Renormalons at the boundaries between perturbative and non-perturbative QCD

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- The attempt to anticipate the Round Table discussions of Session V (02.03.05)

Which Physics with High Intensity Medium Energy Accelerators?

- Two questions in one
  - (I) "Which Physics?"
  - (II) "Which Medium Energy Accelerators?"

•• Two answers to two questions

(I) "Medium energy QCD"

1. perturpative asymptotic freedom effects
2. non-perturbative $O(1/Q^2)$ effects

(II) "Which Medium energy Accelrators?

Experimental situation

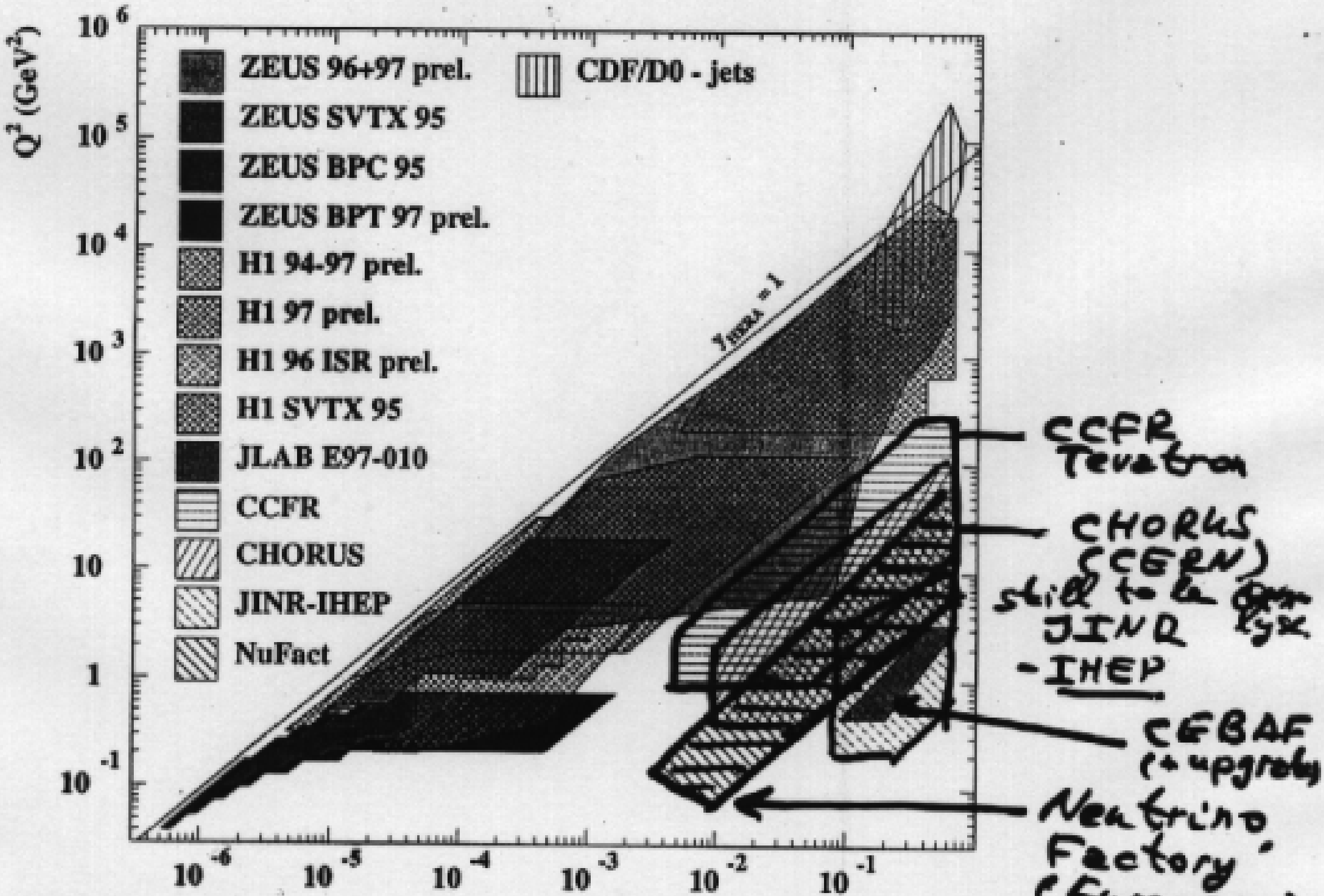


Figure 1: *Kinematic regions in x-Q^2 for cross-section measurements in deep inelastic ep scattering, ν scattering and for triple differential jet cross-section measurements in $p\bar{p}$ collisions (from Ref. [9]).*

+ NuTeV data (Tevatron)
+ Nomad data (CERN)
+ Chorus data (CERN)

A lot of data for νN DIS

(II) – two answers

1) <u>Neutrino</u> <u>factory</u> <u>on</u> <u>the</u> <u>agenda</u> (Europe, <u>Japan</u>, USA)

2) + Low energy and large x data for F_2, g_1, etc. are on the agenda (<u>CEBAF</u>) <u>TJNAF</u>
additional information for $x \to 1$

Physical quantities to be studied:

Bjorken sum rule of DIS νN

$$Bjn(Q^2) = \int_0^1 [F_1^{\nu p}(x,Q^2) - F_1^{\nu n}(x,Q^2)]\, dx$$

Gross-Llewellyn-Smith sum rule of νN DIS

$$GLS(Q^2) = \frac{1}{2} \int_0^1 [F_3^{\nu p}(x,Q^2) + F_3^{\bar{\nu} n}(x,Q^2)]\, dx$$

may be extracted from data of Neutrino factory

Bjorken sum rule of polarized charged lepton-nucleon DIS

$$Bjp(Q^2) = \int_0^1 [g_1^{lp}(x,Q^2) - g_1^{ln}(x,Q^2)]\, dx$$

Extracted from SLAC, CERN, CEBAF data

For neutrino may be extracted from Neutrino factory data

What is the status of theoretical prediction at "medium energies"?

1) In QCD all 3 sum rules are calculated up to α_s^3-terms; estimates of α_s^4-terms are available

2) Twist-4 $1/Q^2$ non-perturbative corrections are estimated using 3-point function QCD SRs and instanton model (results agree).

So: within truncated PT

$$SR(Q^2) = \sum_{i=0}^{K} d_i \left(\frac{\alpha_s}{\pi}\right)^i - \underbrace{\frac{A}{Q^2}}_{NP\text{-contribution}}$$

$K \leq 4$

At low and moderate Q^2 PT and NP-effects may correlate

Moreover:

PT series is asymtotic

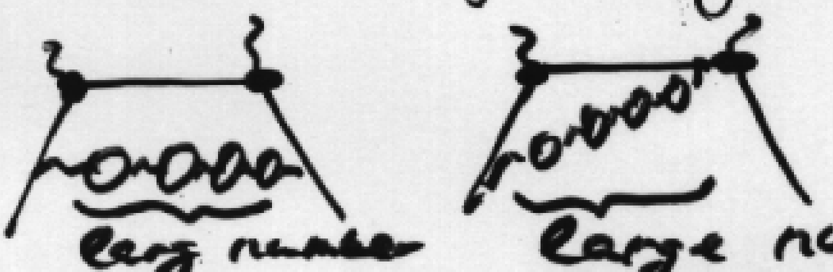
$$SR(Q^2) = \sum_{i=0}^{\infty} d_i \left(\frac{\alpha_s}{\pi}\right)^i$$

$$d_n = a^n n! n^b$$

It is possible to get the impressi on both d_n, ~~and~~ (d_i - truncated) and A using renormalons calculus, which allows to get relation between 3 sum rules (previously unknown)!

Renormalons

Subset of diagrams



Broadhurst, Kataev (93)

same diagrams Bjp; GLS

$$\frac{1}{3}\left|\frac{g_A}{g_V}\right| C_{Bjp}(Q^2) = \int_0^1 \left[g_1^{lp}(x,Q^2) - g_1^{ln}(x,Q^2)\right] dx$$

$$3\, C_{GLS}(Q^2) = \frac{1}{2}\int_0^1 \left[F_3^{\nu p}(x,Q^2) + F_3^{\nu n}(x,Q^2)\right] dx$$

$$C_{Bjp} = C_{GLS} = 1 + \frac{C_F}{T_F N_F}\sum_{n\geq 1}^{\infty} K_n \big(\underbrace{T_F N_F a_s}_{\delta}\big)^n$$

$C_F = 4/3$; $T_f = 1/2$;

$K_n = (-1)^n n!\, n^b \left(1 + O(\tfrac{1}{n})\right)$ at $n \to \infty$

$$K(\delta) = \sum_{n=0}^{\infty} K_n \frac{\delta^n}{n!}\,; \quad K(\delta) = \left(\frac{3+\delta}{2(1+\delta)}\right) u(\delta)$$

$$\boxed{u(\delta) = -\frac{2\exp(5\delta/3)}{(1-\delta)(1-\delta^2/4)}}$$

$$K_n = \lim_{\delta\to 0}\left(-\frac{4}{3}\frac{d}{d\delta}\right) K(\delta)$$

$$C_{Bjp}(a) = \int_0^\infty d\delta\, e^{-\delta/a_s} K(\delta)$$

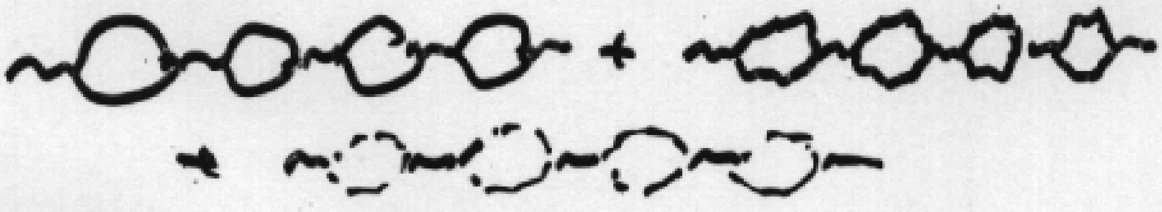
$u(\delta)$ – Borel immage of Bjorken unpolarized sum rule

$$C_{Bj\,unp} = \int_0^1 \left[F_1^{\nu n}(x,Q^2) - F_1^{\nu p}(x,Q^2)\right] dx$$

Broadhurst, Kataev (02)

In QCD

The renormalon chain is



How to take into account other sets of graphs?

Naive Nonabelianization procedure 'ansatz'

$$N_F = -\frac{3}{2}\beta_0^{QCD} = -\frac{3}{2}\left(\frac{11}{3}C_A - \frac{4}{3}T_F N_F\right) = \beta_0^{QCD}$$

$$\delta = T_F N_F \bar{a}_s \rightarrow \beta_0^{QCD}\bar{a}_s$$

 → missed

$$K_n = n!\, n^b \left(1 + O\left(\frac{1}{n}\right)\right)$$

$$C_{GLSR} = \int_0^\infty e^{-\delta/\beta_0 a_s} K(\delta)\, d\delta \quad \text{or}$$

$$C_{Bjp} = \int_0^\infty e^{-\delta/\beta_0 a_s} u(\delta)\, d\delta$$

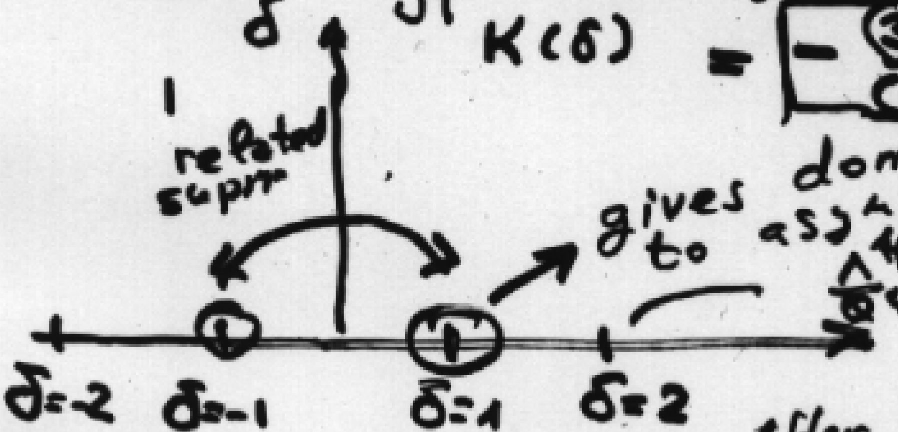
These sum rules are closely related due to similarity of $K(\delta)$ and $u(\delta)$

Indeed: $K(\delta) = \left(\frac{3+\delta}{2(1+\delta)}\right) u(\delta) =$

For Bjp SR we have

$$K(\delta) = \boxed{-\frac{(3+\delta)\exp(5\delta/3)}{(1-\delta^2)(1-\delta^2/4)}}$$

dominating contr. to asymptotic behaviour.

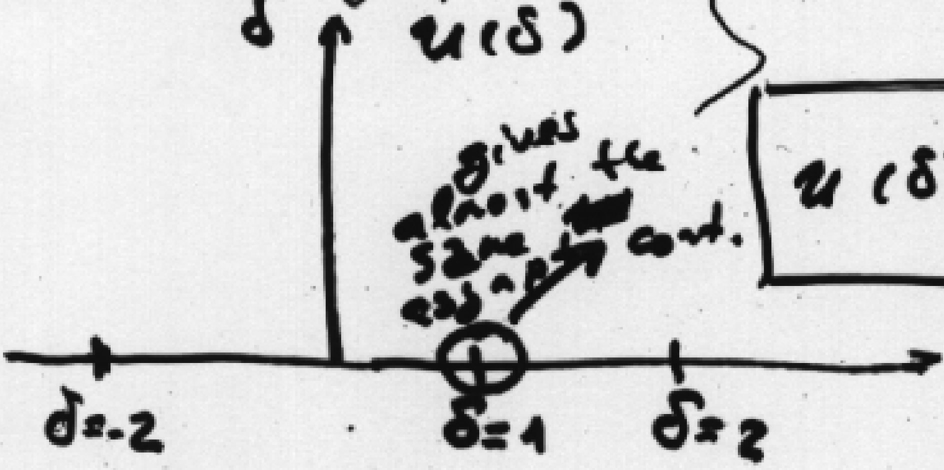


After "Borel integration" gives $\frac{\Lambda^2}{Q^2}$-corr.

$\frac{1}{2}\exp(-10/\beta) = 0.018.$

For Bjunp SR there is the feature:

$$u(\delta) = \boxed{-\frac{2\exp(5\delta/3)}{(1-\delta)(1-\delta^2/4)}}$$



So: perturbative structure is related and non-pert. also.

In fact this becomes more clear after appl. of NNA

$$\delta = \beta_{0f}\, t = (-)\frac{2}{3} N_f\, t \quad \text{in large } N_f$$

$$\delta = \beta_0^{QCD}\, t \quad \text{after } N_f \to -\frac{3}{2}\beta_0^{QCD}$$

$\overline{MS}$-scheme sign-alternating series
(manifestation of one ~~...~~ renormalon
with fermion bubbles

Bjp SR:

$$\sum_n K_n x^n = -\underline{\underline{3x}} + \underline{\underline{8x^2}} - \frac{920}{27}x^3 + \frac{38720}{243}x^4$$

$$- \frac{238976}{243}x^5 + \frac{130862080}{19683}x^6 - \ldots$$

$$x = T_f N_f \overline{a}_s$$

Bjunp:

$$\sum_n u_n x^n = -2x + \frac{64}{9}x^2 - \frac{2480}{81}x^3 + \frac{113920}{729}x^4$$

$$- \frac{6195698}{6561}x^5 + \frac{395898880}{590449}x^6 - \ldots$$

x and x^2-terms are in agreement with the explicit results, obtained ~~in the~~ previously

x^3-terms : material for the Naive Nonabelianization guesses.

Application of NNA procedure

$N_f \to N_f - \frac{33}{2} = -\frac{3}{2}\beta_0^{QCD}$, BK (02)

$\beta_0^{QCD} = 11 - \frac{2}{3} N_f$

Exact result vs NNA in the spirit of Lovett-Turner, Maxwell (95)

$C_{Bjp} = 1 + \sum_{n\geq 1} d_n \left(\frac{\alpha_s}{\pi}\right)^n$ for $\overline{MS}$

$d_1 = -\frac{2}{3}$

$d_2 = -\underline{3.833} + 0.29630 N_f$

Chetyrkin, Gorishny, Larin, Tkachov (84)

$d_3 = -\underline{\underline{36.155}} + \underline{\underline{6.3313}} N_f - 0.15947 N_f^2$

Larin, Tkachov, Vermaseren (91)

$d_2^{NNA} = -\underline{4.8885} + 0.29630 N_f$

$d_3^{NNA} = -\underline{\underline{43.414}} + \underline{\underline{5.2623}} N_f - 0.15947 N_f^2$

(?!) $d_4^{NNA} = -457.02 + 83.094 N_f - 5.0360 N_f^2 + 0.10174 N_f^3$

Series sign alternation in N_f BK(02)

$C_{Bjp} = 1 + \sum_{n\geq 1} \overline{d}_n \left(\frac{\alpha_s}{\pi}\right)^n$ $\overline{d}_1 = -1$

$\overline{d}_2 = -\underline{4.5833} + 0.33333 N_f$

Gorishny, Larin (86)

$\overline{d}_3 = -\underline{\underline{41.440}} + \underline{\underline{7.6070}} N_f - 0.17747 N_f^2$

Larin, Vermaseren (91)

$\overline{d}_2^{NNA} = -\underline{5.5} + 0.33333 N_f$

$\overline{d}_3^{NNA} = -\underline{\underline{48.316}} + \underline{5.8565} N_f - 0.17747 N_f^2$

$\overline{d}_4^{NNA} = -466.00 + 84.728 N_f - 5.1950 N_f^2 + 0.10774 N_f^3$

a) $\frac{1}{Q^2}$ - corrections are negative

both in Bjp and Bjunp.

in agreement with other methods

b) They should be closed numerically to each other. and to the $\frac{A}{Q^2}$ CORRECTION To The Gross - Llewellyn Smith SR.

Since:

$$Bjunp(Q^2) = \int_0^\infty e^{-\delta/\beta_0 a_s} U(\delta)\, d\delta$$

$$Bjp(Q^2) = \int_0^\infty e^{-\delta/\beta_0 a_s} K(\delta)\, d\delta$$

At $\delta = 1$ The residues are identical.

$$GLS(Q^2) = \int_0^\infty e^{-\delta/\beta_0 a_s} K(\delta)\, d\delta$$

identical 3 point fun. SR.

$A_{Bjpr} \approx -0.071$; $A_{GLS} \approx -0.098$

$A_{Bjunp} \approx -0.13$

Conclusions

1) Renormalons calculus contain information on both asymptotic PT series and NP-effects.

2) Renormalon calculus allow us to show, that theoretical expressions for BjpP, GLS and Bjunp SRs are related (The last fact in PT sector at the α_s^3 level was discovered within scheme-invariant approach by Gardi; Karliner (98))

3) Two question to be ans.
   1) To find physical explanation of the relation of differ SRs (theoretical - none rigorous)
   2) To use the results in the analysis of exp. data (still to come).