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Semi-phenomenological approach to the estimate of CP effects in $K^{\pm} \to 3\pi$ decays.

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1. An observation of CP effects in $K^{\pm} \to 3\pi$ decays would allow to understand better how the mechanisms of ~~CP~~ work.

Now the Collaboration NA48/2 is ready to begin a search for such effect with accuracy $\delta\left(\frac{g^+ - g^-}{g^+ + g^-}\right) \le 2 \cdot 10^{-4}$ where $g^{\pm}$ are the slope parameters charaterising the energy distribution of "odd" pions at Dalitz-plot. (For $K^+ \to 3\pi$ and $K^- \to 3\pi$)

Contrary to the case of $K_L \to 2\pi$ decay where CP violates both in $\Delta S = 2$ and $\Delta S = 1$ transitions, in $K^{\pm} \to 3\pi$, only the last (so-called "direct") CP violation takes place.

Experimentally, an existance of the direct ~~CP~~ in $K_L \to 2\pi$ decays predicted by SM and characterized by the parameter ε' is established: $\frac{\varepsilon'}{\varepsilon} = (1.66 \pm 0.16)\, 10^{-3}$

But the large uncertainties in the theoretical predictions

$$\frac{\varepsilon'}{\varepsilon} = \left(17^{+14}_{-10}\right)10^{-4} \text{ Bertolini et al'98}$$

$$\frac{\varepsilon'}{\varepsilon} = (1.5 \div 31.6)\,10^{-4} \text{ Hambye et al'2000}$$

do not allow to affirm that a contribution of the sources of $\cancel{CP}$ beyond the K-M phase is excluded.

The direct $\cancel{CP}$ in $K_L \to 2\pi$ and $K^\pm \to 3\pi$ is the originated by different compositions of the CP-odd parameters. Therefore, a study of $K^\pm \to 3\pi$ will allow to fix these parameters with a better accuracy.

To avoid the uncertainties of the theoretical calculation of the ingredients of the theory, we shall use the following procedure.

Working with some technics, we obtain the theoretical expessions for $K_L \to 2\pi$ and $K^\pm \to 3\pi$ amplitudes. They contain the compositions of one and the same set of parameters.

And calculating $(g^+ - g^-)$ we shall use the magnitudes of these parameters extracted from data on $K_L \to 2\pi$ decays.

2. The scheme of calculations

A theory of $\Delta S = 1$ non-leptonic decays is based on the effective lagrangian (Shifman, Vainshtein, Zakharov '77)

$$L(\Delta S=1) = \sqrt{2}\, G_F \sin\theta_c \cos\theta_c \sum c_i O_i$$

where

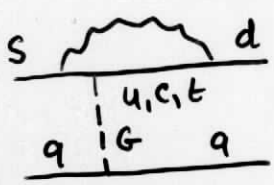
$$O_1 = \bar{s}_L \gamma_\mu d_L \cdot \bar{u}_L \gamma_\mu u_L - \bar{s}_L \gamma_\mu u_L \cdot \bar{u}_L \gamma_\mu d \qquad (\{8_f\},\ \Delta I = \tfrac{1}{2})$$

$$O_2 = \bar{s}_L \gamma_\mu d_L \cdot \bar{u}_L \gamma_\mu u_L + \bar{s}_L \gamma_\mu u \cdot \bar{u}_L \gamma_\mu d_L + 2\bar{s}_L \gamma_\mu d_L \cdot \bar{d}_L \gamma_\mu d + 2\bar{s}_L \gamma_\mu d_L \cdot \bar{s}_L \gamma_\mu s_L \qquad (\{8_d\},\ \Delta I = \tfrac{1}{2})$$

$$O_3 = \bar{s}_L \gamma_\mu d_L \cdot \bar{u}_L \gamma_\mu u_L + \bar{s}_L \gamma_\mu u_L \cdot \bar{u}_L \gamma_\mu d_L + 2\bar{s}_L \gamma_\mu d_L \cdot \bar{d}_L \gamma_\mu d_L - 3\bar{s}_L \gamma_\mu d_L \cdot \bar{s}_L \gamma_\mu s_L \qquad (\{27\},\ \Delta I = \tfrac{1}{2})$$

$$\underline{\underline{O_4}} = \bar{s}_L \gamma_\mu d_L \cdot \bar{u}_L \gamma_\mu u_L + \bar{s}_L \gamma_\mu u_L \cdot \bar{u}_L \gamma_\mu d_L - \bar{s}_L \gamma_\mu d_L \cdot \bar{d}_L \gamma_\mu d_L \qquad (\{27\},\ \underline{\underline{\Delta I = \tfrac{3}{2}}})$$

$$\Delta I = \tfrac{1}{2} \begin{cases} O_5 = \bar{s}_L \gamma_\mu \lambda^a d_L \left(\sum_{q=u,d,s} \bar{q}_R \gamma_\mu \lambda^a q_R \right) \\[2ex] O_6 = \bar{s}_L \gamma_\mu d_L \left(\sum_{q=u,d,s} \bar{q}_R \gamma_\mu q_R \right) \end{cases}$$



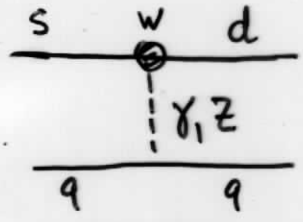
QCD "penguin"

$\Downarrow$

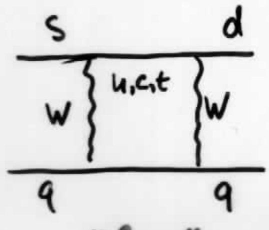
dressed by gluons

This set is sufficient for calculation of the CP even parts of the amplitudes under consideration.

To calculate the CP-odd parts, it is necessary to add the so-called electro-weak contributions originated by the operators O_7, O_8:



"EWP"



"Box"

$$\Delta I = \tfrac{1}{2}, \tfrac{3}{2} \begin{cases} O_7 = \frac{3}{2}\, \bar{s}\gamma_\mu(1+\gamma_5)d \left[\frac{2}{3}\bar{u}\gamma_\mu(1-\gamma_5)u - \frac{1}{3}\bar{d}\gamma_\mu(1-\gamma_5)d - \frac{1}{3}\bar{s}\gamma_\mu(1-\gamma_5)s \right] \\ O_8 = -12 \sum\limits_{q=u,d,s} e_q (\bar{s}_L q_R)(\bar{q}_R d_L)\ , \quad e_q = \left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right) \end{cases}$$

Only c_5, c_6, c_7 and c_8 have the imaginary parts necessary for $\cancel{CP}$.

A bosonization of these operators can be done using the special linear $U(3)_L \otimes U(3)_R$ σ model

$$L^{strong} = \frac{1}{2} \mathrm{Tr}\left(\partial_\mu U\, \partial_\mu U^+\right) - c\, \mathrm{Tr}\left(UU^+ - A^2 t_0^2\right)^2 +$$

$$+ \frac{F_\pi}{2\sqrt{2}} \mathrm{Tr}\left[(U+U^+)M\right] + \Delta L_{ps}(U_1) - c\xi \left(\mathrm{Tr}\left(UU^+ - A^2 t_0^2\right)\right)^2$$

E.Sh. Nucl. Phys. <u>B 409</u>, 87 (1993)

In this lagrangian $M = \begin{pmatrix} \mu_1^2 & & \\ & \mu_2^2 & \\ & & \mu_3^2 \end{pmatrix}$

$$\mu_1^2 = \mu_2^2 = m_\pi^2 \ ; \quad \mu_3^2 = 2m_K^2 - m_\pi^2$$

$\boxed{U = \hat{\sigma} + i\hat{\pi}}$, σ is 3×3 matrix of the scalar parners of pions

$$\hat{\pi} = \begin{pmatrix} \frac{\pi_0}{\sqrt{3}} + \frac{\pi_8}{\sqrt{6}} + \frac{\pi_3}{\sqrt{2}}\ , & \pi^+\ , & K^+ \\ \pi^-\ , & \frac{\pi_0}{\sqrt{3}} + \frac{\pi_8}{\sqrt{6}} - \frac{\pi_3}{\sqrt{2}}\ , & K^0 \\ K^-\ , & \bar{K}^0\ , & \frac{\pi_0}{\sqrt{3}} - \frac{2\pi_8}{\sqrt{6}} \end{pmatrix}$$

The last term of L^{strong} characterized a strength of mixing between $\bar{q}q$ and $(G_{\mu\nu})^2$ states.

In terms of the physical mesonic fields, the diquark compositions entering O_i can be written as follows

$$\bar q_j \gamma_\mu (1+\gamma_5) q_k = i\left\{\partial_\mu U \cdot U^+ - U \partial_\mu U^+\right\}_{kj}$$

$$\bar q_i (1+\gamma_5) q_k = -\sqrt{2}\,\frac{F_\pi m_\pi^2}{m_u+m_d}\, U_{kj} \quad , \quad F_\pi = 93\ \text{MeV}$$

Using also the relations between matrices in the colour space

$$\delta^\alpha_\beta \delta^\gamma_\delta = \frac{1}{3}\delta^\alpha_\delta \delta^\gamma_\beta + \frac{1}{2}\lambda^\alpha_\delta \lambda^\gamma_\beta$$

$$\lambda^\alpha_\beta \lambda^\gamma_\delta = \frac{16}{9}\delta^\alpha_\delta \delta^\gamma_\beta - \frac{1}{3}\lambda^\alpha_\delta \lambda^\gamma_\beta$$

and the Fierz transformation relation

$$\bar s \gamma_\mu (1+\gamma_5) d \cdot \bar q \gamma_\mu (1-\gamma_5) q = -2\, \bar s (1-\gamma_5) q \cdot \bar q (1+\gamma_5) d$$

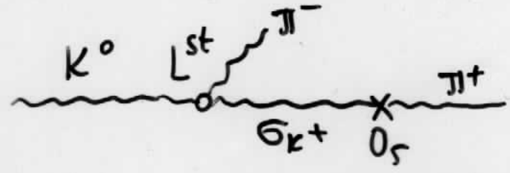
we come to the following mesonic form of O_5: (at $\xi=0$)

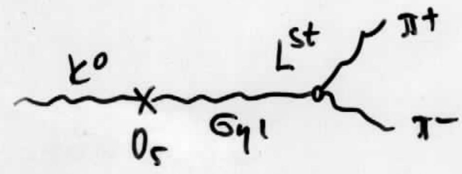
$$(O_5)^{P\text{-odd}} = -i\,\frac{32}{9} F_\pi^2 \Lambda^2 \left[\sigma_{K^+}\pi^- - K^+\sigma_{\pi^-} + \sigma_{K^0}\left(\frac{2\eta}{\sqrt{3}} - \frac{\eta'}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}}\right) + \right.$$

$$\left. + \frac{1}{\sqrt{2}} K^0 \sigma_3 - \frac{1}{\sqrt{2}} K^0 \sigma_{\eta'}\right] \otimes \frac{1}{4};$$

where $\Lambda^2 = m^2_{\sigma_\pi} - m^2_\pi \cong 0.94\,(\text{GeV})^2$; $\quad \sigma_\pi = a_0(980)$

Therefore, a contribution of O_5 in $K^0 \to \pi^+\pi^-$ transition is given by the diagrams





The coupling constants $g_{\sigma_{K^+}K^0\pi^-}$, $g_{\sigma_{\eta'}\pi^+\pi^-}$ and $m_{\sigma_{K^+}}$, $m_{\sigma_{\eta'}}$ are calculated in [E.Sh. Nucl. Phys.'93]

Representing $M(K\to 2\pi)$ in the form

$$M(K_1^0 \to \pi^+\pi^-) = A_0 e^{i\delta_0} - A_2 e^{i\delta_2}$$

$$M(K_1^0 \to \pi^0\pi^0) = A_0 e^{i\delta} + 2A_2 e^{i\delta_2}$$

$$M(K_1^+ \to \pi^+\pi^0) = -\frac{3}{2} A_2 e^{i\delta_2}$$

we obtain

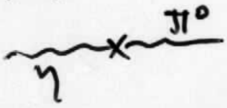
$$A_0 = \frac{G_F F_\pi \sin\theta_c \cos\theta_c}{\sqrt{2}} (m_K^2 - m_\pi^2)\left[c_1 - c_2 - c_3 + \frac{32}{9}\beta\left(\mathrm{Re}\,\tilde{c}_5 + i\,\mathrm{Im}\,\tilde{c}_5\right) - \Delta\right]$$

$$A_2 = \frac{G_F F_\pi \sin\theta_c \cos\theta_c}{\sqrt{2}} (m_K^2 - m_\pi^2)\left[c_4 + i\,\frac{2\beta\Lambda^2 \,\mathrm{Im}\,\tilde{c}_7}{3R(m_K^2 - m_\pi^2)} - \Delta\right]$$

$$\tilde{c}_5 = c_5 + \frac{3}{16}c_6\,;\quad \tilde{c}_7 = c_7 + 3c_8\,;\quad \beta = \frac{2m_\pi^4}{(m_u+m_d)^2\Lambda^2}\,;\quad R = F_K/F_\pi.$$

Δ – for contribution of transition

$$K^0 \to \pi^0\eta(\eta') \to \pi^0\pi^0$$



Isospin breakdown

The contributions from $\tilde{c}_7 O_7$ into $\mathrm{Re}\,A_0$ and $\mathrm{Im}\,A_0$ are small because $\tilde{c}_7/\tilde{c}_5 \sim \alpha_{em}$ and we have neglected these corrections.

From data on widths of $K\to 2\pi$ decays we obtain

$$c_4 = 0.328\,;\quad c_1 - c_2 - c_3 + \frac{32}{9}\beta\,\mathrm{Re}\,\tilde{c}_5 = -10.13$$

At $c_1 - c_2 - c_3 = -2.89$ (SVZ'77, Okun "Leptons and Quarks")

and $\beta = 6.68 \quad \Rightarrow \quad \mathrm{Re}\,\tilde{c}_5 = -0.305$

! From expession for A_2, it is seen that $O_{7,8}$ breek the Chiral Theory rule according to which the mesonic amplitudes must be proportional p^2: For this reason this contribution is enlarged by factor $\frac{\Lambda^2}{m_K^2}$.

Using the general relation

$$\varepsilon' = i\, e^{i(\delta_2-\delta_0)} \left[-\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0} + \frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} \right] \cdot \left| \frac{A_2}{A_0} \right|$$

and the experimental value $\varepsilon' = (3.4 \pm 0.45)\,10^{-6}$

we come to the relation

$$-\frac{\mathrm{Im}\,\tilde{C}_5}{\mathrm{Re}\,\tilde{C}_5} \left(1 - \Omega_{\eta\eta'} + 20.66\, \frac{\mathrm{Im}\,\tilde{C}_7}{\mathrm{Im}\,\tilde{C}_5} \right) = 1.48 \cdot 10^{-4}$$

The naive estimate gives

$$-\frac{\mathrm{Im}\,\tilde{C}_5}{\mathrm{Re}\,\tilde{C}_5} \approx 1.7\, S_2 S_3 \sin\delta$$

where S_2, S_3 and δ are the parameters of CKM matrix

At $\quad 4.6 \cdot 10^{-4} \le S_2 S_3 \le 6.7 \cdot 10^{-4}$ (Landsberg' 2002)

$$\frac{\mathrm{Im}\,\tilde{C}_5}{\mathrm{Re}\,\tilde{C}_5} = (-9.6 \pm 1.8)\,10^{-4} \sin\delta$$

$$\frac{\mathrm{Im}\,\tilde{C}_7}{\mathrm{Im}\,\tilde{C}_5} = \begin{cases} -0.026 & \text{for } \Omega_{\eta\eta'} = 0.3 \\ -0.041 & \text{for } \Omega_{\eta\eta'} = 0 \end{cases}$$

3. Decay $K^\pm \to \pi^\pm \pi^\pm \pi^\mp$

Neglecting CP-odd part and using $\{\xi=0;\ p^2\}$ approximation

$$M\left(K^+(k)\to\pi^+(p_1)\pi^+(p_2)\pi^-(p_3)\right)=\frac{G_F\sin\theta_c\cos\theta_c}{2\sqrt{2}}\left\{\left[c_1-c_2-c_3-c_4+\right.\right.$$
$$\left.\left.+\frac{32}{9}\beta\tilde{c}_5\right]\left(\frac{2}{3}m_K^2+s_0-s_3\right)+9c_4\,(s_0-s_3)\right\};$$

$$M\left(K^+(k)\to\pi^0(p_1)\pi^0(p_2)\pi^+(p_3)\right)=\frac{G_F\sin\theta_c\cos\theta_c}{2\sqrt{2}}\left\{\left[c_1-c_2-c_3-c_4+\right.\right.$$
$$\left.\left.+\frac{32}{9}\beta\tilde{c}_5\right](s_3-m_\pi^2)+\frac{9}{2}c_4\,(s_0-s_3)\right\};$$

where $s_i=(k-p_i)^2$ and $s_0=\frac{1}{3}m_K^2+m_\pi^2$.

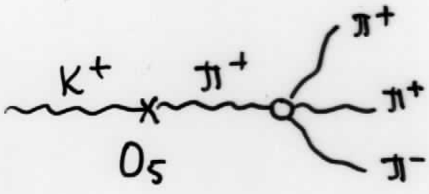
It is not difficult to check that these expressions can be rewritten in the form, obtained by methods of current algebre and soft-pion technics (Vainshtein, Zakharov'70)

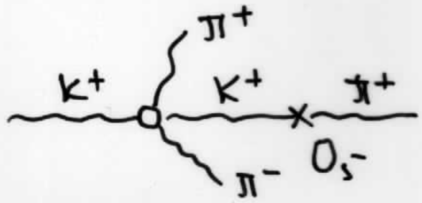
$$M\left(K^+\to\pi^+\pi^+\pi^-(p_3)\right)=\frac{i}{3F_\pi}M\left(K_1^0\to\pi^+\pi^-\right)\left[1+y+6\zeta y\right]$$

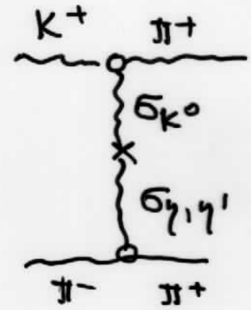
$$M\left(K^+\to\pi^0\pi^0\pi^+(p_3)\right)=\frac{i}{6F_\pi}M\left(K_1^0\to\pi^+\pi^-\right)\left[1-2y+6\zeta y\right]$$

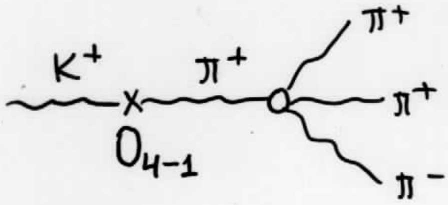
where

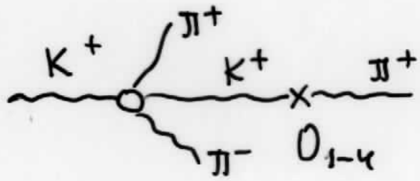
$$y=\frac{3E_3}{m_K}-1;\quad \zeta=-\frac{M(K^+\to\pi^0\pi^+)}{M(K_1\to\pi^+\pi^-)}=\frac{3c_4}{2(c_1-c_2-c_3-c_4+\frac{32}{9}c_5\tilde{\beta})}$$

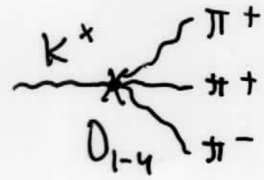


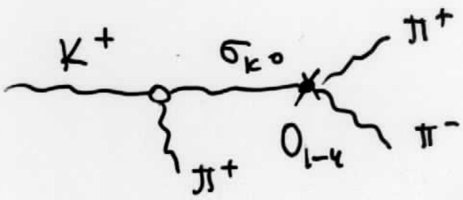


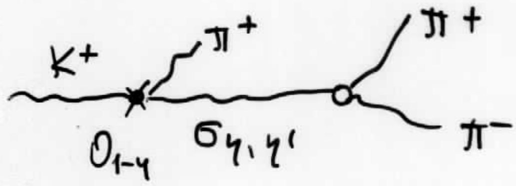












$$a\mathcal{L}(O_5)^{P\text{-even}} = -\frac{2F_\pi^2 m_\pi^4}{(m_u+m_d)^2}\Big[K^+\pi^- - \frac{1}{\sqrt{2}}K^0\pi^0 + \sigma_{K^0}\sigma_{\eta'}\Big(\frac{2}{\sqrt{3}}\cos\theta_S - \frac{\sin\theta_S}{\sqrt{6}}\Big) - \sigma_{K^0}\sigma_\eta\Big(\frac{2}{\sqrt{3}}\sin\theta_S + \frac{1}{\sqrt{6}}\cos\theta_S\Big) + \dots\Big]$$

$$a\mathcal{L}(O_{1\text{-}4})^{P\text{-even}} = F_\pi^2\,\partial_\mu K^+\,\partial_\mu \pi^- + \dots$$

Taking into account the CP-odd contribution produced by $\mathrm{Im}\,\tilde{c}_{5,7}$ we obtain

$$M(K^+ \to \pi^+\pi^+\pi^-(p_3)) = k\left[1 - i a_{KM} + \frac{1}{2} g Y (1 - i b_{KM})\right]$$

where $$k = \frac{G_F \sin\theta_c \cos\theta_c \cdot \frac{2}{3} m_K^2 C_0}{2\sqrt{2}}$$

$$a_{KM} = \left[\frac{32}{9}\beta\,\mathrm{Im}\,\tilde{c}_5 + 4\beta\,\mathrm{Im}\,\tilde{c}_7\left(\frac{3\Lambda^2}{2m_K^2} + \frac{2}{2R-1}\right)\right] \Big/ C_0$$

$$b_{KM} = \left[\frac{32}{9}\beta\,\mathrm{Im}\,\tilde{c}_5 + 4\beta\,\mathrm{Im}\,\tilde{c}_7 \cdot \frac{2}{2R-1}\right] \Big/ (C_0 + 9c_4)$$

$$g = -\frac{3m_\pi^2}{2m_K^2} \cdot \frac{C_0 + 9c_4}{C_0}$$

$$C_0 = c_1 - c_2 - c_3 - c_4 + \frac{32}{9}\beta\,\mathrm{Re}\,\tilde{c}_5 = -10.46$$

Important point

As the field K^+ is the complex one and its phase is arbitrary, we can replace K^+ by $K^+ \cdot \frac{1 + i a_{KM}}{\sqrt{1 + a_{KM}^2}}$

Then

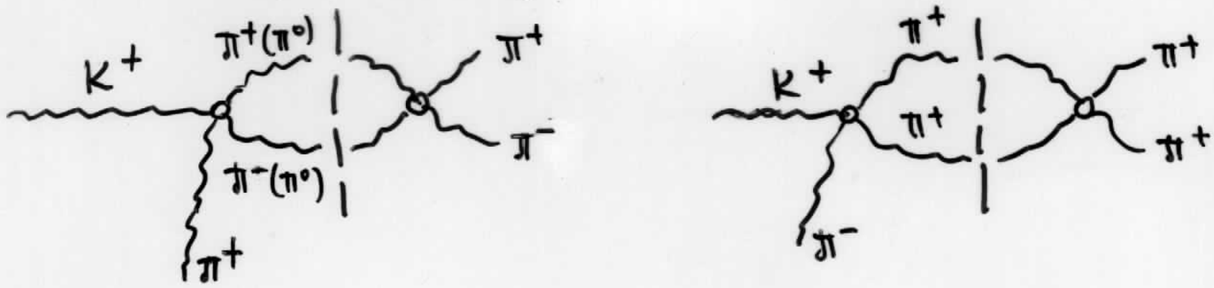
$$M(K^+ \to \pi^+\pi^+\pi^-(p_3)) = k\left[1 + \frac{1}{2} g Y (1 - i(b_{KM} - a_{KM}) + \dots\right]$$

Though this expression contains the imaginary CP-odd part, it does not lead to observable CP effects. To become observable, this part must interfere with CP-even imaginary part arising due to rescattering of the final pions.

Then

$$M(K^+ \to \pi^+\pi^+\pi^-) = k\left[1 + \underset{\substack{\uparrow \\ \text{CP-even}}}{ia} + \tfrac{1}{2} g Y \left(1 + \underset{\substack{\uparrow \\ \text{CP-even}}}{ib} - i\underbrace{(b_{KM} - a_{KM})}_{\text{CP-odd}}\right) + \dots\right]$$

The CP-even imaginary part can be found calculating the diagrams



Using

$$M\left(\pi^+(r_2)\pi^-(r_3)\to\pi^+(p_2)\pi^-(p_3)\right)=\frac{1}{F_\pi^2}\left[(p_2+p_3)^2+(r_2-p_2)^2-2\mu^2\right]$$

$$M\left(\pi^0(r_2)\pi^0(r_3)\to\pi^+(p_2)\pi^-(p_3)\right)=\frac{1}{F_\pi^2}\left[(p_2+p_3)^2-\mu^2\right]$$

$$M\left(\pi^+(r_1)\pi^+(r_2)\to\pi^+(p_1)\pi^+(p_2)\right)=\frac{1}{F_\pi^2}\left[(r_1-p_1)^2+(r_1-p_2)^2-2\mu^2\right]$$

we find

$$\boxed{a=0.12065\ ;\ b=0.714}$$

The slope parameters $g^\pm$ are defined by the relations:

$$\left|M\left(K^\pm(k)\to\pi^+(p_1)\pi^+(p_2)\pi^-(p_3)\right|^2\sim1+g^\pm Y+h^\pm Y^2+k^\pm X^2\right.$$

where $Y=\frac{s_3-s_0}{m_\pi^2}$; $X=\frac{s_1-s_2}{m_\pi^2}$; $s_i=(k-p_i)^2$

Therefore

$$\left|M\left(K^+\to\pi^+\pi^+\pi^-(p_3)\right)\right|^2\sim1+\frac{g}{1+a^2}Y\left(1+ab-a(b_{KM}-a_{KM})\right)$$

$$\left|M\left(K^-\to\pi^-\pi^-\pi^+(p_3)\right)\right|^2\sim1+\frac{g}{1+a^2}Y\left(1+ab+a(b_{KM}-a_{KM})\right)$$

$$\boxed{\frac{g^+-g^-}{g^++g^-}=-\frac{a(b_{KM}-a_{KM})}{1+ab}}$$

Substituting the numerical values for c_i, Λ^2, R, a, b we get

$$\left(\frac{g^+-g^-}{g^++g^-}\right)_{\{\xi=0;\, p^2\}} = 0.030\,\frac{\mathrm{Im}\,\tilde{c}_5}{\mathrm{Re}\,\tilde{c}_5}\left(1-14{,}9\,\frac{\mathrm{Im}\,\tilde{c}_7}{\mathrm{Im}\,\tilde{c}_5}\right)$$

4. The role of the p^4 corrections

Our technics allows to estimate the p^4 contributions to different parts of the amplitude rather well.

As the corrections due to mixing between $\bar{q}q$ and $(G^a_{\mu\nu})^2$ are of the same order as p^4 corrections, we shall take them into account putting

$$\xi = -0.225$$

Such a mixing changes the coupling constants of $\sigma_i\pi\pi(K)$ vertices and m_{σ_i}. (E.Sh.? 93)

As a result

$$K(\xi=-0.225;\, p^2+p^4) = 1.175\, K(\xi=0;\, p^2)$$

$$g(\xi=-0.225;\, p^2+p^4) = 1.244\, g(\xi=0;\, p^2)$$

The theoretical width and slope parameter become very close to their experimental values!

These corrections give also considerable contribution into π-π scattering shifts, leading to good agreement of the calculated δ_0^0, δ_1^1 and δ_0^2 with the experimental ones (E.Sh.'93).

And this is important, because we need to know the imaginary CP-even part of the amplitude.

$$a(\xi=-0.225;\ p^2+p^4) = 0.16265$$

$$b(\xi=-0.225;\ p^2+p^4) = 0.762$$

$$r_g \equiv \left(\frac{g^+-g^-}{g^++g^-}\right)_{\{\xi=-0.225;\ p^2+p^4\}} = 0.039\,\frac{\mathrm{Im}\,\tilde{c}_5}{\mathrm{Re}\,\tilde{c}_5}\left(1-11.95\,\frac{\mathrm{Im}\,\tilde{c}_7}{\mathrm{Im}\,\tilde{c}_5}\right)$$

At the magnitudes of the rest parameters determined previously we have

$$\left(\frac{g^+-g^-}{g^++g^-}\right) = (0.52 \pm 0.10)\,10^{-4}$$

Compare: L. Maiani, N. Paver "The second DAΦNE Physics Handbook"

$$\frac{g^+-g^-}{g^++g^-} = (0.23 \pm 0.06)\,10^{-5}$$

If $\dfrac{\mathrm{Im}\,\tilde{c}_5}{\mathrm{Re}\,\tilde{c}_5} > 1.7 \sin\theta_2 \sin\theta_3 \sin\delta$, then $r_g > 0.6 \cdot 10^{-4}$