrow ^{136}\text{Ba}^{++} + e^- + e^-$  will be optically tagged with lasers. Background only from  $2\nu\beta\beta$  decay.

$$T_{1/2}^{0\nu} \simeq 1 \cdot 10^{28} \text{ years}; \quad |m_{ee}| \simeq (1.3 - 3.7) 10^{-2} \text{ eV}$$

GENIUS 1 ton 86 % enreached  $^{76}\text{Ge}$  in liquid nitrogen (creostat and shielding)

$$T_{1/2}^{0\nu} \simeq 1 \cdot 10^{28} \text{ years}; \quad |m_{ee}| \simeq (1.3 - 5.0) 10^{-2} \text{ eV}$$

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MAJORANA 500 kg of 86 % enriched  $^{76}\text{Ge}$   
background from  $^{68}\text{Ge} \rightarrow e^+ + \nu_e + ^{68}\text{Ga}$ .

can be suppressed by the segmentation and  
pulse shape analysis

$$T_{1/2}^{0\nu} \simeq 4 \cdot 10^{27} \text{ years}; \quad |m_{ee}| \simeq (2.1 - 7.0) 10^{-2} \text{ eV}$$

*Effective Majorana mass  $|m_{ee}|$*

### THREE-NEUTRINO MIXING

$$\nu_{lL} = \sum_{i=1}^3 U_{li} \nu_{iL}$$

LSND ? will be checked by the MiniBooNE

$$m_{ee} = \sum_{i=1}^3 U_{ei}^2 m_i$$

#### I. Neutrino masses

From neutrino oscillations

$$\Delta m_{21}^2 \simeq \Delta m_{\text{sol}}^2 \quad |\Delta m_{32}^2| \simeq \Delta m_{\text{atm}}^2$$

For the minimal mass upper bounds

From Troitsk and Mainz tritium experiments

$$m_{\text{min}} \leq 2.05 \text{ (2.3) eV}$$

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From WMAP and Sloan Digital Survey data

$$m_{\min} \leq 0.6 \text{ eV}$$

II. 1-2 and 1-3 mixing angles

$$U_{e1}^2 = \cos^2 \theta_{13} \cos^2 \theta_{12} e^{2i\alpha_1}$$

$$U_{e2}^2 = \cos^2 \theta_{13} \sin^2 \theta_{12} e^{2i\alpha_2}$$

$$U_{e3}^2 = \sin^2 \theta_{13} e^{2i\alpha_3}$$

$$\sin^2 \theta_{12} \simeq \sin^2 \theta_{\text{sol}} \quad \sin^2 \theta_{13} \leq 5 \cdot 10^{-2}$$

III. Majorana phases

In the case of CP invariance

$$U_{ei} = U_{ei}^* \eta_i$$

$$\eta_i = i \rho_i \quad \rho_i = \pm 1 \text{ is CP-parity of } \nu_i$$

$$|m_{ee}| = \left| \sum_{i=1}^3 |U_{ei}|^2 \rho_i m_i \right|$$

Cancellations take place for different CP  
parities

$|m_{ee}| \text{ strongly depends}$

on pattern of neutrino mass spectrum and the  
minimal neutrino mass

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## Three “standard” spectra

### I. Hierarchy of masses $m_1 \ll m_2 \ll m_3$

Neutrino masses are known from oscillation data

$$m_2 \simeq \sqrt{\Delta m_{\text{sol}}^2} \simeq 8.4 \cdot 10^{-3} \text{eV}.$$

$$m_3 \simeq \sqrt{\Delta m_{\text{atm}}^2} \simeq 4.5 \cdot 10^{-2} \text{eV}.$$

$$|m_{ee}| \simeq \left| \sin^2 \theta_{\text{sol}} \sqrt{\Delta m_{\text{sol}}^2} + e^{i\alpha_{32}} \sin^2 \theta_{13} \sqrt{\Delta m_{\text{atm}}^2} \right|$$

The first term is small. Contribution of “large”  $\sqrt{\Delta m_{\text{atm}}^2}$  is suppressed by the smallness of  $\sin^2 \theta_{13}$

Upper bound

$$|m_{ee}| \leq 4.6 \cdot 10^{-3} \text{eV}$$

significantly smaller than the sensitivity of the future experiments

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## II. Inverted hierarchy of masses $m_3 \ll m_1 < m_2$

$$m_2 \simeq m_1 \simeq \sqrt{\Delta m_{\text{atm}}^2}; m_3 \ll \sqrt{\Delta m_{\text{atm}}^2}$$

Effective Majorana mass

$$|m_{ee}| \simeq \sqrt{\Delta m_{\text{atm}}^2} (1 - \sin^2 2\theta_{\text{sol}} \sin^2 \alpha_{21})^{\frac{1}{2}}$$

$\alpha_{21} = \alpha_2 - \alpha_1; \alpha_{21} = 0, \pm \frac{\pi}{2}$  in the CP case

Important:  $\sin^2 2\theta_{\text{sol}} < 1$  (5.4  $\sigma$ )

$|m_{ee}|_{\text{min}}$  can not be equal to zero

$$\cos 2\theta_{\text{sol}} \sqrt{\Delta m_{\text{atm}}^2} \leq |m_{ee}| \leq \sqrt{\Delta m_{\text{atm}}^2}$$

Using solar data

$$0.4 \sqrt{\Delta m_{\text{atm}}^2} \leq |m_{ee}| \leq \sqrt{\Delta m_{\text{atm}}^2}$$

Scale of  $|m_{ee}|$  is determined by  $\sqrt{\Delta m_{\text{atm}}^2}$

For the best-fit values of the parameters

$$0.8 \cdot 10^{-2} \text{eV} \leq |m_{ee}| \leq 5.5 \cdot 10^{-2} \text{eV}$$

Can be reached in the future experiments

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### III. Practically degenerate neutrino masses

In two previous cases  $m_{\min}$  was small and neutrino masses were determined by  $\sqrt{\Delta m_{\text{atm}}^2}$   
 $\sqrt{\Delta m_{\text{sol}}^2}$

If  $m_{\min} \gg \sqrt{\Delta m_{\text{atm}}^2}$

$m_1 \simeq m_2 \simeq m_3$  and  $m_{\min}$  unknown

Effective Majorana mass

$$|m_{ee}| \simeq m_{\min} (1 - \sin^2 2\theta_{\text{sol}} \sin^2 \alpha)^{\frac{1}{2}}$$

From the solar data

$$0.42 m_{\min} \leq |m_{ee}| \leq m_{\min}$$

The scale of  $|m_{ee}|$  is determined by  $m_{\min}$

The sensitivity of the future tritium KATRIN  
experiment

$$m_{\min} \simeq 0.2 \text{ eV}$$

From the measurement  $|m_{ee}|$  an information on  
 $m_{\min}$  can be inferred

$$|m_{ee}| \leq m_{\min} \leq 2.38 |m_{ee}|$$

$|m_{ee}|$  for arbitrary  $m_{\min}$  Fig.

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## *Nuclear Matrix elements*

Complicated nuclear problem

two methods: QRPA and shell model

The results of different calculations differ by  
factor  $\geq 3$

$$6.8 \cdot 10^{26} \text{y} \leq T_{1/2}^{0\nu}({}^{76}\text{Ge}) \leq 70.8 \cdot 10^{26} \text{y}$$
$$\text{at } |m_{ee}| = 5 \cdot 10^{-2} \text{eV}$$

Recently important progress in QRPA

(V.A. Rodin et al Phys. Rev. C 68 (2003)  
044302)

For  ${}^{76}\text{Ge}$ ,  ${}^{100}\text{Mo}$ ,  ${}^{130}\text{Te}$  and  ${}^{136}\text{Xe}$   $0\nu\beta\beta$  NME  
were calculated

the constant  $g_{pp}$  (particle-particle interaction)  
was fixed by measured half-life of the  $2\nu\beta\beta$ -  
decay.

NME are stable under the change of the  
nuclear potential and the number of single  
particle states



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## A POSSIBLE TEST OF NME CALCULATIONS

NME of the  $0\nu\beta\beta$ - decay can not be connected  
with other measurable

$$|m_{ee}| = \left( \frac{1}{T_{1/2}^{0\nu} |M^{0\nu}(A, Z)|^2 G^{0\nu}(E_0, Z)} \right)^{1/2}$$

Assume  $0\nu\beta\beta$ - decay of several nuclei is  
observed

A model is compatible with the data if

$$|m_{ee}|_{A_1, Z_1} \simeq |m_{ee}|_{A_2, Z_2}$$

For models  $m_1$  and  $m_2$

$$|m_{\beta\beta}|_{A, Z}^{m_2} = |m_{\beta\beta}|_{A, Z}^{m_1} \eta_{A, Z}(m_2/m_1)$$

$$\eta_{A, Z}(m_2/m_1) = \frac{|M^{0\nu}(A, Z)|_{m_2}}{|M^{0\nu}(A, Z)|_{m_1}}$$

Using existing calculations we can see that

$$\eta_{A, Z}^A \text{ depends on } (A, Z)$$

However, observation of the  $0\nu\beta\beta$ - decay of two  
nuclei can be not enough to distinguish models

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| Nucleus           | $\eta(NSM/QRPA)$ |
|-------------------|------------------|
| $^{76}\text{Ge}$  | 1.75             |
| $^{130}\text{Te}$ | 1.45             |
| $^{136}\text{Xe}$ | 2.54             |

The latest NSM and QRPA calculations were  
used

If  $0\nu\beta\beta$ -decay of  $^{76}\text{Ge}$  and  $^{130}\text{Te}$  is observed  
and the same  $|m_{ee}|$  will be obtained with  
QRPA nuclear matrix elements, it will be  
difficult to exclude NSM

Observation of  $0\nu\beta\beta$ -decay of more than two  
nuclei is needed

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## *Conclusion*

- The establishment of the nature of  $\nu_i$  (Majorana or Dirac?) will have a profound importance for the understanding of small neutrino mass and mixing physics
- Far the most sensitive process is  $0\nu\beta\beta$ -decay
- Today's limit  $|m_{ee}| \leq (0.3 - 1.2) \text{ eV}$
- Goal of future experiments (15 proposals)  $|m_{ee}| \simeq \text{a few } 10^{-2} \text{ eV}$
- If  $0\nu\beta\beta$ -decay is observed, the pattern of the neutrino mass spectrum can be revealed.
- Calculation of the nuclear matrix elements is a complicated nuclear problem and is a theoretical challenge
- Observation of  $0\nu\beta\beta$ -decay of several nuclei the only possibility to test calculations