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$\alpha$  **DETERMINATION FROM  $\ell, \bar{\ell}$   $B$   
DECAYS**

*20 La Thuile...*

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G.G. Ovanesyan, M.V., JETP Letters **81** (2005) 361

M.V., Phys. Atom. Nucl., **69** (2006) 679

A.B.Kaidalov, M.V., hep-ph/0603013

# ITEP TH

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## Theoretical Laboratories

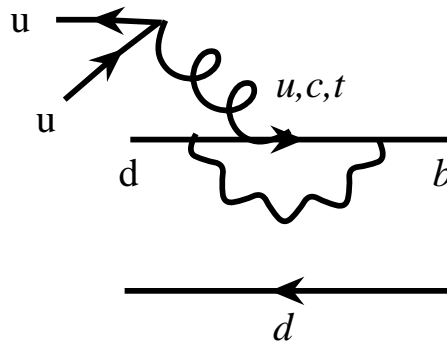
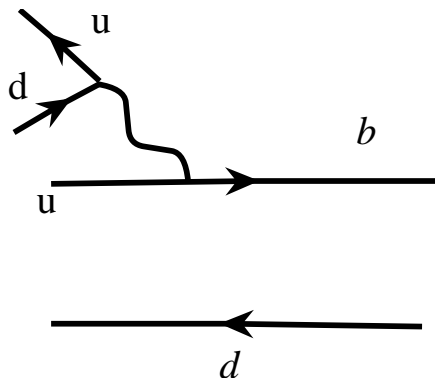


# PLAN

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- $P/T$  - small parameter
- $B \rightarrow \pi\pi$  in tree approximation:  $A_0, A_2, \delta, \alpha$
- factorisation: pro and contro
- $K \rightarrow \pi\pi, D \rightarrow \pi\pi, B \rightarrow D\pi$
- penguin corrections:  $C_{+-}, C_{00}, \alpha$
- Conclusions

$$b \rightarrow ud\bar{u}$$



$B \rightarrow \pi\pi, B \rightarrow \rho\rho, B \rightarrow \rho\pi$  decays

How large is penguin?

$$c_1 = 1.09, c_2 = -0.21;$$

$$c_3 = 0.013, c_4 = -0.032, c_5 = 0.009, c_6 = -0.037$$

# $P/T$ - small

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$$P/T = 0 \implies \sin 2\alpha^T = S_{+-}$$

$$B \rightarrow \pi^+ \pi^- : \alpha_{BABAR}^T = (99 \pm 5)^\circ$$

$$B \rightarrow \rho^+ \rho^- : \alpha^T = (96 \pm 7)^\circ$$

$$B \rightarrow \pi^\pm \rho^\mp : \alpha^T = (94 \pm 4)^\circ, \text{ or } (86 \pm 4)^\circ$$

values of  $\alpha$  from all 3 decays agree with each other and with global CKM fit result.

R.Aleksan, F.Buccella, A.Le Yaouanc, L.Oliver, O.Pene, J.-C.Raynal (1995):

$$\Delta\alpha^{\pi\pi} > \Delta\alpha^{\rho\rho} > \Delta\alpha^{\pi\rho}$$

Perturbation theory over  $P/T$

# $B \rightarrow \pi\pi$ exp data

	BABAR	Belle	Heavy Flavor Averaging Group
$B_{+-}$	$5.5 \pm 0.5$	$4.4 \pm 0.7$	$5.0 \pm 0.4$
$B_{00}$	$1.17 \pm 0.33$	$2.3 \pm 0.5$	$1.45 \pm 0.29$
$B_{+0}$	$5.8 \pm 0.7$	$5.0 \pm 1.3$	$5.5 \pm 0.6$
$S_{+-}$	$-0.30 \pm 0.17$	$-0.67 \pm 0.16$	$-0.50 \pm 0.12$
$C_{+-}$	$-0.09 \pm 0.15$	$-0.56 \pm 0.13$	$-0.37 \pm 0.10$
$C_{00}$	$-0.12 \pm 0.56$	$-0.44 \pm 0.56$	$-0.28 \pm 0.39$

$C_{+-}$ : BABAR - little penguin; Belle - big penguin. Average?

Ambitions: the same level of understanding as in  $K \rightarrow \pi\pi$  decays

# $B \rightarrow \pi\pi$ phenomenology

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$$M_{\bar{B}_d \rightarrow \pi^+ \pi^-} = \frac{G_F}{\sqrt{2}} |V_{ub} V_{ud}^*| m_B^2 f_\pi f_+(0) \left\{ e^{-i\gamma} \frac{1}{2\sqrt{3}} A_2 e^{i\delta} + e^{-i\gamma} \frac{1}{\sqrt{6}} A_0 + \left| \frac{V_{td}^* V_{tb}}{V_{ub} V_{ud}^*} \right| e^{i\beta} P e^{i\delta_p} \right\}, \quad (1)$$

$$M_{\bar{B}_d \rightarrow \pi^0 \pi^0} = \frac{G_F}{\sqrt{2}} |V_{ub} V_{ud}^*| m_B^2 f_\pi f_+(0) \left\{ -e^{-i\gamma} \frac{1}{\sqrt{3}} A_2 e^{i\delta} + e^{-i\gamma} \frac{1}{\sqrt{6}} A_0 + \left| \frac{V_{td}^* V_{tb}}{V_{ub} V_{ud}^*} \right| e^{i\beta} P e^{i\delta_p} \right\}, \quad (2)$$

$$M_{\bar{B}_u \rightarrow \pi^- \pi^0} = \frac{G_F}{\sqrt{2}} |V_{ub} V_{ud}^*| m_B^2 f_\pi f_+(0) \left\{ \frac{\sqrt{3}}{2\sqrt{2}} e^{-i\gamma} A_2 e^{i\delta} \right\} \quad (3)$$

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# tree approximation: $A_0, A_2, \delta, \alpha$

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3 parameters from 3 equations ( $B_{+-}, B_{00}, B_{+0}$ ):

$$A_0 = 1.53 \pm 0.23, A_2 = 1.60 \pm 0.20, \delta = \pm(53^\circ \pm 7^\circ)$$

$$A_0^f = 1.54, \quad A_2^f = 1.35 \quad \text{but } \delta \dots$$

$$\sin 2\alpha^T = S_{+-} ,$$

$$\alpha_{\text{BABAR}}^T = 99^\circ \pm 5^\circ , \quad \alpha_{\text{Belle}}^T = 111^\circ \pm 6^\circ , \quad \alpha_{\text{average}}^T = 105^\circ \pm 4^\circ .$$



# FSI in $K \rightarrow \pi\pi, D \rightarrow \pi\pi, B \rightarrow D\pi$

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$K$  decays: 3 decay probabilities, or Watson theorem:

$$\delta_0^K = 35^\circ \pm 3^\circ, \delta_2^K = -7^\circ \pm 0.2^\circ, \delta^K = 42^\circ \pm 4^\circ$$

$D$  decays: 3 decay probabilities (Watson theorem is not applicable): factorisation also good for moduli of decay amplitudes, while  $\delta_2^D - \delta_0^D = 86^\circ \pm 4^\circ(!?)$

$1/M$  scaling of FSI phases?

$(D\pi) : I = 1/2$  or  $3/2$  ; 3 decay probabilities:  $\delta_{D\pi} = 30^\circ \pm 7^\circ$

So: FSI phases can be L A R G E

# penguin corrections

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To avoid shifts of  $B_{+-}$  and  $B_{00}$  we should shift  $A_0$  and  $\delta$  :

$$A_0 \rightarrow A_0 + \tilde{A}_0, \quad \delta \rightarrow \delta + \tilde{\delta},$$

$$\tilde{A}_0 = \sqrt{6} \left| \frac{V_{td}}{V_{ub}} \right| \cos \alpha \cos \delta_p P,$$

$$\tilde{\delta} = -\sqrt{6} \left| \frac{V_{td}}{V_{ub}} \right| \cos \alpha \sin \delta_p P / A_0,$$

where only the terms linear in  $P$  are taken into account

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In the factorisation approach we have:

$$P^f = -a_4 - \frac{2m_\pi^2}{(m_u + m_d)m_b} a_6 = 0.06 \quad ,$$

$a_4 = c_4 + c_3/3$ ,  $a_6 = c_6 + c_5/3$  and shifts of  $A_0$  and  $\delta$  are small:

$$-0.12 < \tilde{A}_0 < 0.12 \quad , \quad -4^\circ < \tilde{\delta} < 4^\circ$$

for

$$A_0 = 1.5 \quad , \quad -1 < \cos \delta_p \quad , \quad \sin \delta_p < 1 \quad \text{and} \quad 70^\circ < \alpha < 110^\circ$$

# $C_{+-}, C_{00}$

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In linear in  $P$  approximation for direct CP asymmetries we obtain:

$$C_{+-} = -2P \sin \alpha \left| \frac{V_{td}}{V_{ub}} \right| \frac{\cos(\delta_p + \varphi)}{\sqrt{\frac{1}{12} A_2^2 + \frac{1}{6} A_0^2 + \frac{1}{3\sqrt{2}} A_0 A_2 \cos \delta}} =$$
$$-4.7P \cos(\delta_p + 68^\circ)$$

$$C_{00} = -2P \sin \alpha \left| \frac{V_{td}}{V_{ub}} \right| \frac{\cos(\delta_p + \psi)}{\sqrt{\frac{1}{3} A_2^2 + \frac{1}{6} A_0^2 - \frac{\sqrt{2}}{3} \cos \delta A_0 A_2}} = 6.2P \cos \delta_p$$

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where

$$\varphi = \arccos \frac{\frac{1}{\sqrt{3}} A_2 \sin \delta}{\sqrt{\frac{1}{3} A_2^2 + \frac{2}{3} A_0^2 + \frac{2\sqrt{2}}{3} A_0 A_2 \cos \delta}} \approx 68^\circ ,$$

$$\psi = \arccos \frac{-\frac{2}{\sqrt{3}} A_2 \sin \delta}{\sqrt{\frac{4}{3} A_2^2 + \frac{2}{3} A_0^2 - \frac{4\sqrt{2}}{3} A_0 A_2 \cos \delta}} \approx 175^\circ$$

From experimental values of  $C_{ik}$  we can determine  $P$  and  $\delta_p$   
- Gronau-London pass;

since experimental uncertainty in  $C_{00}$  is big while Belle and  
BABAR contradicts each other in  $C_{+-}$  this pass is (tempo-  
rary) closed

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Let us look which values of direct asymmetries follow from our formulas. With  $P = P_f$  we get:

$$C_{+-} = -0.28 \cos(\delta_p + 68^\circ) ,$$

and for the theoretically motivated value  $\delta_p \leq 30^\circ$  we obtain:

$$0 > C_{+-} > -0.10 ,$$

which is close to BABAR result.

For direct CP asymmetry in  $B_d \rightarrow \pi^0 \pi^0$  decay we get:

$$C_{00} = 0.4 \cos \delta_p ,$$

which differs in sign from  $C_{+-}$  and is rather big. It is very interesting to check these predictions experimentally.

# $\alpha$

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$S_{+-}$  is not changed when penguins are taken into account:

$$\alpha = \alpha^T + \tilde{\alpha} \ ,$$

$$\tilde{\alpha} = - \left| \frac{V_{td}}{V_{ub}} \right| P(1 + C_{+-}) \sin \alpha \frac{\cos(\delta_p - \kappa)}{\sqrt{\frac{1}{12}A_2^2 + \frac{1}{6}A_0^2 + \frac{1}{3\sqrt{2}}A_0A_2 \cos \delta}} \ ,$$

$$\kappa = \frac{\pi}{2} - \varphi \ .$$

$$\tilde{\alpha} = -2.4(1 + C_{+-})P \cos(\delta_p - \kappa) = -0.14(1 + C_{+-}) \cos(\delta_p - 22^\circ) \ ,$$

$$\tilde{\alpha}_{\text{BABAR}} \approx -7^\circ \ , \ \alpha_{\text{BABAR}} = \alpha_{\text{BABAR}}^T + \tilde{\alpha}_{\text{BABAR}} = 92^\circ \pm 5^\circ \ .$$

# Conclusions

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$$\tilde{\alpha}_{\text{average}} = -5^\circ, \quad \alpha_{\text{average}} = \alpha_{\text{average}}^T + \tilde{\alpha}_{\text{average}} = 100^\circ \pm 4^\circ.$$

Theoretical uncertainty of the value of  $\alpha$  can be estimated in the following way. Let us suppose that the accuracy of the factorisation calculation of the penguin amplitude is 50%:

$$\tilde{\alpha}_{\text{average}} = -5^\circ \pm 3_{\text{theor}}^\circ,$$

$$\alpha_{\text{average}} = 100^\circ \pm 4_{\text{exp}}^\circ \pm 3_{\text{theor}}^\circ,$$

The model independent isospin analysis of  $B \rightarrow \rho\rho$  decays performed by BABAR gives:

$$\alpha_{BABAR}^{\rho\rho} = 100^\circ \pm 13^\circ,$$



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The global CKM fit results are:

$$\alpha_{CKMFitter} = 98^\circ \pm 8^\circ, \quad \alpha_{UTfit} = (97_{-19}^{+13})^\circ$$

- The moduli of the amplitudes  $A_0$  and  $A_2$  are given with good accuracy by factorisation; FSI phase shift is very large,  $\delta \approx 50^\circ$ .
- Theoretical uncertainty of the value of  $\alpha$  extracted from  $B \rightarrow \pi\pi$  data on  $S_{+-}$  is at the level of few degrees.
- Resolution of the contradiction of Belle and BABAR data on CP asymmetries are very important both for checking the correctness of our approach ( $C$ ) and determination of angle  $\alpha$  ( $S$ ).