Neutrino motion and radiation in matter

Alexander Studenikin

Moscow State University

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La Thuile

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Matter effects in neutrino flavour oscillations

- L.Wolfenstein,
  Neutrino oscillations in matter, Phys.Rev.D 17 (1978) 2369;
- S.Mikheyev, A.Smirnov,
  Resonance amplification of neutrino oscillations in matter and the spectroscopy of the solar neutrino , Sov.J.Nucl.Phys.42 (1985) 913.

$$\boxed{P_{\nu_e\to\nu_\mu}(x) = \sin^2 2\theta_{eff}\sin^2\frac{\pi x}{L_{eff}},}$$

effective mixing angle, θ_{eff}, and the effective oscillation length, L_{eff}, are given by

$$\sin^2 2\theta_{eff} = \frac{\Delta^2\sin^2 2\theta}{\left(\Delta\cos 2\theta - A\right)^2 + \Delta^2\sin^2 2\theta},\quad L_{eff} = \frac{2\pi}{\sqrt{\left(\Delta\cos 2\theta - A\right)^2 + \Delta^2\sin^2 2\theta}},\quad \Delta = \delta m_\nu^2/2|\vec{p}|$$

θ is the vacuum mixing angle , $\boxed{A = \sqrt{2}G_F n}$, n ← the particle number density

MSW effect

Matter effects in neutrino spin (spin-flavour) oscillations

- C.-S.Lim, W.Marciano,
  Resonant spin-flavour precession of solar and supernova neutrinos, Phys.Rev.D37 (1988) 1368;

- E.Akhmedov,
  Resonant amplification of neutrino spin rotation in matter and the solar-neutrino problem, Phys.Lett.B213 (1988) 64.

$$\boxed{P_{\nu_L \to \nu_R}(x) = \sin^2 2\theta_{eff} \sin^2 \frac{\pi x}{L_{eff}}}$$

neutrino magnetic moment

$$\sin^2 2\theta_{eff} = \frac{(2\mu B_\perp)^2}{(2\mu B_\perp)^2 + \Omega^2}, \quad L_{eff} = \frac{2\pi}{\sqrt{\Omega^2 + (2\mu B_\perp)^2}},$$

magnetic field

$$\boxed{\Omega = \frac{\delta m_\nu^2}{2|\vec{p}|} A(\theta_{vac}) - \sqrt{2} G_F n_{eff},}$$

$$\boxed{\Omega = 0}$$

← resonance in neutrino spin-flavour oscillations

Outline

- “Quantum approach” to neutrino motion in matter
    - **Modified Dirac equation** for neutrino in background matter
- Exact solution of **modified Dirac equation** in matter
    - Neutrino **wave function** and **energy spectrum** in matter
- Quantum theory of ***neutrino spin light*** in matter
    - Transition **rate,** radiation **power,** photon’s **energy** and **polarization**

Standard model electroweak interaction of a flavour neutrino in matter (*f* = *e*)

Interaction Lagrangian (it is supposed that **matter contains only electrons**)

$$L_{int} = -\frac{g}{4\cos\theta_W}\left[\bar{\nu}_e\gamma^{\mu}(1+\gamma_5)\nu_e - \bar{e}\gamma^{\mu}(1-4\sin^2\theta_W+\gamma_5)e\right]Z_{\mu}$$

$$-\frac{g}{2\sqrt{2}}\bar{\nu}_e\gamma^{\mu}(1+\gamma_5)eW_{\mu}^{+} - \frac{g}{2\sqrt{2}}\bar{e}\gamma^{\mu}(1+\gamma_5)\nu_e W_{\mu}^{-}$$

→ **Charged current** interactions contribution to neutrino potential in matter

$$\Delta L_{eff}^{CC} = \sqrt{2}\,G_F\left\langle\bar{e}\gamma^{\mu}(1+\gamma_5)e\right\rangle\left(\bar{\nu}_e\gamma^{\mu}\frac{1+\gamma_5}{2}\nu_e\right)$$

→ **Neutral current** interactions contribution to neutrino potential in matter

$$\Delta L_{eff}^{NC} = -\frac{G_F}{\sqrt{2}}\left\langle\bar{e}\gamma^{\mu}[(1-4\sin\theta_W^2)+\gamma_5]e\right\rangle\left(\bar{\nu}_e\gamma^{\mu}\frac{1+\gamma_5}{2}\nu_e\right)$$

Matter current and polarization

When the **electron field bilinear**

$$\left\langle \bar{e}\gamma^{\mu}(1+\gamma_5)e \right\rangle$$

is **averaged** over the background

$$\langle \bar{e}\gamma_0 e\rangle \sim density\,,$$

$$\langle \bar{e}\gamma_i e\rangle \sim velocity\,, \qquad i = 1, 2, 3$$

$$\langle \bar{e}\gamma_\mu\gamma_5 e\rangle \sim spin\,,$$

it can be replaced by the **matter** (electrons) **current**

☆ $$\boxed{j^{\mu} = (n, n\mathbf{v}),}$$

invariant number density

speed of matter

and **polarization**

☆ $$\boxed{\lambda^{\mu} = \left(n(\boldsymbol{\zeta}\mathbf{v}), n\boldsymbol{\zeta}\sqrt{1-v^2} + \frac{n\mathbf{v}(\boldsymbol{\zeta}\mathbf{v})}{1+\sqrt{1-v^2}}\right)}$$

Modified Dirac equation for neutrino in matter

Addition to the vacuum neutrino Lagrangian

$$\Delta L_{eff} = \Delta L_{eff}^{CC} + \Delta L_{eff}^{NC} = -f^{\mu}\left(\bar{\nu}\gamma_{\mu}\frac{1+\gamma^{5}}{2}\nu\right)$$

matter current

where $$f^{\mu} = \frac{G_F}{\sqrt{2}}\left((1+4\sin^2\theta_W)j^{\mu} - \lambda^{\mu}\right)$$

matter polarization

$$\boxed{\left\{i\gamma_{\mu}\partial^{\mu} - \frac{1}{2}\gamma_{\mu}(1+\gamma_5)f^{\mu} - m\right\}\Psi(x) = 0}$$

It is suppose that there is a macroscopic amount of electrons in the scale of a neutrino de Broglie wave length. Therefore, **the interaction of a neutrino with the matter (electrons) is coherent.**

L.Chang, R.Zia,'88; J.Panteleone,'91; K.Kiers, N.Weiss, M.Tytgat,'97-'98; P.Manheim,'88; D.Nötzold, G.Raffelt,'88; J.Nieves,'89; W.Naxton, W-M.Zhang'91; M.Kachelriess,'98; A.Kusenko, M.Postma,'02;

A.Studenikin, A.Ternov, '04

This is the most general equation of motion of a neutrino in which the effective potential accounts for both the **charged** and **neutral-current** interactions with the background matter and also for the possible effects of the matter **motion** and **polarization**.

Neutrino wave function and energy spectrum in matter (I)

In the **rest frame** of **unpolarized matter**

$$\boxed{\left\{i\gamma_\mu\partial^\mu - \frac{1}{2}\gamma_\mu(1+\gamma_5)f^\mu - m\right\}\Psi(x) = 0}$$

$$f^\mu = \frac{1}{2\sqrt{2}}\tilde{G}_F(n,0,0,0), \quad \tilde{G}_F = G_F(1+4\sin^2\theta_W)$$

The Hamiltonian form of the equation:

$$\boxed{i\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \hat{H}_{matt}\Psi(\mathbf{r},t)},$$

where

$$\boxed{\hat{H}_{matt} = \hat{\boldsymbol{\alpha}}\mathbf{p} + \hat{\beta}m + \hat{V}_{matt},}$$

$$\boxed{\hat{V}_{matt} = \frac{1}{2\sqrt{2}}(1+\gamma_5)\tilde{G}_F n}$$

number density of background matter (electrons)

$$\hat{\boldsymbol{\alpha}} = \begin{pmatrix} 0 & \hat{\boldsymbol{\sigma}} \\ \hat{\boldsymbol{\sigma}} & 0 \end{pmatrix} = \gamma_0\boldsymbol{\gamma}, \quad \hat{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \gamma_0,$$

$$\tilde{G}_F = G_F(1+4\sin^2\theta_W)$$

The form of the Hamiltonian implies that the **operators of the momentum,** $\hat{\mathbf{p}}$, and **longitudinal polarization,** $\hat{\Sigma}\mathbf{p}/p$, are the **integrals of motion:**

$$\boxed{\frac{\hat{\Sigma}\mathbf{p}}{p}\Psi(\mathbf{r},t) = s\Psi(\mathbf{r},t),} \quad \hat{\Sigma} = \begin{pmatrix} \hat{\boldsymbol{\sigma}} & 0 \\ 0 & \hat{\boldsymbol{\sigma}} \end{pmatrix}, \quad \boxed{s = \pm 1}$$

positive-helicity

negative-helicity

In the relativistic limit the **negative-helicity** neutrino state is dominated by the **left-handed chiral** state: $\nu_- \approx \nu_L$, $\nu_+ \approx \nu_R$.

Stationary states

$$\Psi(\mathbf{r},t) = e^{-i(E_\varepsilon t - \mathbf{pr})} u(\mathbf{p}, E_\varepsilon),$$

neutrino wave function in matter

and

$$E_\varepsilon = \varepsilon\sqrt{\mathbf{p}^2\left(1 - s\alpha\frac{m}{p}\right)^2 + m^2} + \alpha m$$

neutrino energy spectrum in matter

$s = \pm 1$ for two **helicity** states ,

where the **matter density parameter** $\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m}$,

J.Panteleone, 1991
(if NC interaction were left out)

$\frac{1}{2\sqrt{2}}\tilde{G}_F n \sim 1\ eV$ for $n = 10^{37}\ cm^{-3}$ ← density of matter in a neutron star

Neutrino energy in the background matter depends on the state of the neutrino longitudinal polarization (helicity), i.e. in the relativistic case the left-handed and right-handed neutrinos with equal momenta have different energies.

Neutrino wave function in matter (II)

$$\boxed{\Psi_{\varepsilon,\mathbf{p},s}(\mathbf{r},t)=\frac{e^{-i(E_\varepsilon t-\mathbf{pr})}}{2L^{\frac{3}{2}}}\begin{pmatrix}\sqrt{1+\frac{m}{E_\varepsilon-\alpha m}}\ \sqrt{1+s\frac{p_3}{p}}\\ s\sqrt{1+\frac{m}{E_\varepsilon-\alpha m}}\ \sqrt{1-s\frac{p_3}{p}}\ \ e^{i\delta}\\ s\varepsilon\sqrt{1-\frac{m}{E_\varepsilon-\alpha m}}\ \sqrt{1+s\frac{p_3}{p}}\\ \varepsilon\sqrt{1-\frac{m}{E_\varepsilon-\alpha m}}\ \ \sqrt{1-s\frac{p_3}{p}}\ e^{i\delta}\end{pmatrix},}$$

$$\boxed{E_\varepsilon-\alpha m=\varepsilon\sqrt{\mathbf{p}^2\Big(1-s\alpha\frac{m}{p}\Big)^2+m^2}}$$

$$\delta\ =\ \mathrm{arctan}\,(p_2/p_1)$$

The quantity $\boxed{\varepsilon\ =\ \pm 1}$ splits the solutions into the two branches that in the limit of **vanishing matter density,** $\boxed{\alpha\ \longrightarrow\ 0,}$ reproduce the **positive** and **negative-frequency** solutions, respectively.

Modified Dirac equation for matter composed of electrons, protons and neutrons (I)

The generalizations of the modified Dirac equation for more complicated matter compositions and the other flavour neutrinos are just straightforward.

For matter composed of **electrons** , **protons** and **neutrons** :

$$f^{\mu} = \frac{G_F}{\sqrt{2}} \sum_{f=e,p,n} j_f^{\mu} q_f^{(1)} + \lambda_f^{\mu} q_f^{(2)}$$

where

$$q_f^{(1)} =(I_{3L}^{(f)} - 2Q^{(f)} \sin^2 \theta_W + \delta_{ef}), \quad q_f^{(2)} = -(I_{3L}^{(f)} + \delta_{ef}), \quad \delta_{ef} = \begin{cases} 1 & \text{for} f = e, \\ 0 & \text{for} f = n, p \end{cases}$$

isospin third component

electric charge of a fermion f

current

polarization

$$\left\{ i\gamma_{\mu}\partial^{\mu} - \frac{1}{2}\gamma_{\mu}(1+\gamma_5) f^{\mu} - m \right\} \Psi(x) = 0$$

An important note (I)

The modified Dirac equation
for a neutrino in the background matter
(and the obtained exact solution and energy spectrum)
establish a basis for an effective method
in investigations of different phenomena
that can appear when neutrinos are moving in media.

similar to the Furry representation of quantum electrodynamics

Neutrino and antineutrino energy spectra in matter

For the fixed magnitude of the neutrino momentum $\mathbf{P}$ there are the two values for the "positive sign" $\varepsilon = +1$ energies

$$E^{s=+1} = \sqrt{\mathbf{p}^2\Big(1 - \alpha\frac{m}{p}\Big)^2 + m^2} + \alpha m,$$

positive-helicity neutrino energy

$$E^{s=-1} = \sqrt{\mathbf{p}^2\Big(1 + \alpha\frac{m}{p}\Big)^2 + m^2} + \alpha m$$

negative-helicity neutrino energy

particle (neutrino) energies in matter

The two other values of the energy for the "negative sign" $\varepsilon = -1$ correspond to the antiparticle solutions. By changing the sign of the energy, we obtain the values

$$\tilde{E}^{s=+1} = \sqrt{\mathbf{p}^2\Big(1 - \alpha\frac{m}{p}\Big)^2 + m^2} - \alpha m,$$

positive-helicity antineutrino energy

$$\tilde{E}^{s=-1} = \sqrt{\mathbf{p}^2\Big(1 + \alpha\frac{m}{p}\Big)^2 + m^2} - \alpha m$$

negative-helicity antineutrino energy

antiparticle (antineutrino) energies in matter

Neutrino processes in matter

- **Neutrino reflection from interface between vacuum and matter**
- **Neutrino trapping in matter**
- **Neutrino-antineutrino pair annihilation at interface between vacuum and matter**
- **Spontaneous neutrino-antineutrino pair creation in matter**

L.Chang, R.Zia,’88
A.Loeb,’90
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A.Kusenko, M.Postma,’02
H.Koers,’04
A.Studenikin, A.Ternov,’04

Neutrino reflection from interface between vacuum and matter

$1 < \alpha_1 < 2$

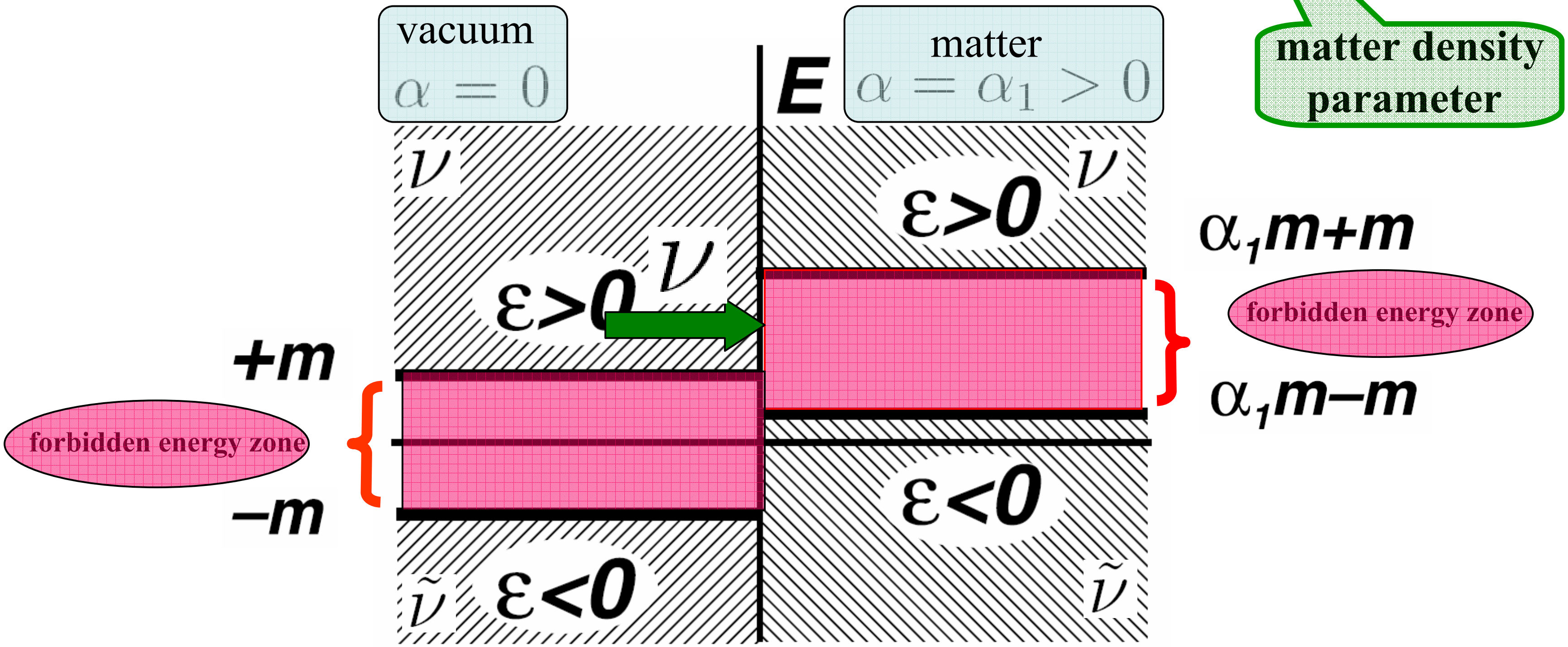
If the neutrino energy in **vacuum** E

is less than the neutrino minimal energy in **medium** $\alpha_1 m + m$

$$\boxed{m \leq E < \alpha_1 m + m}$$

$$\boxed{\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m}}$$

matter density parameter



then the appropriate energy level inside the medium is not accessible for neutrino

neutrino **is reflected** from the interface.

Neutrino trapping in matter

$1 < \alpha_1 < 2$

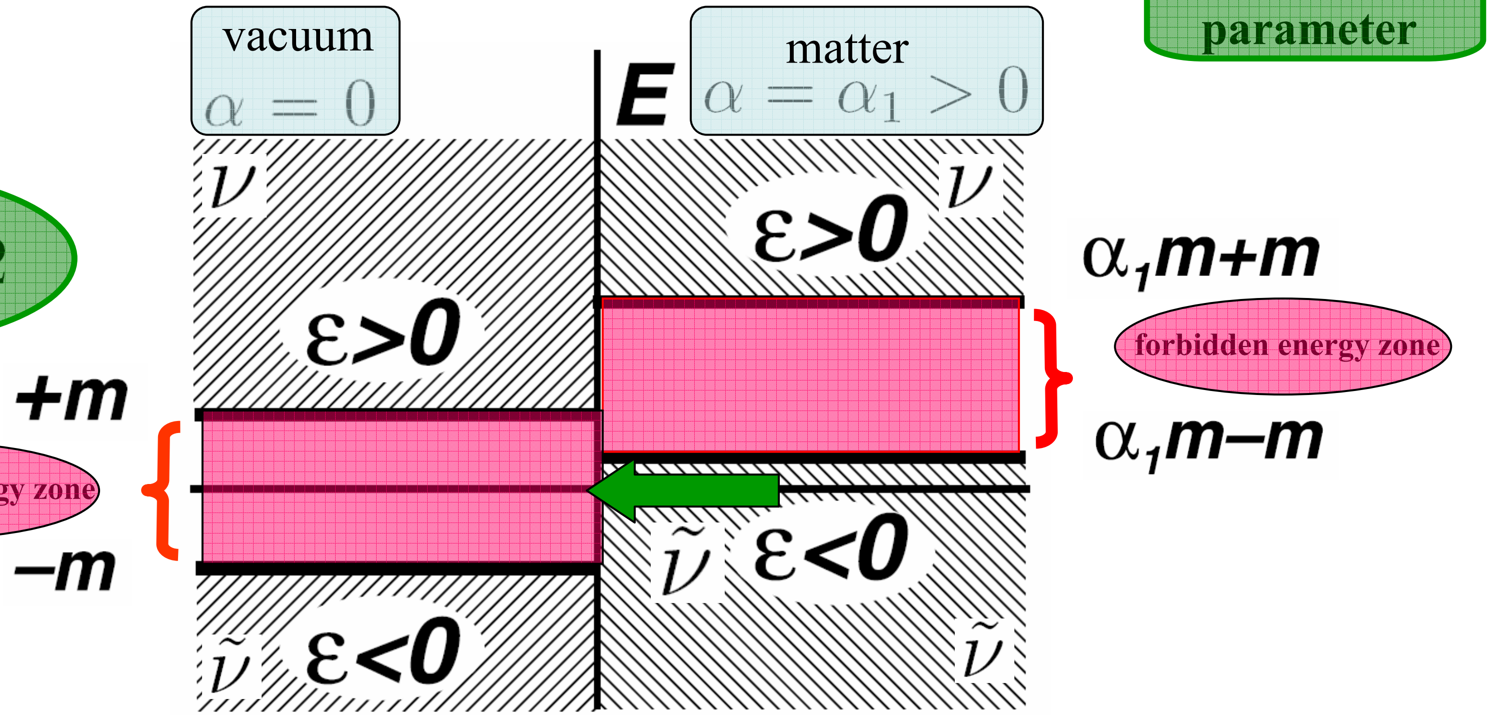
Antineutrino in **medium** with energy $\boxed{|\alpha_1 m - m| \le E < m}$

$$\boxed{\alpha = \frac{1}{2\sqrt{2}} \tilde{G}_F \frac{n}{m}}$$

matter density parameter

can not escape from the medium because this particular range of energies exactly falls on the forbidden energy zone in **vacuum** :

$1 < \alpha_1 < 2$



Antineutrino has not enough energy
to survive in **vacuum**

→ it is **trapped** inside the medium.

Neutrino-antineutrino pair annihilation at interface between vacuum and matter

Consider a **neutrino** with energy $\boxed{m < E \le \alpha_2 m - m}$

$\alpha_2 \ge 2$

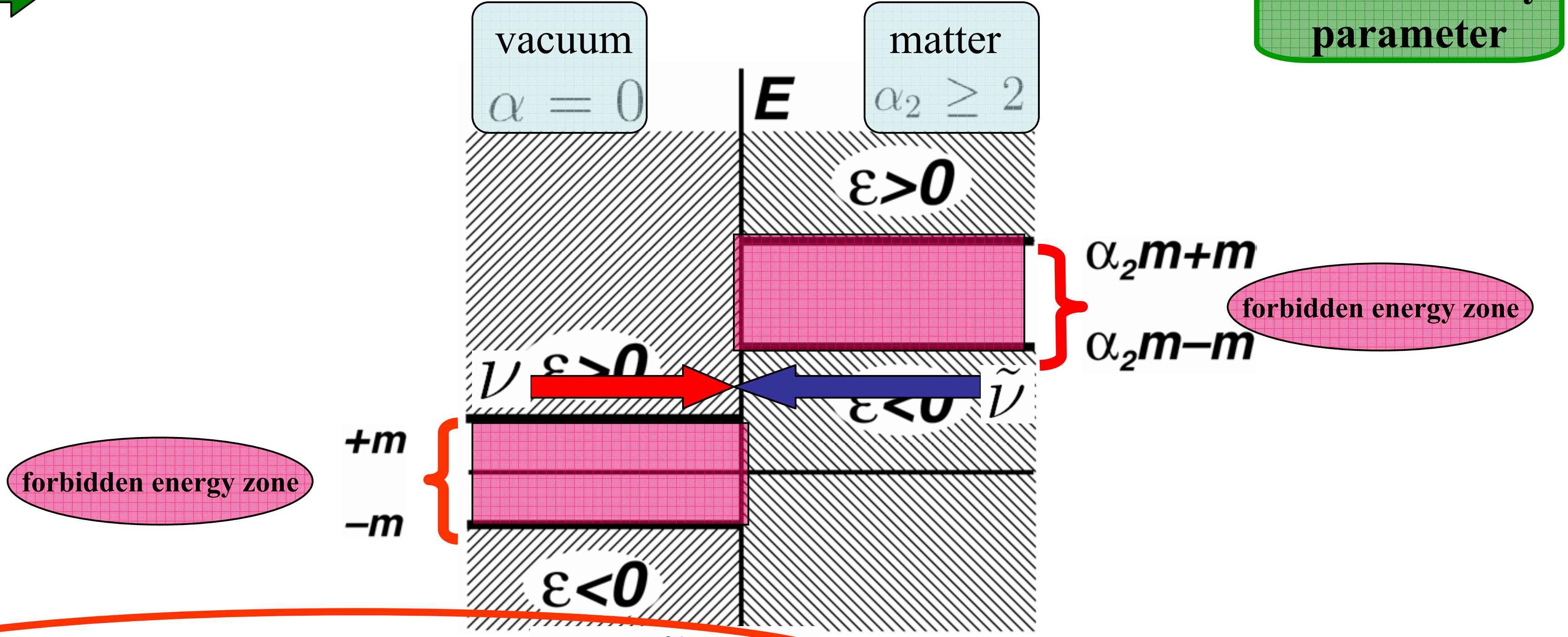
$$\boxed{\alpha_2 = \frac{1}{2\sqrt{2}}\tilde{G}_F \frac{n}{m}}$$

matter density parameter

propagating in vacuum towards the interface with matter.

If not all of “negative sign” energy levels are occupied and, in particular, the level with energy exactly equal to E

➡ an **antineutrino** exist in matter :



neutrino-**antineutrino** annihilation $\nu + \tilde{\nu} = \gamma$ at the interface of vacuum and matter.

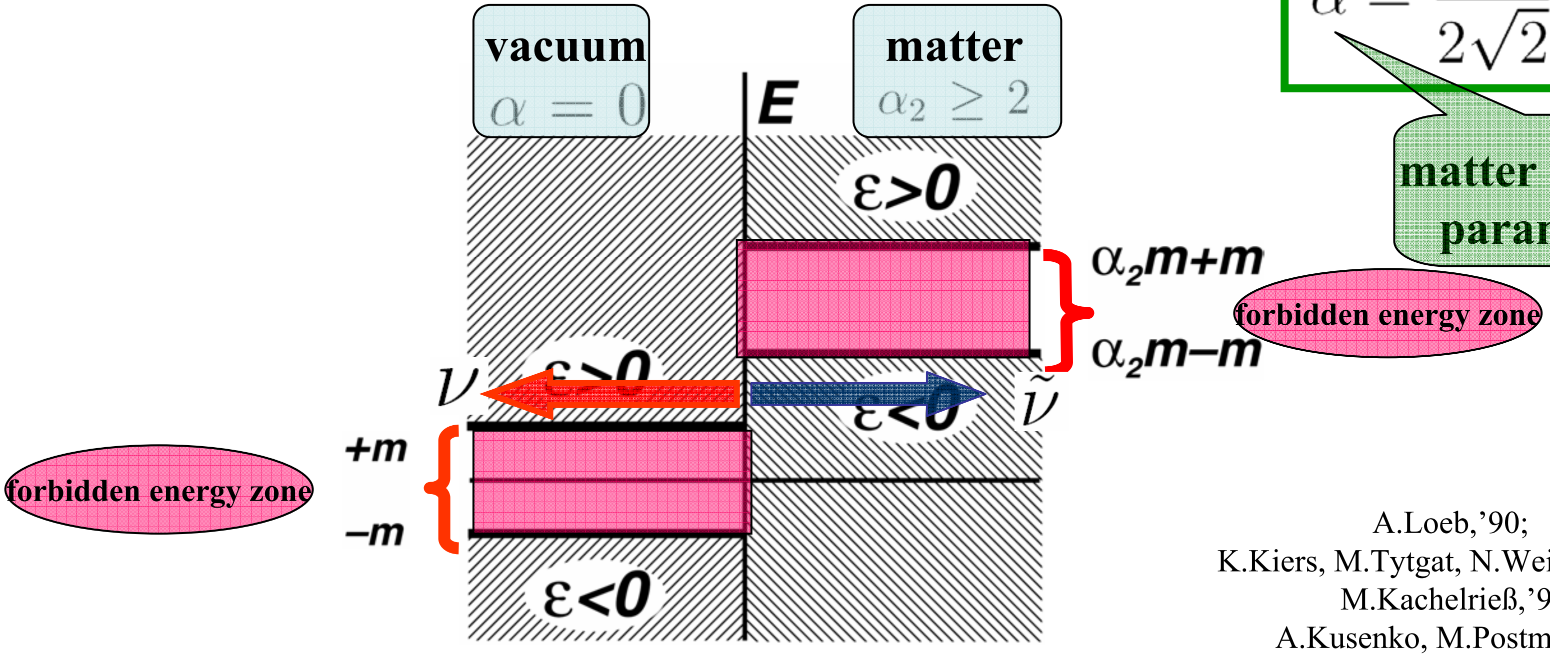
Spontaneous neutrino-antineutrino pair creation in matter

$\alpha_2 \geq 2$

"Negative sign" energy levels in matter have their counterparts in **"positive sign"** energy levels in vacuum:

$$\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F \frac{n}{m}$$

matter density parameter



A.Loeb,'90;
K.Kiers, M.Tytgat, N.Weiss,'97-'98;
M.Kachelrieß,'98;
A.Kusenko, M.Postma,'02;
H.Koers,'04

Neutrino-antineutrino pair creation can be interpreted as a process of appearance of particle state of in the **"positive sign"** energy range accompanied by appearance of the hole state in the **"negative sign"** energy sea.

⟺

Spontaneous electron-positron pair creation according to Klein's paradox of electrodynamics.

Spin Light of Neutrino in matter

Quantum theory of $SL\nu$

A.Studenikin, A.Ternov, *Phys.Lett.B* **608** (2005) 107;

A.Grigoriev, A.Studenikin, A.Ternov, *Grav. & Cosm.* (2005), in press;

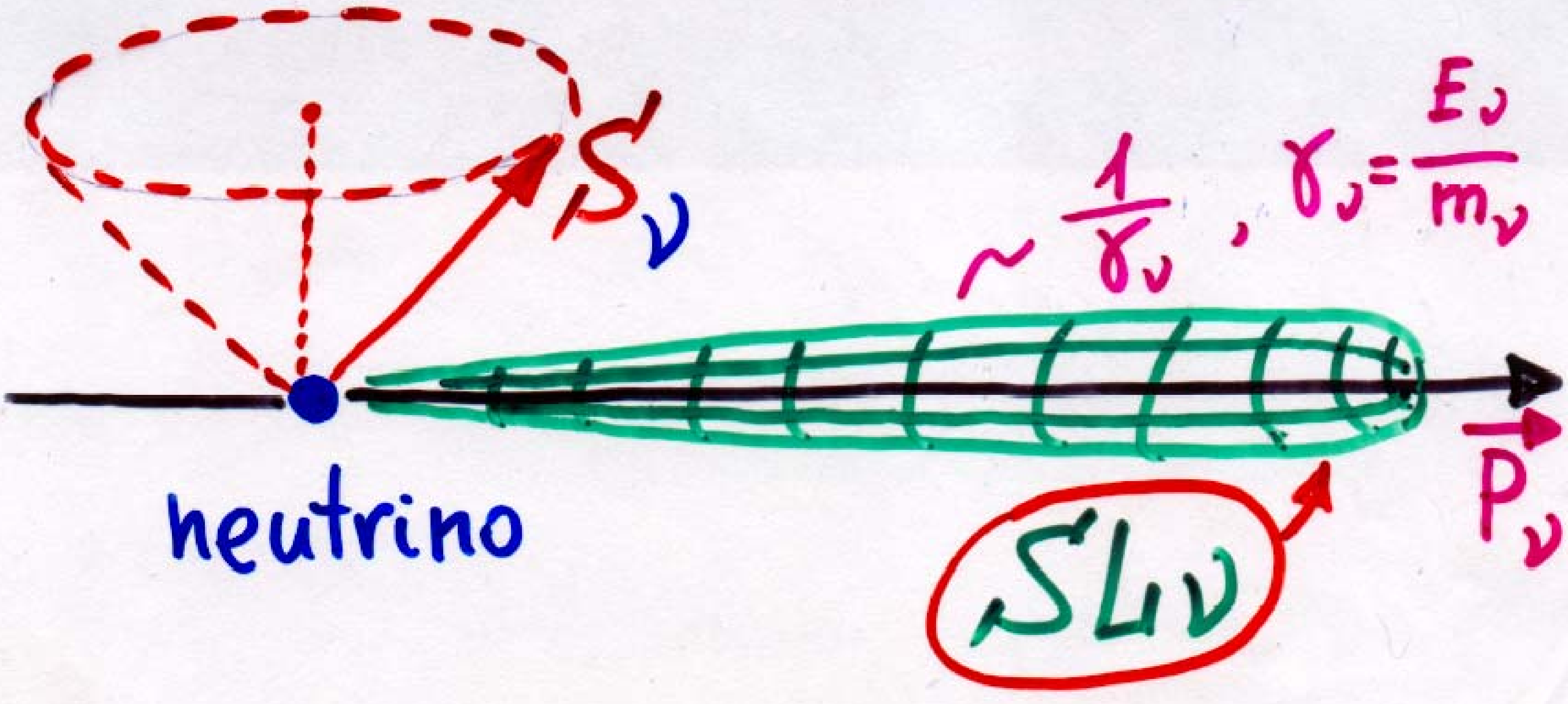
A.Grigoriev, A.Studenikin, A.Ternov, hep-ph/0502210, hep-ph/0502231;

A.Studenikin, A.Ternov, hep-ph/0410296, hep-ph/0410297.

Quasi-classical theory of spin light of neutrino in matter

A.Lobanov, A.Studenikin, '03-'04

Neutrino spin procession in background environment

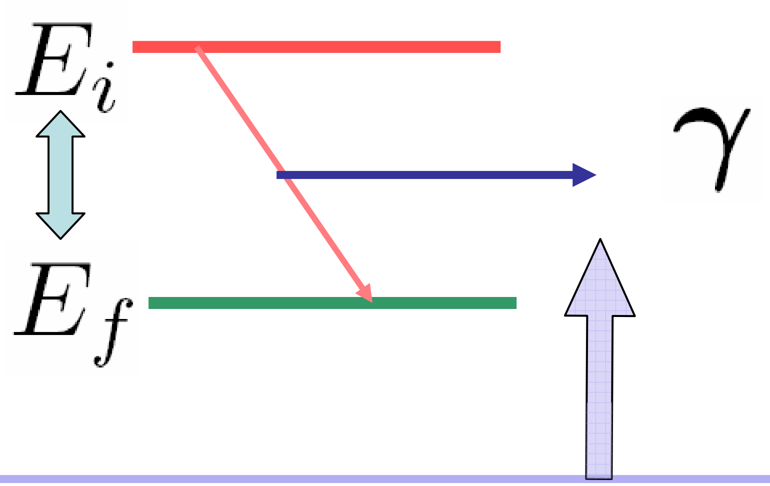


Quantum theory of spin light of neutrino (I)

Quantum treatment of *spin light of neutrino* in matter
showns that this process originates from the **two subdivided phenomena:**

$SL\nu$

the shift of the neutrino energy levels in the presence of the background matter, which is different for the two opposite neutrino helicity states,



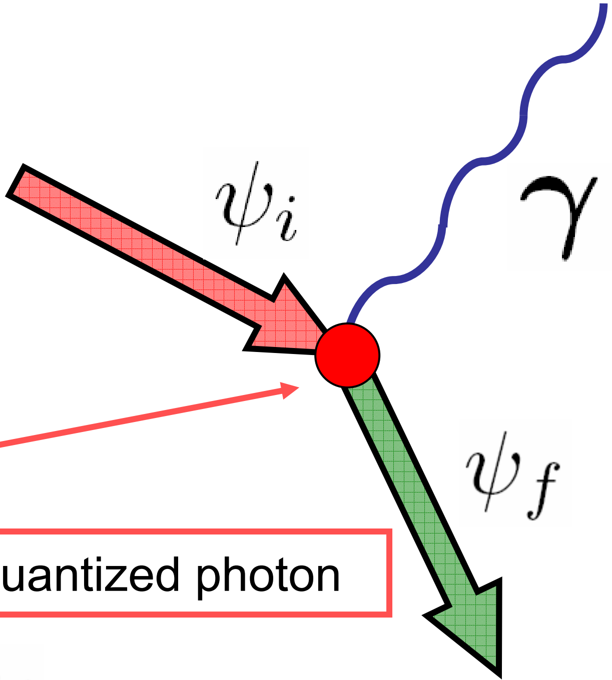
$$E = \sqrt{\mathbf{p}^2\left(1 - s\alpha\frac{m}{p}\right)^2 + m^2} + \alpha m$$

$$s = \pm 1$$

the radiation of the photon in the process of the neutrino transition from the **"exited" helicity state** to the **low-lying helicity state** in matter

neutrino-spin self-polarization effect in the matter

Quantum theory of spin light of neutrino $SL\nu$



Within the **quantum approach**, the corresponding Feynman diagram is the one-photon emission diagram with the initial and final neutrino states described by the **"broad lines"** that account for the neutrino interaction with matter.

Neutrino magnetic moment interaction with quantized photon

the amplitude of the transition $\psi_i \longrightarrow \psi_f$

$$S_{fi} = -\mu\sqrt{4\pi}\int d^4x\,\bar{\psi}_f(x)(\hat{\mathbf{\Gamma}}\mathbf{e}^*)\frac{e^{ikx}}{\sqrt{2\omega L^3}}\psi_i(x)\ ,$$

$$\hat{\mathbf{\Gamma}} = i\omega\{[\mathbf{\Sigma}\times\boldsymbol{\varkappa}] + i\gamma^5\mathbf{\Sigma}\}\ ,$$

$k^\mu = (\omega, \mathbf{k}), \boldsymbol{\varkappa} = \mathbf{k}/\omega$ momentum

$\mathbf{e}^*$ polarization of photon

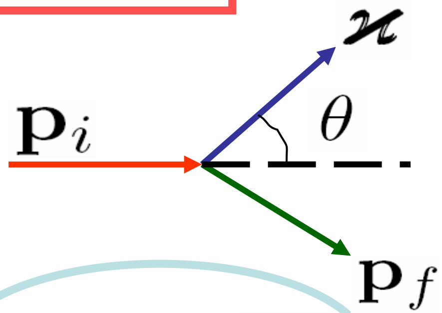
Spin light of neutrino photon's energy

$SL\boldsymbol{\nu}$ transition amplitude after integration :

$$S_{fi} = -\mu\sqrt{\frac{2\pi}{\omega L^3}}\,2\pi\delta(E_f - E_i + \omega)\int d^3x\bar{\psi}_f(\mathbf{r})(\hat{\mathbf{\Gamma}}\mathbf{e}^*)e^{i\mathbf{kr}}\psi_i(\mathbf{r})$$

Energy-momentum conservation

$$E_i = E_f + \omega, \qquad \mathbf{p}_i = \mathbf{p}_f + \boldsymbol{\varkappa}$$



For **electron neutrino** moving in matter composed of **electrons**

$$\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m} > 0$$

$$\omega = \frac{2\alpha m p_i\left[(E_i - \alpha m) - (p_i + \alpha m)\cos\theta\right]}{(E_i - \alpha m - p_i\cos\theta)^2 - (\alpha m)^2}$$

photon's energy

- In the radiation process: $s_i = -1 \Longrightarrow s_f = +1$ **neutrino self-polarization**
- For not very high densities of matter , $\tilde{G}_F n/m \ll 1$, in the linear approximation over α

$$\omega = \frac{\beta}{1 - \beta\cos\theta}\omega_0 \quad , \qquad \omega_0 = \frac{\tilde{G}_F}{\sqrt{2}}n\beta$$

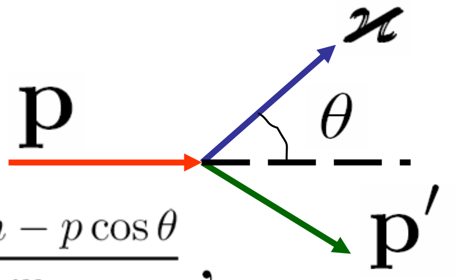
neutrino speed in vacuum

Spin light transition rate (I)

The matter density parameter $\alpha = \frac{1}{2\sqrt{2}} \tilde{G}_F \frac{n}{m}$ is accounted for exactly:

$$\boxed{\Gamma = \mu^2 \int_0^\pi \omega^3 \big[(\tilde{\beta}\tilde{\beta}' + 1)(1 - y\cos\theta) - (\tilde{\beta} + \tilde{\beta}')(\cos\theta - y)\big] \frac{\sin\theta}{1 + \tilde{\beta}' y} d\theta} \; ,$$

θ is the angle between the **initial neutrino** and **photon** momenta:



$$\tilde{\beta} = \frac{p + \alpha m}{E - \alpha m} \; , \quad \tilde{\beta}' = \frac{p' - \alpha m}{E' - \alpha m} \; , \quad y = \frac{\omega - p\cos\theta}{p'}, \quad K = \frac{E - \alpha m - p\cos\theta}{\alpha m} \; ,$$

$E' = E - \omega$ energy of

$p' = K\omega - p$ **final neutrino** momentum

energy of **initial neutrino** $E = \sqrt{\mathbf{p}^2 \left(1 - s\alpha \frac{m}{p}\right)^2 + m^2} + \alpha m$

Non-trivial dependence on the matter density parameter α :

$$\alpha = \frac{1}{2\sqrt{2}} \tilde{G}_F \frac{n}{m}$$

$$\boxed{\Gamma \simeq \frac{64}{3} \frac{\mu^2 \alpha^3 p^3 m}{E_0}}$$

for low densities $\alpha \ll 1,$

$E_0 = \sqrt{p^2 + m^2}$

$$\frac{1}{2\sqrt{2}} \tilde{G}_F n \sim 1 \; eV$$

$$n = 10^{37} \;\; cm^{-3}.$$

Spin light transition rate (II)

Performing the integration over the photon's angle θ

one obtains for the **spin light of neutrino** rate in **matter** :

$$\begin{aligned}
\Gamma = & \frac{1}{2\left(E-p\right)^{2}\left(E+p-2\alpha m\right)^{2}\left(E-\alpha m\right)p^{2}} \\
& \times \Big\{\left(E^{2}-p^{2}\right)^{2}\left(p^{2}-6\alpha^{2}m^{2}+\right. \\
& \left. 6E\alpha m-3E^{2}\right)\left(\left(E-2\alpha m\right)^{2}-p^{2}\right)^{2} \\
& \times \ln\left[\frac{\left(E+p\right)\left(E-p-2\alpha m\right)}{\left(E-p\right)\left(E+p-2\alpha m\right)}\right] \\
& + 4\alpha m p\left[16\alpha^{5}m^{5}E\left(3E^{2}-5p^{2}\right)\right. \\
& - 8\alpha^{4}m^{4}\left(15E^{4}-24E^{2}p^{2}+p^{4}\right) \\
& + 4\alpha^{3}m^{3}E\left(33E^{4}-58E^{2}p^{2}+17p^{4}\right) \\
& - 2\alpha^{2}m^{2}\left(39E^{2}-p^{2}\right)\left(E^{2}-p^{2}\right)^{2} \\
& + 12\alpha m E\left(2E^{2}-p^{2}\right)\left(E^{2}-p^{2}\right)^{2} \\
& \left. - \ \left(3E^{2}-p^{2}\right)\left(E^{2}-p^{2}\right)^{3}\right]\Big\},
\end{aligned}$$

where the matter density parameter

$$\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_{F}\frac{n}{m} > 0 \quad ,$$

and the initial neutrino energy

$$\boxed{E = \sqrt{\mathbf{p}^{2}\left(1 - s\alpha\frac{m}{p}\right)^{2} + m^{2}} + \alpha m}$$

$$s_{i} = -1 \ .$$

Spin light transition rate (III)

$SL\nu$

transition rate for different neutrino momentum p and **matter density parameter** $\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m} > 0$

"relativistic" case

$p \gg m$

$$\Gamma = \begin{cases} \frac{64}{3}\mu^2\alpha^3p^2m, & \text{for } \alpha \ll \frac{m}{p}, \\ 4\mu^2\alpha^2m^2p, & \text{for } \frac{m}{p} \ll \alpha \ll \frac{p}{m}, \\ 4\mu^2\alpha^3m^3, & \text{for } \alpha \gg \frac{p}{m}. \end{cases}$$

"non-relativistic" case

$p \ll m$

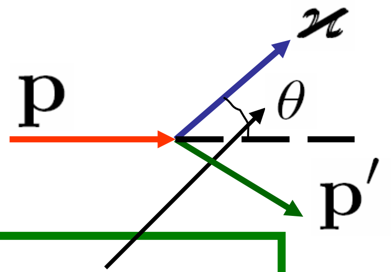
neutrino momentum

mass

$$\Gamma = \begin{cases} \frac{64}{3}\mu^2\alpha^3p^3, & \text{for } \alpha \ll 1, \\ \frac{512}{5}\mu^2\alpha^6p^3, & \text{for } 1 \ll \alpha \ll \frac{m}{p}, \\ 4\mu^2\alpha^3m^3, & \text{for } \alpha \gg \frac{m}{p}. \end{cases}$$

neutrino magnetic moment

Spin light radiation power



$SL\nu$

radiation power angular distribution :

$$I = \mu^2 \int_0^{\pi} \omega^4 \left[(\tilde{\beta}\tilde{\beta}' + 1)(1 - y\cos\theta) - (\tilde{\beta} + \tilde{\beta}')(\cos\theta - y) \right] \frac{\sin\theta}{1 + \tilde{\beta}' y} d\theta$$

$$\tilde{\beta} = \frac{p + \alpha m}{E - \alpha m}, \quad \tilde{\beta}' = \frac{p' - \alpha m}{E' - \alpha m}, \quad y = \frac{\omega - p\cos\theta}{p'}, \quad K = \frac{E - \alpha m - p\cos\theta}{\alpha m}, \quad \omega = \frac{2\alpha m p\left[(E - \alpha m) - (p + \alpha m)\cos\theta\right]}{(E - \alpha m - p\cos\theta)^2 - (\alpha m)^2}$$

“relativistic” case

$$\boxed{p \gg m}$$

$$I = \begin{cases} \frac{128}{3}\mu^2\alpha^4 p^4, & \text{for } \alpha \ll \frac{m}{p}, \\ \frac{4}{3}\mu^2\alpha^2 m^2 p^2, & \text{for } \frac{m}{p} \ll \alpha \ll \frac{p}{m}, \\ 4\mu^2\alpha^4 m^4, & \text{for } \alpha \gg \frac{p}{m}. \end{cases}$$

“non-relativistic” case

$$\boxed{p \ll m}$$

$$I = \begin{cases} \frac{128}{3}\mu^2\alpha^4 p^4, & \text{for } \alpha \ll 1, \\ \frac{1024}{3}\mu^2\alpha^8 p^4, & \text{for } 1 \ll \alpha \ll \frac{m}{p}, \\ 4\mu^2\alpha^4 m^4, & \text{for } \alpha \gg \frac{m}{p}. \end{cases}$$

Spin light photon's average energy

$$\langle\omega\rangle = \frac{\text{radiation power}}{\text{transition rate}} = \frac{I}{\Gamma}$$

"relativistic" case

$\boxed{p \gg m}$

$$\langle\omega\rangle \simeq \begin{cases} 2\alpha\frac{p^2}{m}, & \text{for } \alpha \ll \frac{m}{p}, \\ \frac{1}{3}p, & \text{for } \frac{m}{p} \ll \alpha \ll \frac{p}{m}, \\ \alpha m, & \text{for } \alpha \gg \frac{p}{m}. \end{cases}$$

"non-relativistic" case

$\boxed{p \ll m}$

$$\langle\omega\rangle \simeq \begin{cases} 2p\alpha, & \text{for } \alpha \ll 1, \\ \frac{10}{3}p\alpha^2, & \text{for } 1 \ll \alpha \ll \frac{m}{p}, \\ \alpha m, & \text{for } \alpha \gg \frac{m}{p}. \end{cases}$$

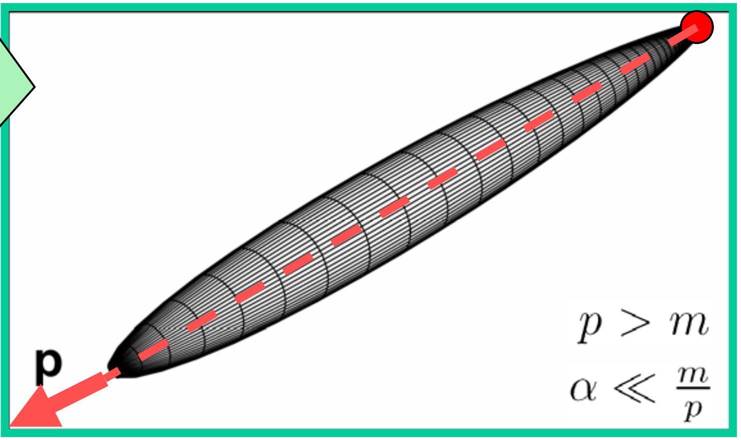
$\alpha \ll \frac{m}{p}$

$\omega = 2.37 \times 10^{-7} \left(\frac{n}{10^{30} cm^{-3}}\right)\left(\frac{E}{m_\nu}\right)^2 eV$ ⇦ energy range of $\boxed{SL\nu}$ span up to gamma-rays

Spatial distribution of radiation power

From the **angular distribution** of $\boxed{SL\nu}$

$$I = \mu^2 \int_0^\pi \omega^4 \left[(\tilde\beta\tilde\beta' + 1)(1 - y\cos\theta) - (\tilde\beta + \tilde\beta')(\cos\theta - y)\right] \frac{\sin\theta}{1 + \tilde\beta' y} d\theta$$



$p > m$

$\alpha \ll \frac{m}{p}$

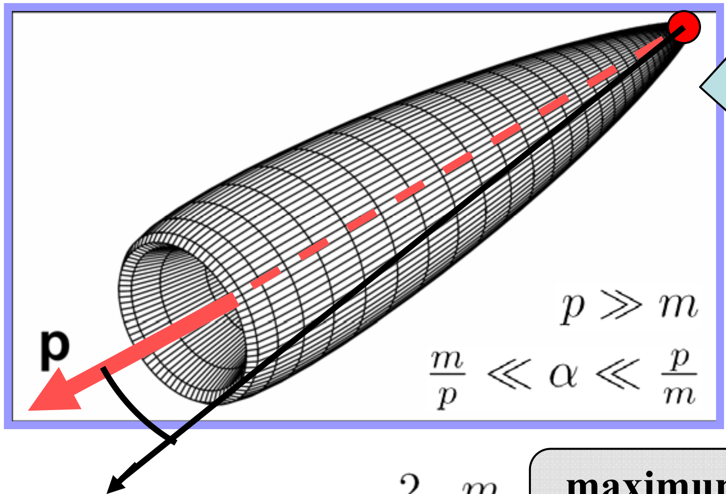
for $p/m = 5$ **and** $\alpha = 0.01$

$\boxed{n \approx 10^{35} cm^{-3}}$

neutrino momentum

mass

matter density



$p \gg m$

$\frac{m}{p} \ll \alpha \ll \frac{p}{m}$

$\cos\theta_{max} \simeq 1 - \frac{2}{3}\alpha\frac{m}{p}$

maximum in radiation power distribution

for $p/m = 10^3$ **and** $\alpha = 100$

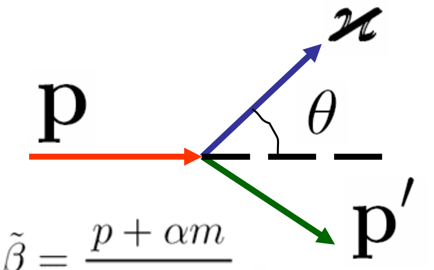
$\boxed{n \approx 10^{39} cm^{-3}}$

increase of matter density

projector-like distribution → **cap-like distribution**

Polarization properties of $SL\nu$ photons (II)

- Radiation power of **circulary polarized** photons:



$$\boxed{I^{(l)} = \mu^2 \int_0^{\pi} \frac{\omega^4}{1+\beta' y} S_l \sin\theta d\theta} \; , \quad \tilde{\beta}' = \frac{p' - \alpha m}{E' - \alpha m} \, , \; \tilde{\beta} = \frac{p + \alpha m}{E - \alpha m} \, ,$$

$$y = \frac{\omega - p\cos\theta}{p'}, \quad K = \frac{E - \alpha m - p\cos\theta}{\alpha m} \, ,$$

$$\omega = \frac{2(E-\alpha m)(K\beta - 1)}{K^2 - 1}$$

where

$$\boxed{S_l = \frac{1}{2}\left(1 + l\beta'\right)\left(1 + l\beta\right)\left(1 - l\cos\theta\right)\left(1 + ly\right)}$$

$l = \pm 1$ correspond to the photon **right** and **left circular polarizations** .

- In the limit of **low matter density** $\alpha \ll 1$:

$$E_0 = \sqrt{p^2 + m^2}$$

$$\boxed{I^{(l)} \simeq \frac{64}{3}\mu^2 \alpha^4 p^4 \left(1 - l\frac{p}{2E_0}\right)} \, , \quad I^{(+1)} > I^{(-1)} \, ,$$ however $I^{(+1)} \sim I^{(-1)}$.

- In **dense matter** $\left(\alpha \gg \frac{m}{p} \text{ for } p \gg m, \text{and } \alpha \gg 1 \text{ for } p \ll m\right)$:

$$\boxed{\begin{array}{lcr} I^{(+1)} & \simeq & I \\ I^{(-1)} & \simeq & 0 \end{array}} \longleftarrow$$

In a dense matter $SL\nu$ is right-circular polarized.

Conclusions

Quantum approach to neutrino propagation in a dense matter

- **Modified Dirac equation** for a neutrino wave function in a background

Exact solution of **modified Dirac equation** in matter

- Neutrino **wave function** and **energy spectrum** in matter

Quantum theory of ***neutrino spin light*** in matter

- Transition **rate,** radiation **power,** photon’s **energy** and **polarization**