UNIVERSITÀ DEGLI STUDI ROMA TRE

DETERMINATION OF V_{US}: RECENT PROGRESSES FROM THEORY

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OUTLINE

1. Motivations for V_{us}

The most important: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

2. K`3 decays: the best determination of V_{us}

* Experiments: several new (very important) results
* Theory: determination of $f_+(0)$
  - ChPT: the relevant LEC at $O(p^6)$ unknown
  - Quark model: Leutwyler and Roos
  - Lattice QCD: first calculation

RÉGION AUTONOME VALLÉE D'AOSTE
Assessorat de l'istruction Publique

ISTITUTO NAZIONALE DI FISICA NUCLEARE

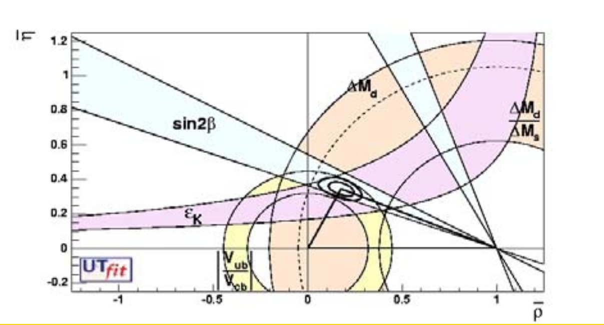
Les Rencontres de Physique de la Vallée d'Aoste

February 27- March 5, 2005

MOTIVATIONS

$V_{us}=\lambda$ is a fundamental SM parameter

The values of $V_{us}=\lambda$ and $V_{cb}=A\lambda^2$ are crucial inputs in the Unitarity Triangle Analysis



V_{ud}, V_{us} provide the most stringent test of unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$$\Delta|V_{ud}|^2 \approx \Delta|V_{us}|^2 \approx 1\cdot10^{-3} \qquad |V_{ub}|^2 \approx 10^{-5}$$

THE "FIRST ROW" UNITARITY TEST:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

Status of V_{ud}:

SFT $0^+ \rightarrow 0^+$:	$|V_{ud}| = 0.9740 \pm 0.0005$	Extremely precise, 9 experiments
n β-decay:	$|V_{ud}| = 0.9750 \pm 0.0017$	→ See next ←
π_{e3}:	$|V_{ud}| = 0.9737 \pm 0.0039$	Theor. clean, but $BR=10^{-8}$ (PIBETA at PSI)

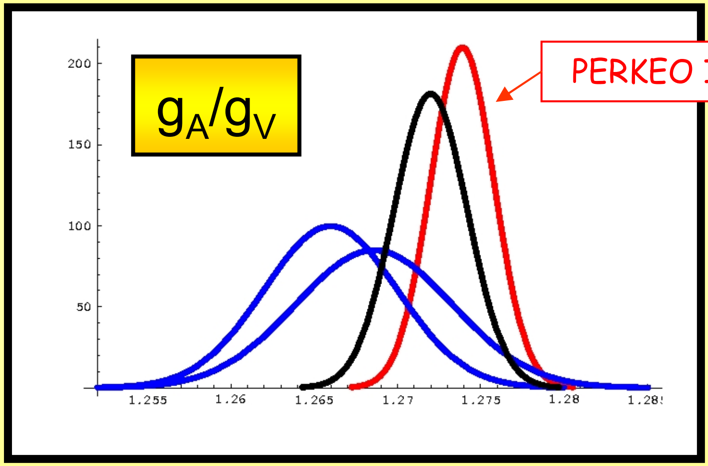
Average: $|V_{ud}| = 0.9740 \pm 0.0005$

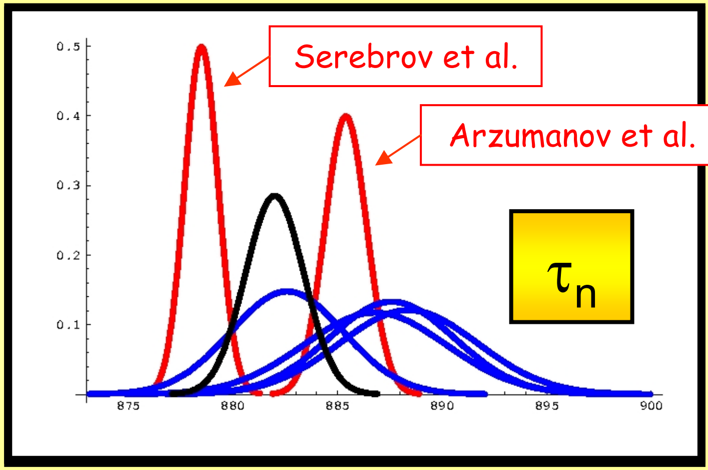
V_{ud} from neutron β-decay:

$$\frac{1}{\tau_n} = \frac{G_F^2 m_e^5}{2\pi^3} |V_{ud}|^2 \left[1+3\,(g_A/g_V)^2\right] f\,(1+RC)^2$$

$f = 1.6887$

$(1+RC) = 1.0390(8)$





g_A/g_V **= 1.2720 ± 0.0022**

τ_n **= 885.7 ± 0.8 sec** PDG 04

τ_n **= 878.5 ± 0.8 sec** Serebrov et al.

$|V_{ud}|$ = 0.9729 ± 0.0016

➡ **$|V_{ud}|$ = 0.9750 ± 0.0017**

➡ **τ_n = 882.0 ± 1.4 sec**

THE "FIRST ROW" UNITARITY TEST:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

SFT $0^+ \rightarrow 0^+$: $|V_{ud}| = 0.9740 \pm 0.0005$ Extremely precise, 9 expts

n β-decay: $|V_{ud}| = 0.9750 \pm 0.0017$ g_V/g_A, will be improved at PERKEO, Heidelb.

π_{e3}: $|V_{ud}| = 0.9737 \pm 0.0039$ Theor. clean, but BR=10^{-8} (PIBETA at PSI)

Average: $|V_{ud}| = 0.9740 \pm 0.0005$

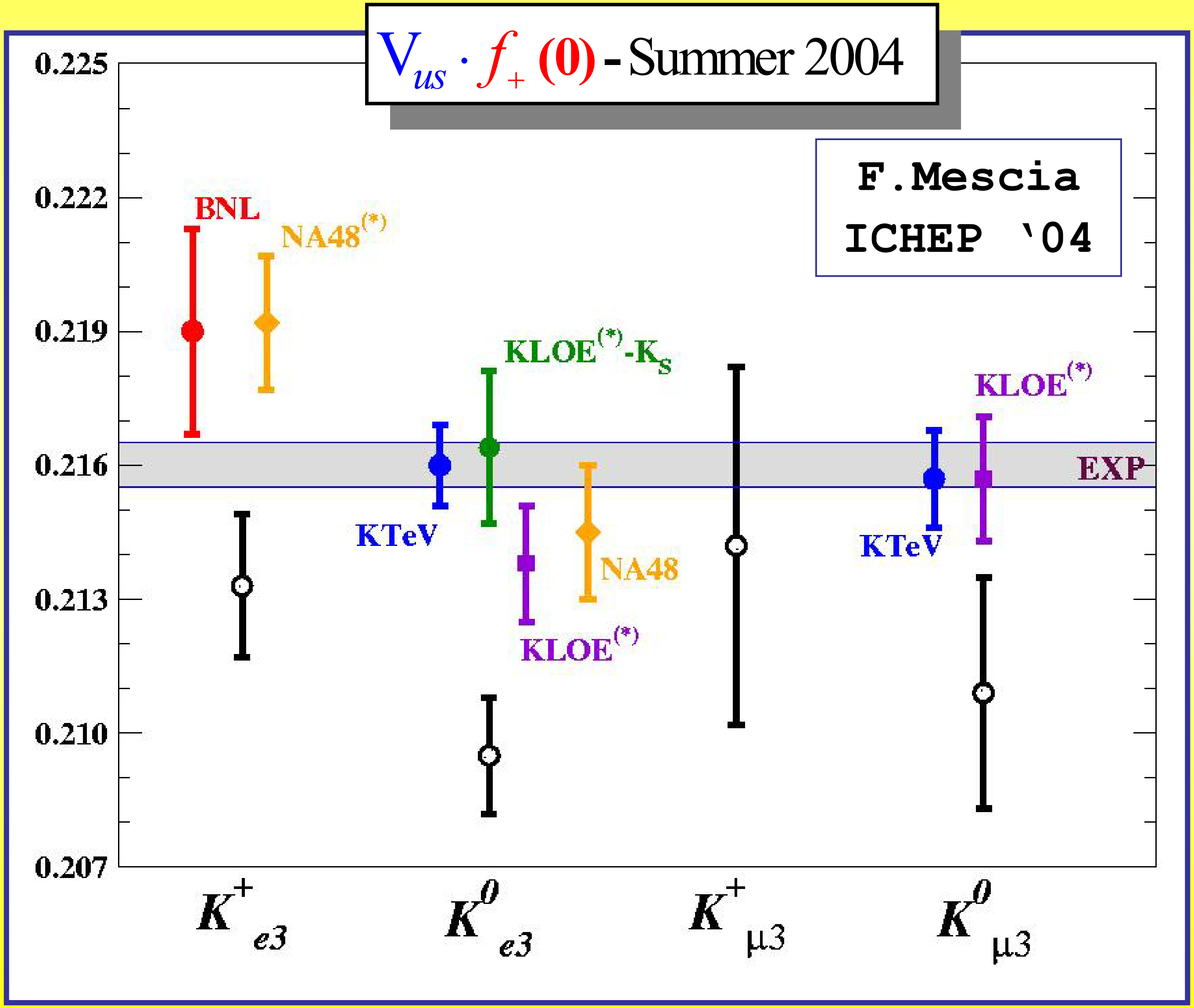
$|V_{us}|^{Unit.} = 0.2265 \pm 0.0021$

PDG 2004 (BNL-E865+old exps)

$|V_{us}|^{Kl3} = 0.2200 \pm 0.0026$

~ 2σ discrepancy

K`3: the NEW experimental results



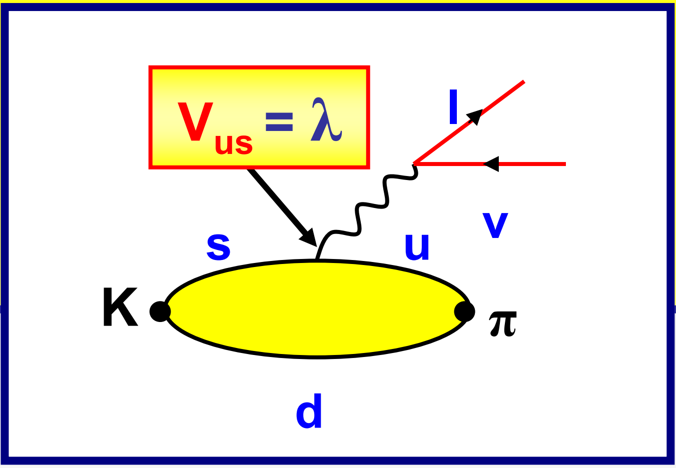
BNL-E865
PRL 91, (2003) 261802

KTeV
PRL 93, (2004) 181802

NA48
PL B602, (2004) 41

KLOE
Presented @ ICHEP 04

K`3 theory



$$\Gamma(K \to \pi e \nu(\gamma)) = \frac{G_F^2 M_K^5}{192\pi^3} \cdot$$

$$\cdot\ C_K^2 |V_{us}|^2\ |f_+^{K^0\pi^-}(0)|^2\, I_K\, S_{ew}\, (1+\delta_{SU(2)}^K + \delta_{em}^{Kl})^2$$

S_{ew}

1.0232(3)

(Sirlin 82')

	$\delta^K_{SU(2)}(\%)$	$\delta^{K\ell}_{\mathrm{em}}(\%)$ 3-body	$\delta^{K\ell}_{\mathrm{em}}(\%)$ full
K^+_{e3}	2.31 ± 0.22	-0.35 ± 0.16	-0.10 ± 0.16
K^0_{e3}	0	$+0.30 \pm 0.10$	$+0.55 \pm 0.10$
$K^+_{\mu3}$	2.31 ± 0.22	-0.05 ± 0.20	$+0.20 \pm 0.20$
$K^0_{\mu3}$	0	$+0.55 \pm 0.20$	$+0.80 \pm 0.20$

$$f_+^{K^+\pi^0}(0) = 1.023\, f_+^{K^0\pi^-}(0)$$

(Andre 04', Cirigliano 02'-04')

Accurately known

$$\frac{\Delta|V_{us}|}{|V_{us}|} \approx 0.2\%$$

$I_K(\lambda_+,\lambda_0)$ **accurately measured:** $\Delta|V_{us}|/|V_{us}| \approx 0.3\%$

THE FORM FACTOR $f_+(0)$:

The Ademollo-Gatto theorem:

$$f_+(0) = 1 - O(m_s - m_u)^2$$

Still, $f_+(0)$ gives the largest uncertainty: $\Delta|V_{us}|/|V_{us}| \approx 1\%$

ChPT

$$f_+(0) = 1 + f_2 + f_4 + O(p^8)$$

Vector Current Conservation

$f_2 = -0.023$
Independent of L_i
(Ademollo-Gatto)

THE LARGEST UNCERTAINTY

"Standard" estimate:
Leutwyler, Roos (1984)
(QUARK MODEL)
$f_4 = -0.016 \pm 0.008$

f_4: the complete ChPT $O(p^6)$-calculation

Post, Schilcher (2001), Bijnens, Talavera (2003):

$$f_4 = \Delta_{loops}(\mu) - \frac{8}{F_\pi^4}\,[C_{12}(\mu) + C_{34}(\mu)]\,(M_K^2 - M_\pi^2)^2$$

$C_{12}(\mu)$ and $C_{34}(\mu)$ can be determined from the slope and the curvature of the scalar form factor:

$$\lambda_0 = \frac{1}{M_K^2 - M_\pi^2}\left(\frac{f_K}{f_\pi} - 1\right) + \frac{d}{dt}\bar{\Delta}(\mu,t) + \frac{8}{f_\pi^4}(2C_{12}(\mu) + C_{34}(\mu))(M_K^2 + M_\pi^2)$$

$$c_0 = \frac{d^2}{dt^2}\bar{\Delta}(\mu,t) - \frac{8}{f_\pi^4}C_{12}(\mu)$$

Experimental data, however, are not accurate enough.

MODEL ESTIMATES OF f_4

	f_4^{LOC}	$\Delta_{loops}(\mu)$	$f_+(0)=1+f_2+f_4$
PDG: LR, 1984 Quark model	- 0.016 (8)		0.961 ±0.008
Bijnens, Talavera '03 Quark model (LR)	- 0.016 (8)	0.015 (6)	0.976 ±0.010
Jamin, Oller, Pich '04 Dispersive analysis	- 0.018 (9)	0.015 (6)	0.974 ±0.011
Cirigliano et al., '04 Resonance saturation	- 0.012 (?)	0.015 (6)	0.980 ± ....

μ =?? $\Delta_{loops}(1\text{GeV})=0.004$ $\Delta_{loops}(M_\rho)=0.015$ $\Delta_{loops}(M_\eta) = 0.031$

The ±0.010 error might well be underestimated...

Lattice QCD calculation of $f_+(0)$

D.Becirevic, G.Isidori, V.L., G.Martinelli, F.Mescia, S.Simula, C.Tarantino, G.Villadoro.

[NPB 705 (2005) 339, hep-ph/0403217; hep-lat/0411016]

VERY CHALLENGING:

A PRECISION OF O(1%) MUST BE REACHED ON THE LATTICE !!

THE STRATEGY:

1) Evaluation of $f_0(q^2_{MAX})$ with high precision (better than 1%)

2) Extrapolation of $f_0(q^2_{MAX})$ to $f_0(0)=f_+(0)$ by evaluating the slope λ_0

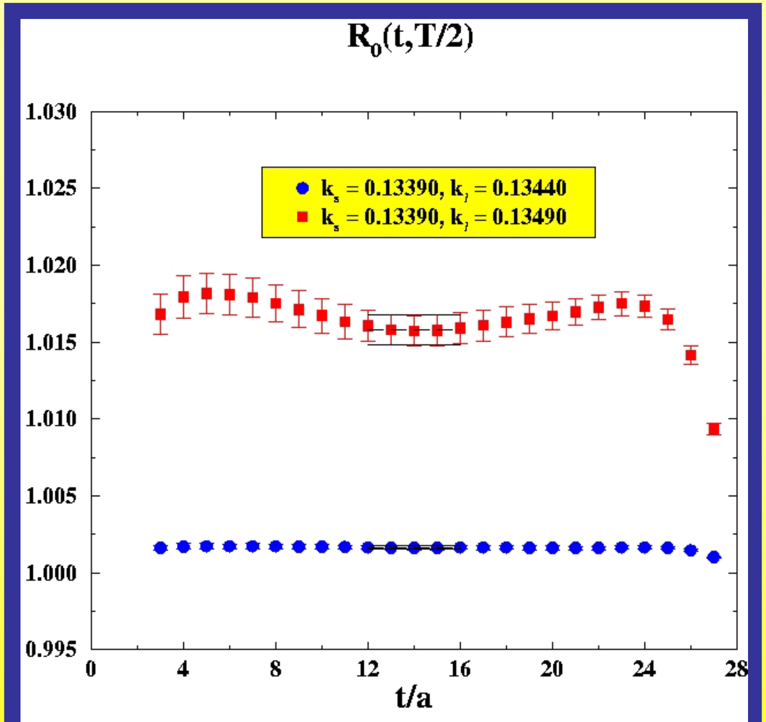
3) Extrapolation to the physical meson masses

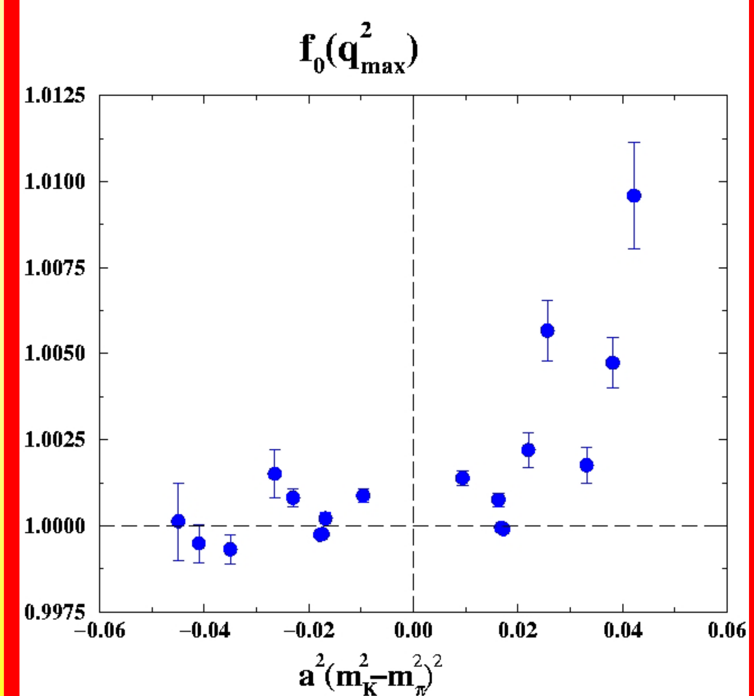
1) Evaluation of $f_0(q^2_{MAX})$

The basic ingredient is a double ratio of correlation functions:

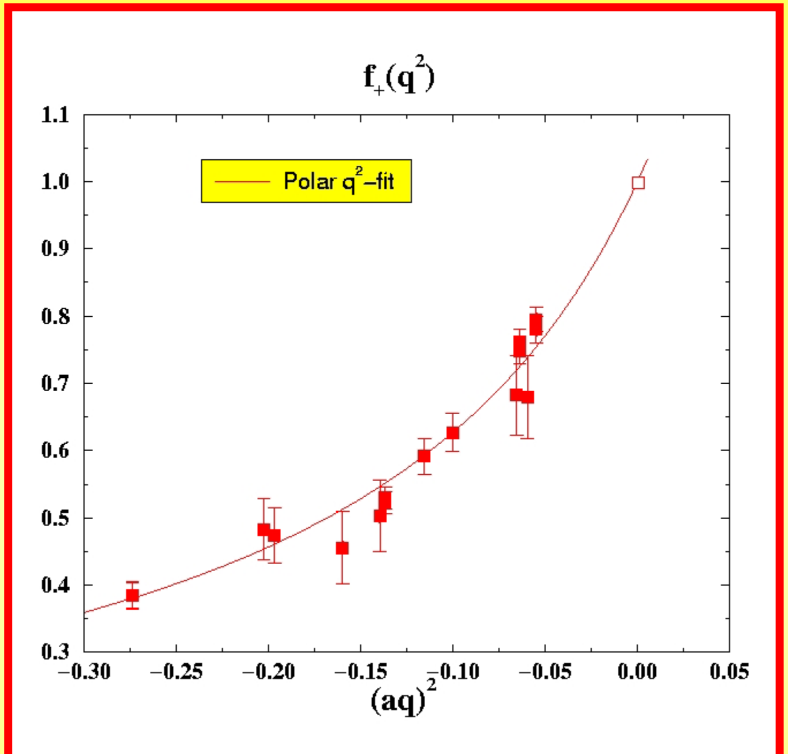
[FNAL]

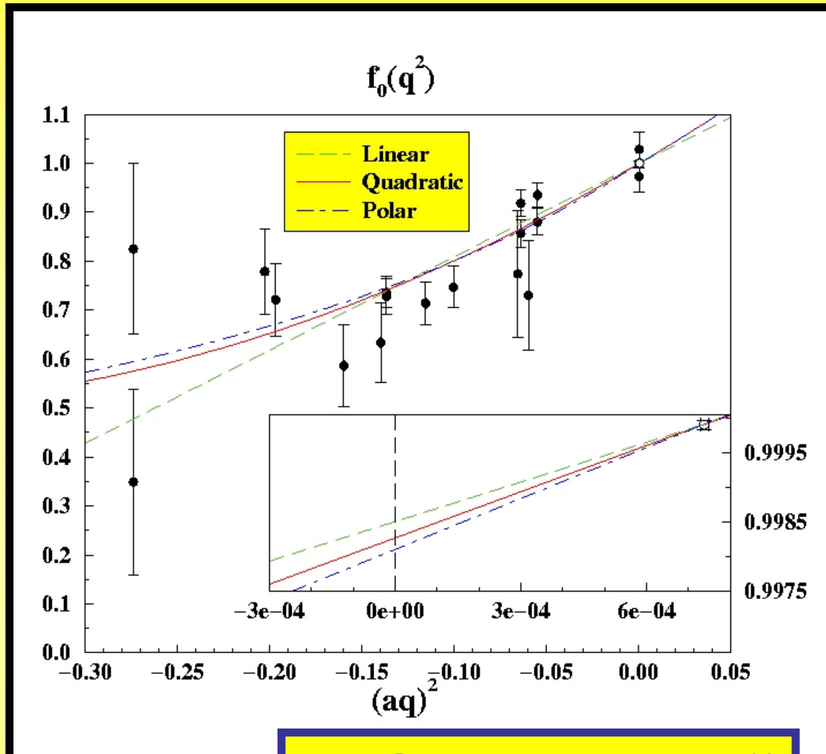
$$R = \frac{\langle \pi | \bar{s}\gamma_0 u | K \rangle \langle K | \bar{u}\gamma_0 s | \pi \rangle}{\langle \pi | \bar{u}\gamma_0 u | \pi \rangle \langle K | \bar{s}\gamma_0 s | K \rangle} = \frac{(M_K + M_\pi)^2}{4 M_K M_\pi} f_0(q^2_{max})^2$$





2) Extrapolation of $f_0(q^2_{MAX})$ to $f_0(0)$





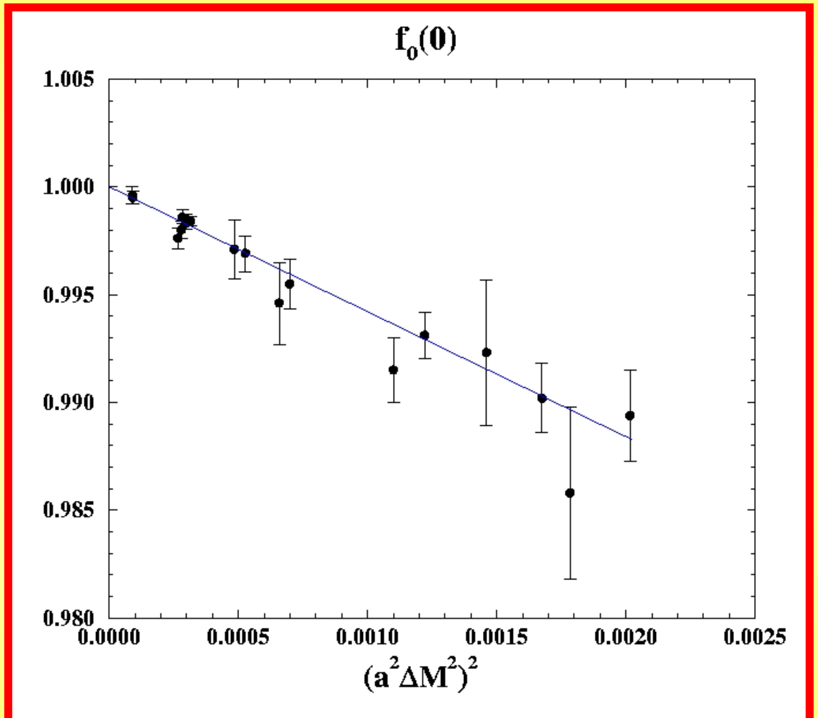
PREDICTION !!

Comparison of polar fits:

LQCD: $\lambda_+ = (25 \pm 2)\,10^{-3}$ $\lambda_0 = (12 \pm 2)\,10^{-3}$

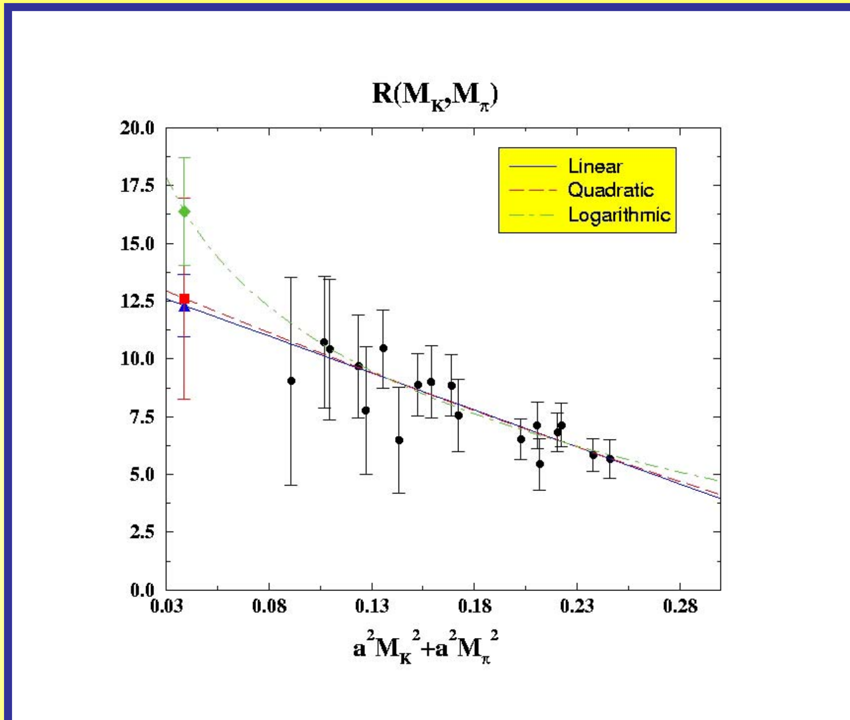
KTeV: $\lambda_+ = (24.11 \pm 0.36)\,10^{-3}$ $\lambda_0 = (13.62 \pm 0.73)\,10^{-3}$

3) Extrapolation to the physical masses



The Ademollo-Gatto theorem

$$f(0) = 1 + O[(M_K^2 - M_\pi^2)]^2$$



$$R = \frac{f_+(0) - 1 - f_2^{QUEN}}{(M_K^2 - M_\pi^2)^2}$$

Computed in Quenched-ChPT

Systematic error: (besides the quenched approximation) mainly from the q^2 and mass dependencies of the form factor

- **Final result:** $\Delta f \equiv f_+(0) - 1 - f_2^{QUEN}$

$$\Delta f = -0.017 \pm 0.005_{stat} \pm 0.007_{syst}$$

Quenching error is not included

- **Leutwyler and Roos:** $\Delta f = -0.016 \pm 0.008$

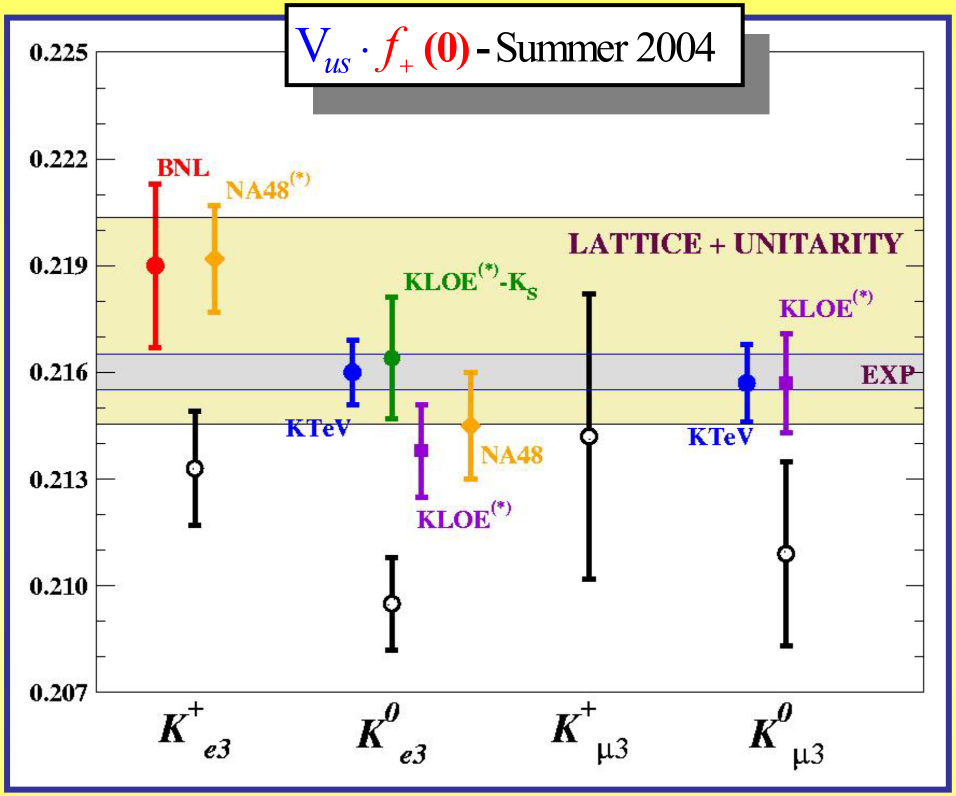
- **Preliminary unquenched result (n_f=2+1):**

$$\Delta f = -0.015 \pm 0.006_{stat} \pm 0.009_{syst}$$

Fermilab Lattice, MILC, HPQCD

$$f_+(0) = 0.960 \pm 0.005_{stat} \pm 0.007_{syst}$$

FIRST ROW UNITARITY



$[V_{us} \cdot f_+(0)]_{EXP} =$
$= 0.2250 \pm 0.0021$

$f_+(0) = 0.960 \pm 0.009$

$V_{us} = 0.2250 \pm 0.0021$

$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0007 \pm 0.0014$

CONCLUSIONS

- ♦ New experimental results on Kl3 decays have superseded rather old measurements. Assuming the Leutwyler and Roos determination of the vector form factor, these results turn out to be in good agreement with the unitarity prediction.

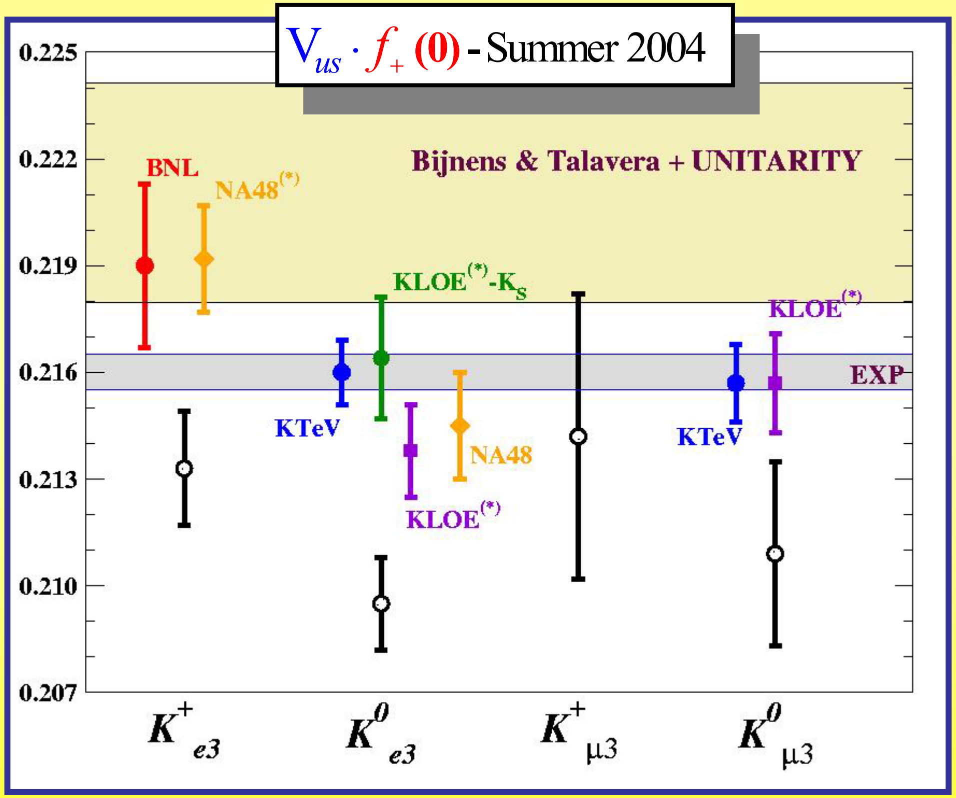
- ♦ From the theoretical point of view, the most important ingredient is the determination of the Kl3 vector form factor

- ♦ ChPT alone is not sufficient to determine this form factor at the required level of accuracy

- ♦ The first Lattice QCD calculation of the Kl3 form factor has been recently performed. The result is in very good agreement with the quark model estimate obtained by Leutwyler and Roos.

BACKUP SLIDES

The unitarity test is less well satisfied when $f_+(0)$ is evaluated by combining the $O(p^6)$ chiral perturbation theory calculation with the quark model estimate of f_4^{LOC}:



$f_+(0) = 0.976 \pm 0.010$

[Bijnens & Talavera]

$V_{us} = 0.2213 \pm 0.0023$

$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2-1 =$

$= -0.0024 \pm 0.0014$

V_{us} from leptonic kaon decays

$$\frac{\Gamma(K \to \mu\bar{\nu}_\mu(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}_\mu(\gamma))} = \frac{|V_{us}|^2 f_K^2 m_K \left(1 - \frac{m_\mu^2}{m_K^2}\right)^2}{|V_{ud}|^2 f_\pi^2 m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2} \, 0.9930(35)$$

[Rad.Corr.]

Precise Lattice QCD calculation of f_K/f_π

(N_f=3) MILC '04

$f_\pi = 129.5 \pm 0.9_{stat} \pm 3.6_{syst}$ MeV

$f_K = 156.6 \pm 1.0_{stat} \pm 3.8_{syst}$ MeV

$f_K/f_\pi = 1.210 \pm 0.004_{stat} \pm 0.013_{syst}$

C.Bernard, update of Marciano 2004: $|V_{us}| = 0.2219(26)$

Agreement with unitarity at 1.4 σ

The dominant source of systematic error comes from the lattice calculation ⇒ VERY DIFFICULT TO REDUCE !!

EXTENSION TO HYPERON DECAYS

- Both the vector and the axial-vector components of the weak current contribute: 6 form factors.
The most important ones are f_1 and g_1.

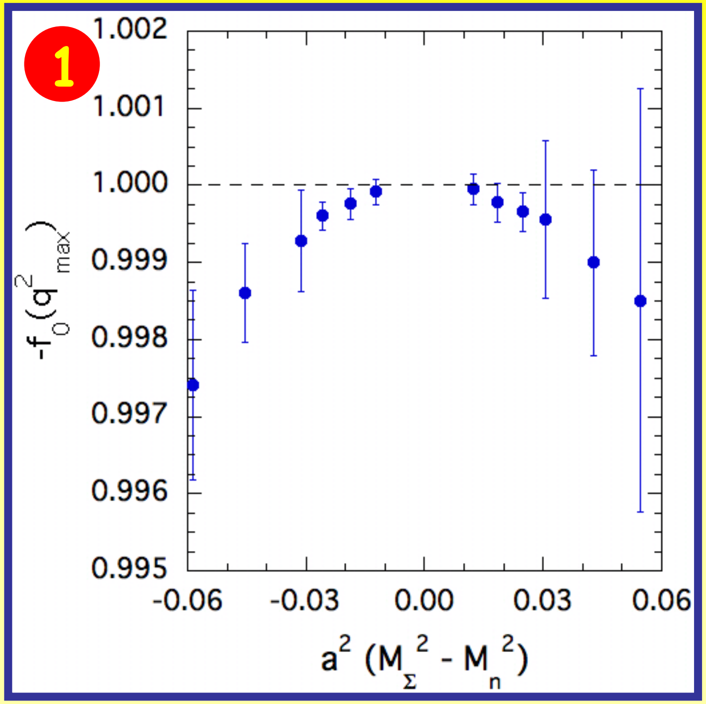
- The axial-vector contributions are renormalized by first order SU(3) breaking effects.

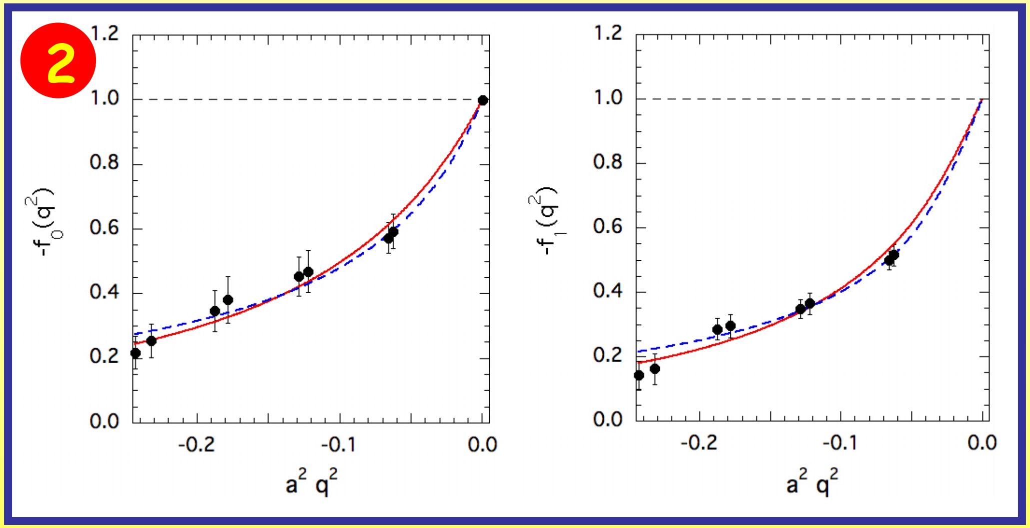
- The Cabibbo, Swallow and Winston analysis (2003):
- the ratio g_1/f_1, which encodes first order SU(3) breaking effects, is extracted from the experimental measurements of decay asymmetries
- second order SU(3) breaking effects in f_1 are neglected

The final result is $|V_{us}|=0.2250(27)$
(good agreement with unitarity)

The $\Sigma^- \to n l \nu$ decay on the lattice:

D.Guadagnoli, G.Martinelli, M.Papinutto, S.Simula
[hep-lat/0409048, hep-lat/0411016]





WORK IN PROGRESS.
MORE DIFFICULT THAN Kl3

3

Extrapolation to physical masses: Heavy-baryon ChPT

- The octet-decuplet splitting (of $O(m_K)$) treated as a large scale
- $1/m_B$ corrections unknown
- Quenching effects unknown

V_{us} from hadronic τ decays

E.Gamiz et al, hep-ph/0408244

Basic ingredient: OPE applied to the spectral moments in hadronic τ decays

Experimental input: measurements of the hadronic spectral moments

Theoretical input/output: $m_{strange}$, V_{us}

Using:
$m_s(2\ \text{GeV}) = (95 \pm 20)$ MeV
(LATTICE+QCDSR)

→

$|V_{us}| = 0.2208 \pm 0.0034$