



 *Physikalisches Institut*

RUPRECHT-KARLS-UNIVERSITÄT
HEIDELBERG



Measurement of $|V_{ub}|$ and $|V_{cb}|$ at BABAR

Rolf Dubitzky
University of Heidelberg

Overview

➤ Introduction

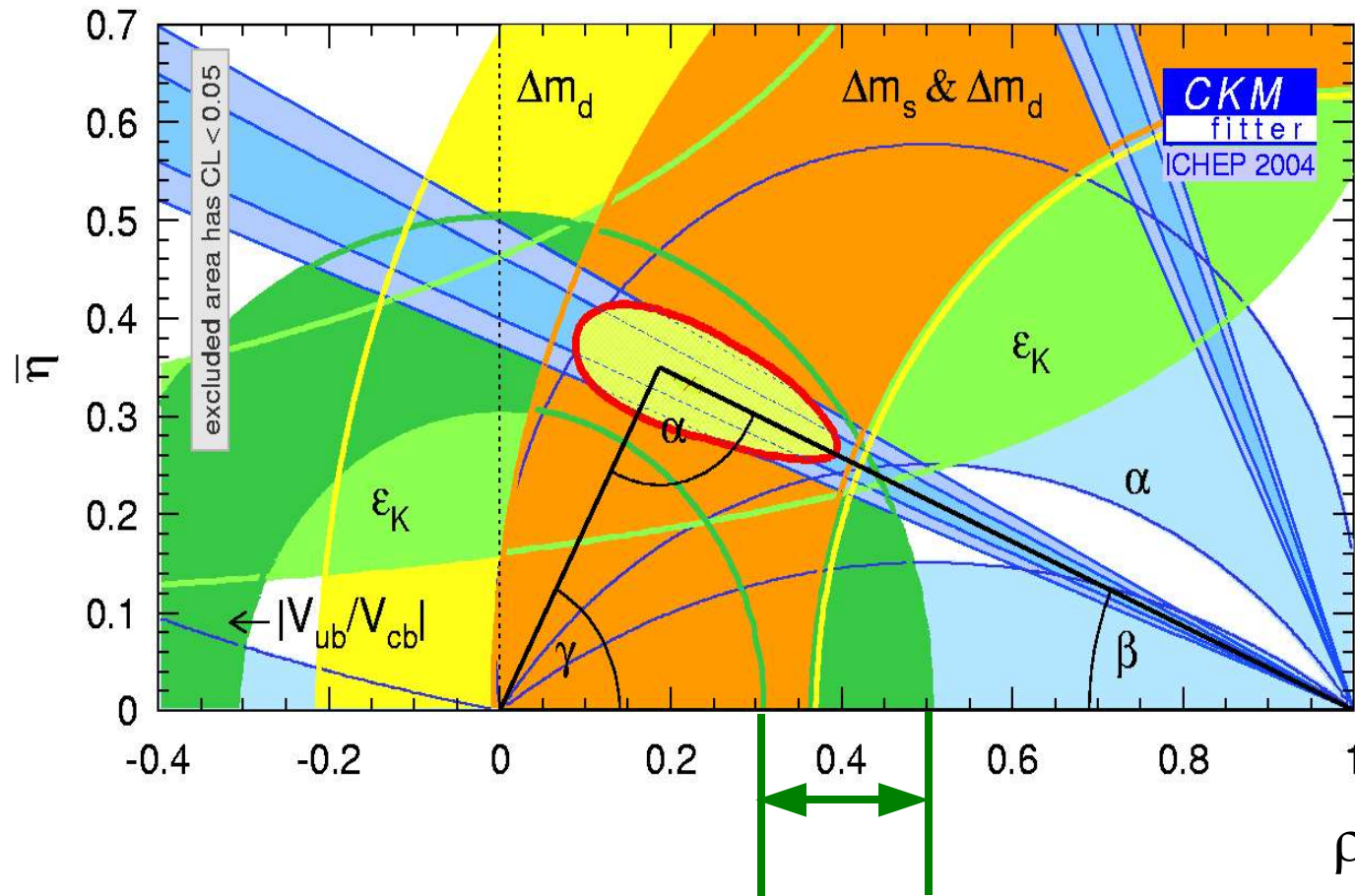
➤ $|V_{cb}|$ from inclusive measurements

- Electron energy and hadronic mass moments
- Combined Fit with Operator Product Expansion

➤ $|V_{ub}|$

- Electron energy endpoint
- E_1 versus q^2
- m_x
- m_x versus q^2
- moments from unfolded m_x spectrum

Motivation

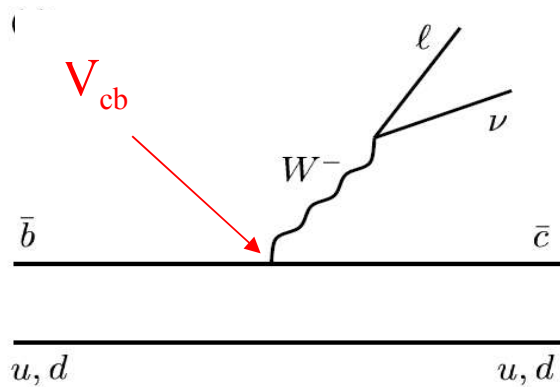


Measurements of $|V_{ub}|$ and $|V_{cb}|$ provide **stringent and redundant consistency tests** of the **unitarity triangle (UT)**

Need to measure $|V_{ub}|/|V_{cb}|$ to better than **10%** to limit unitarity tests

Challenge for semileptonic decays

- Semileptonic B decays provide the best method to measure $|V_{cb}|$ and $|V_{ub}|$

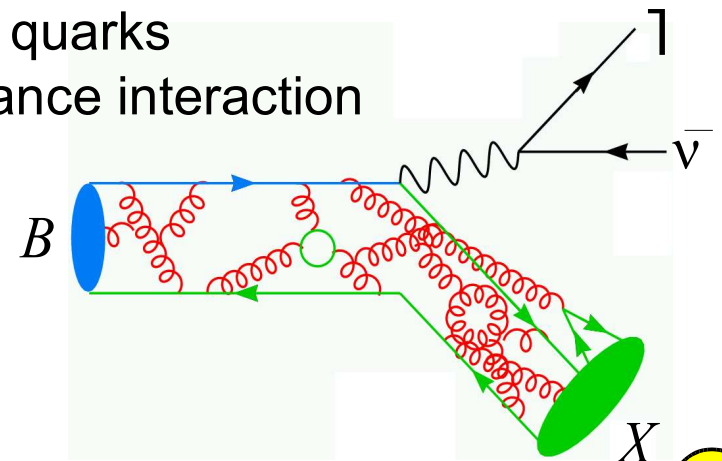


- relatively easy to calculate on parton level
- rates depend on CKM matrix elements and quark masses:

$$\Gamma_u \equiv \Gamma(b \rightarrow u \ell \nu) = \frac{G_F^2}{192\pi^2} |V_{ub}|^2 m_b^5$$

$$\Gamma_c \equiv \Gamma(b \rightarrow c \ell \nu) = \frac{G_F^2}{192\pi^2} |V_{cb}|^2 m_b^2 (m_b - m_c)^3$$

- **BUT:** sensitive to QCD phenomenology of heavy quarks
 - Higher order QCD (α_s^n) corrections to short distance interaction
 - long distance QCD interaction
- Evaluate in theoretical framework of **HQE / OPE**



Heavy Quark Expansion

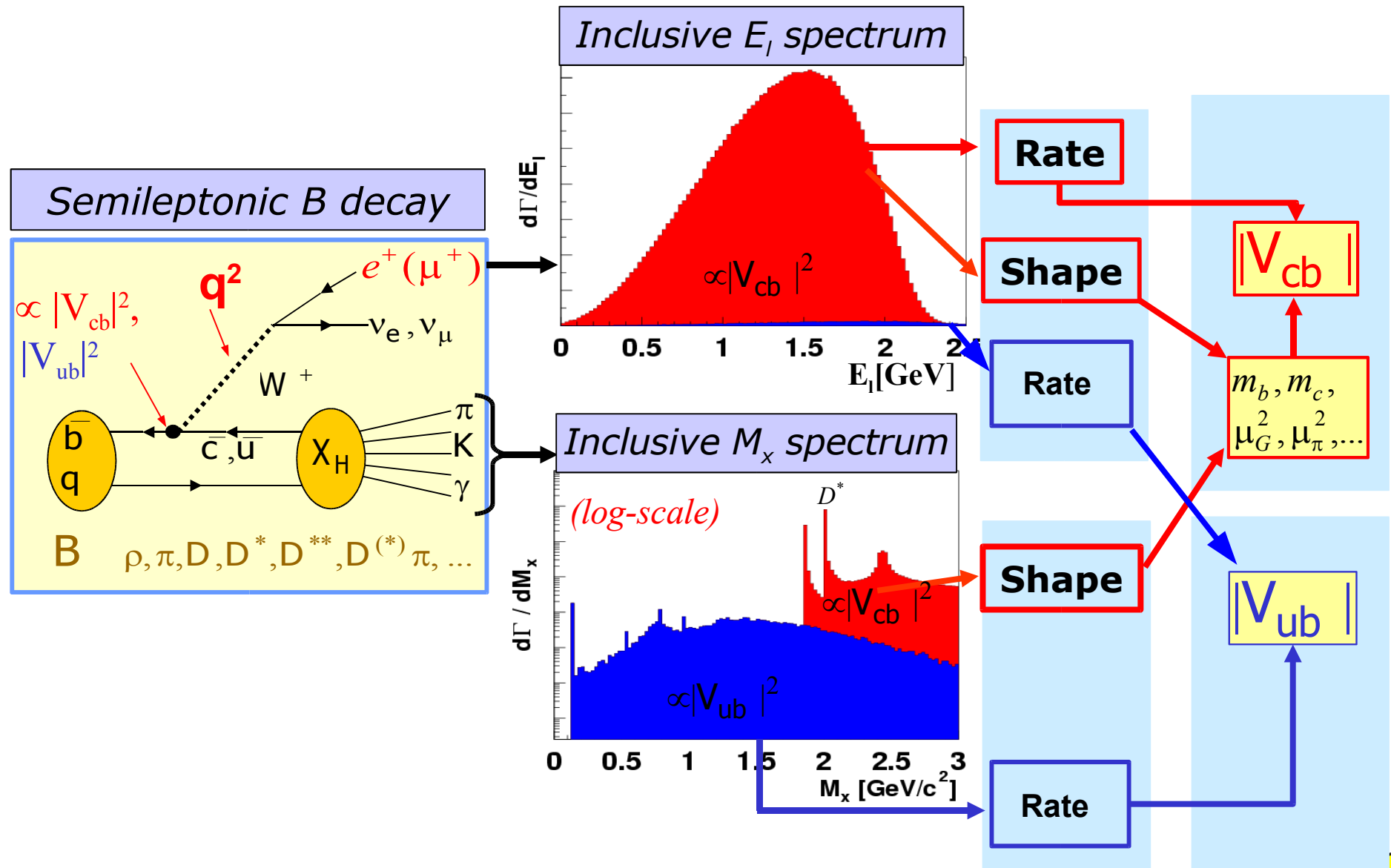
- Evaluate in theoretical framework of **HQE / OPE**
 - expand matrix element in powers of $1/m_b$ in terms of local operators
 - expectation values are properties of the b quark within the B meson
 - ✓ can calculate perturbative corrections (α_s)
 - ✓ can factorize long distance corrections ($1/m_b^n$) into measurable quantities

$$\Gamma_{clv} = \frac{G_F m_b^5}{192\pi^3} |V_{cb}|^2 (1 + A_{ew}) A_{pert} A_{nonpert} \cong |V_{cb}|^2 f_{OPE}(m_b, m_c, a_i)$$

- ✓ when summing over all final states ("**inclusive**")
 - problems with properties of the final state cancel out
- **measure rate and shape of inclusive spectra**, e.g.
 - Lepton energy moments are defined:

$$\langle X^n \rangle (E_{cut}) = \frac{\int (X - X^0)^n \frac{d\Gamma}{dX} dX}{\int \frac{d\Gamma}{dX} dX} \Bigg|_{E_l > E_{cut}} \cong f'_{OPE}(m_b, m_c, a_i)$$

Overview of techniques



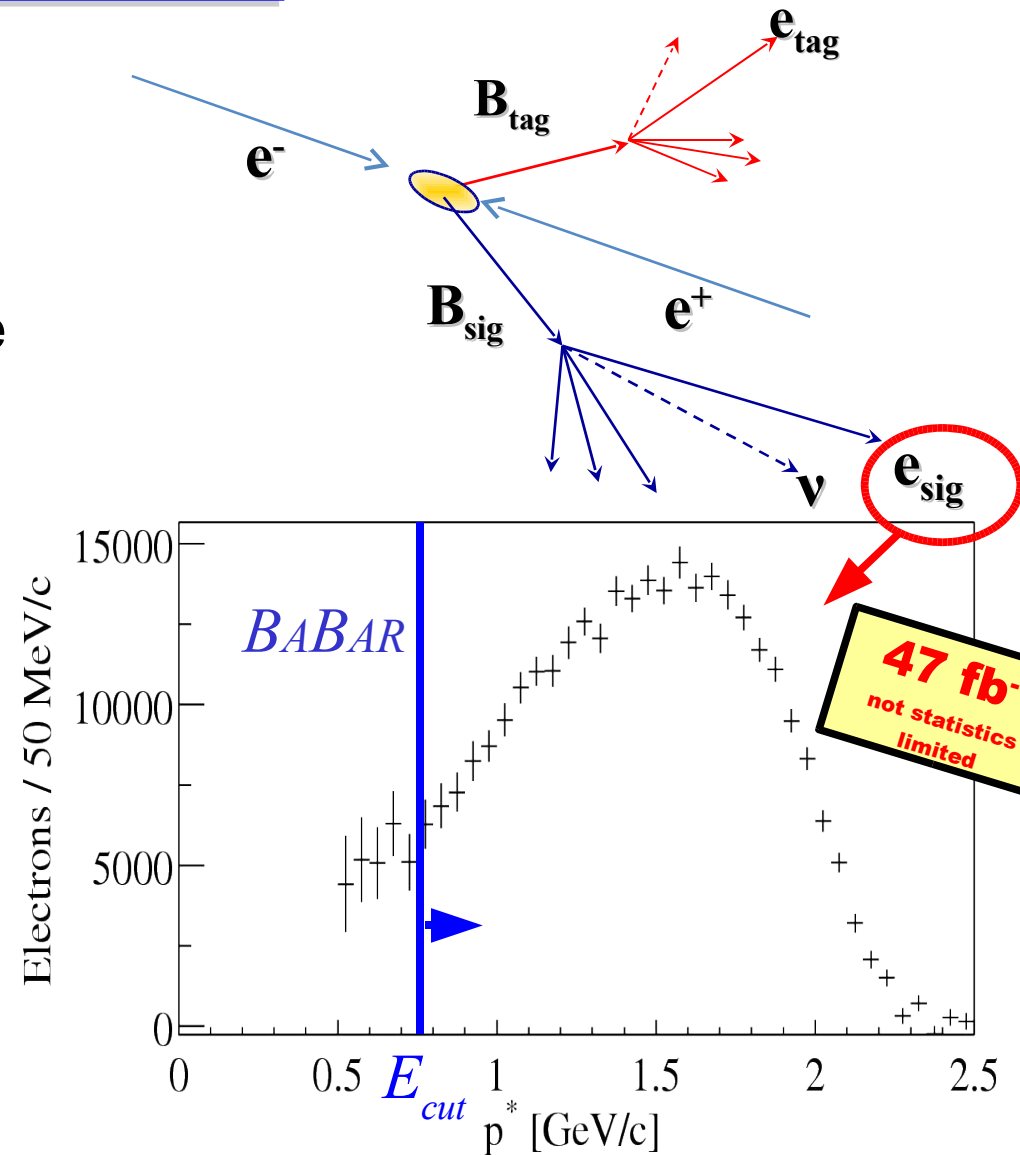
$|V_{cb}|$ - Electron Energy Moments

PRD69:111104

- Select events with 2 electrons
- One electron to "tag" a B event
($1.4 < p^* < 2.3$ GeV in CMS)
- Measure the energy spectrum of the other ("signal") electron
- ✓ Correct for efficiency, B^0 mixing, non-prompt electrons, and detector effects (e.g. Bremsstrahlung)
- ✓ Subtract $B \rightarrow X_u | \nu$ background
- Calculate first four moments for various lower cuts in E_i :

$$E_{cut} = 0.6 \dots 1.5 \text{ GeV}$$

→ Results are input to OPE fit



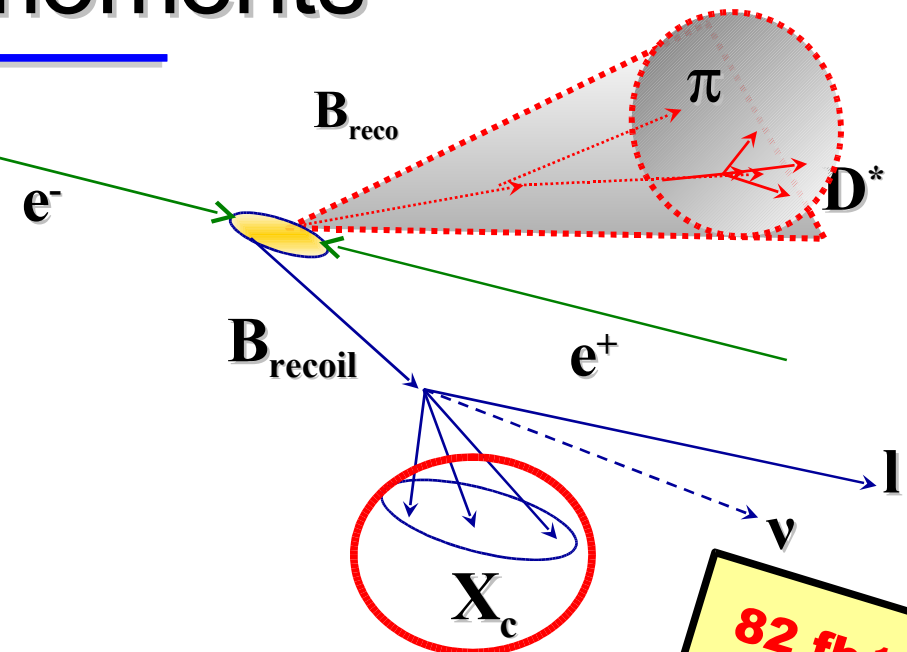
$|V_{cb}|$ - Hadronic mass moments

PRD69:111103

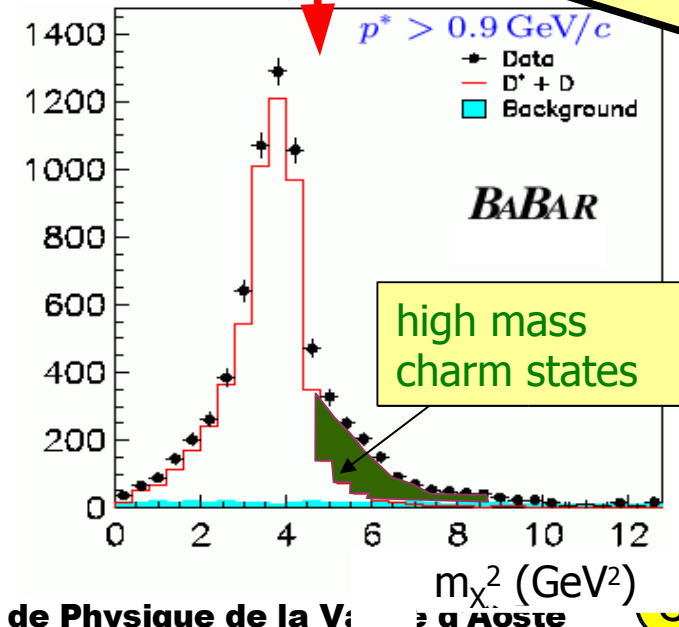
- Select events with **fully-reconstructed** B mesons
- Measure hadronic mass (m_X) of X_c
- Select lepton l with $E_l > E_{cut}$
- Require lepton charge consistent with fully reconstructed B meson
- Use Breco momentum to calculate m_X
- Calculate first four moments for various lower cuts in E_l :

$$E_{cut} = 0.9 \dots 1.6 \text{ GeV}$$

→ Results are input to OPE fit



82 fb⁻¹

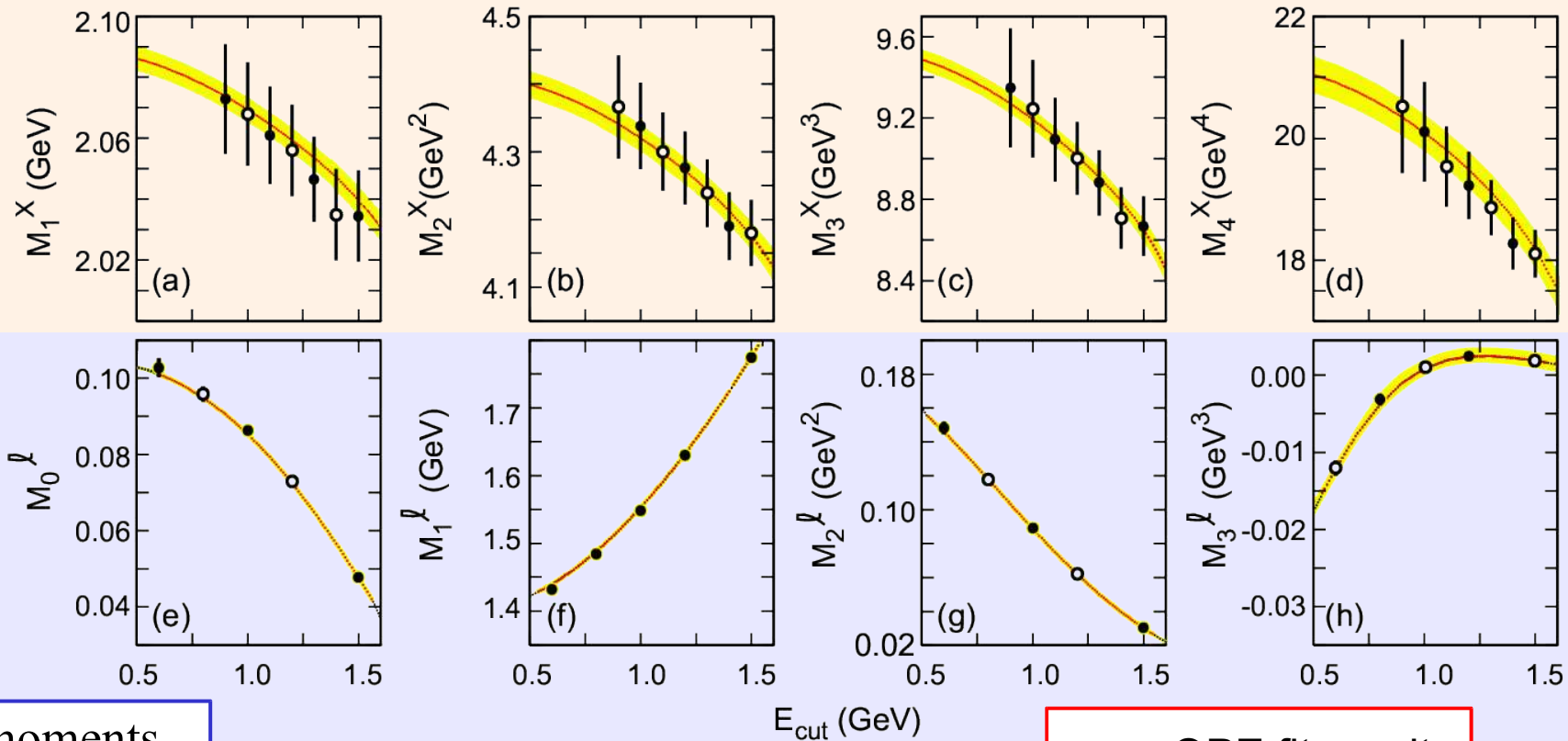


|Vcb| - Combined OPE fit

m_X moments

● = used in fit
○ = not used in fit
(cross check: switch ●↔○)

$\chi^2/\text{ndf} = 15/20$



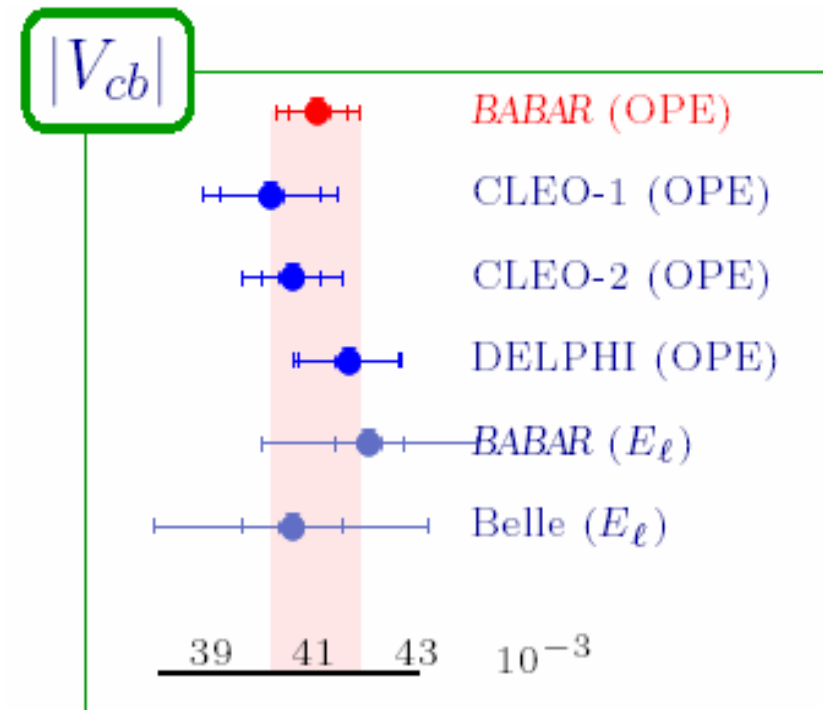
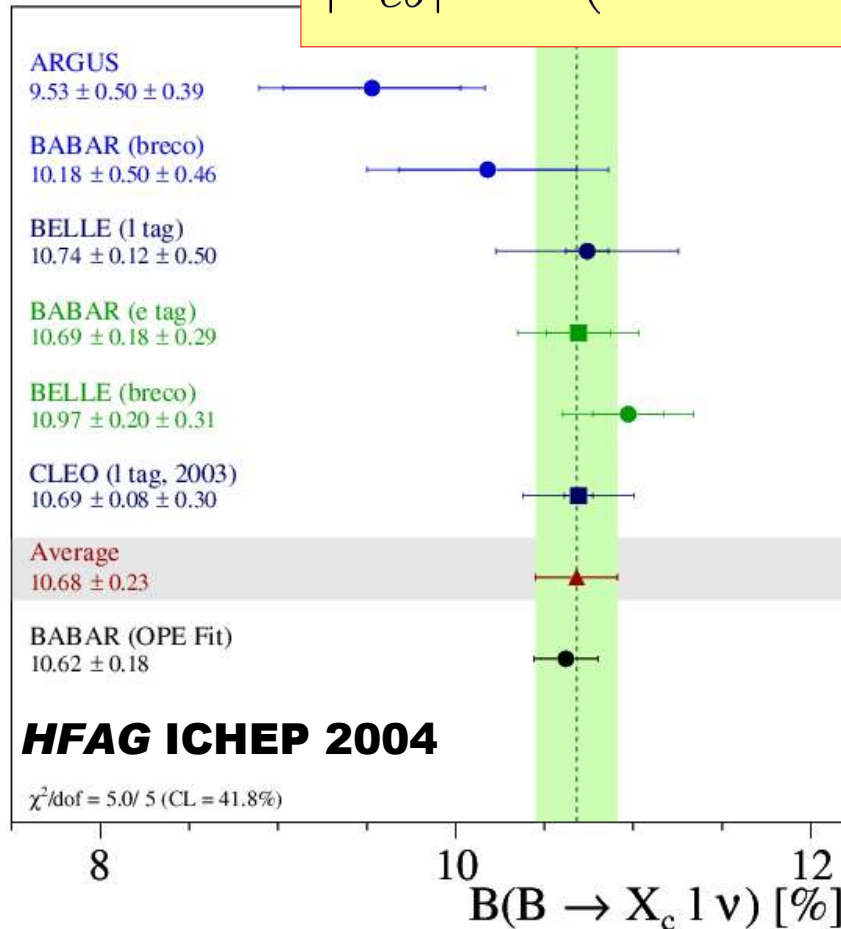
E_ℓ moments

— = OPE fit result
■ = theory errors

Inclusive $|V_{cb}|$ Results

$|V_{cb}|$ is measured up to $\pm 2\%$:

$$|V_{cb}| = (41.4 \pm 0.4_{exp} \pm 0.4_{HQE} \pm 0.6_{th}) \times 10^{-3}$$



$b \rightarrow u \ell \nu$ decays and $|V_{ub}|$

Challenging, because:

- $B \rightarrow X_c \ell \nu$ background 50 times higher
- signal can only be analyzed in limited region of phase space
- need theory to extrapolate to full rate or calculate partial rate

BABAR can present a set of internally consistent analyses, using different approaches to reduce uncertainties and check consistency:

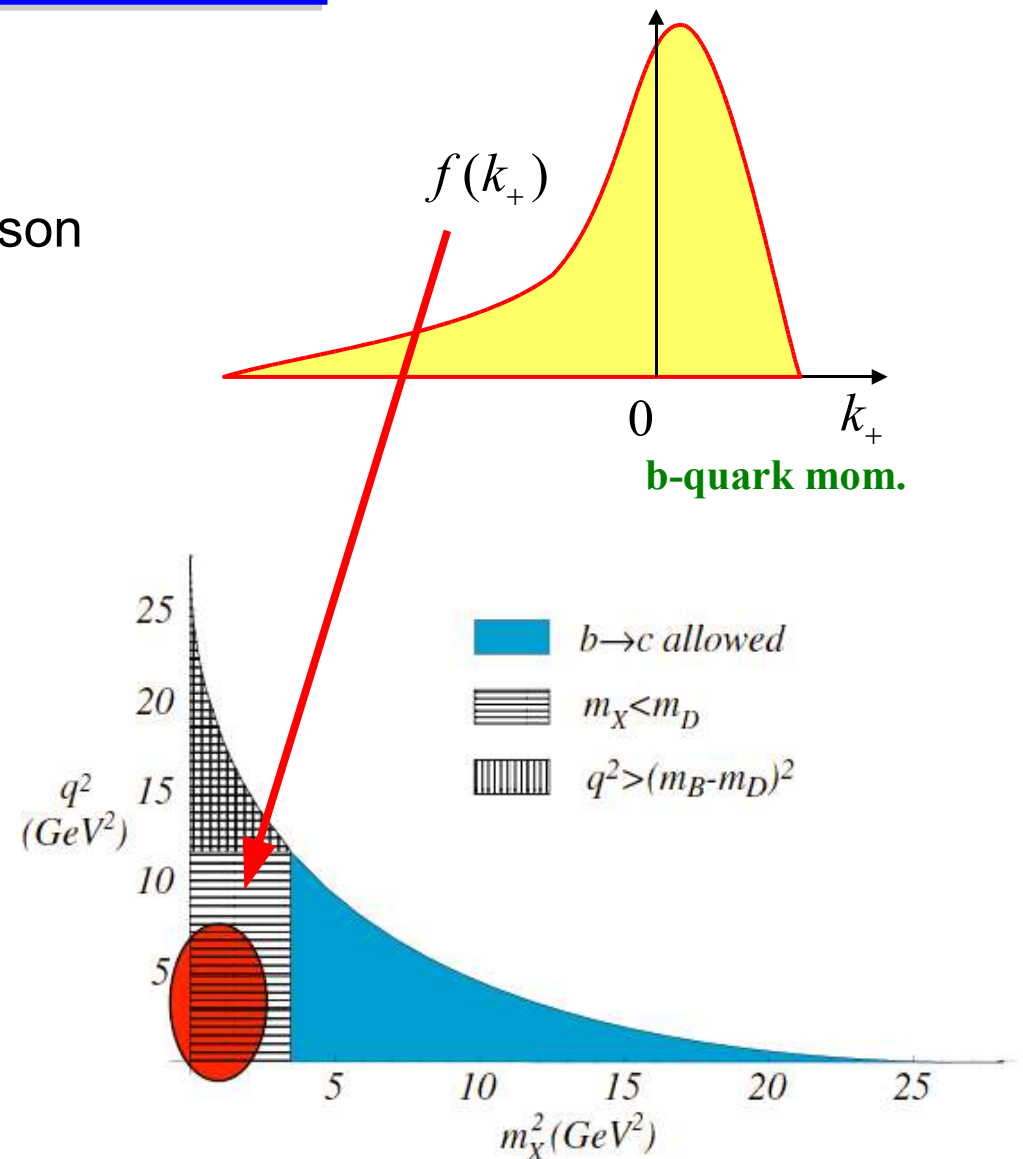
- Lepton endpoint, E_ℓ inclusive, untagged
- E_ℓ vs q^2 add "neutrino reconstruction"
- m_X tag with fully-reconstructed B mesons
- m_X vs q^2 dito, reduce uncertainties in theory
- hadronic moments unfold detector response and calculate M_1, M_2'

Non-perturbative Effects (aka. Shape Function)

- OPE doesn't hold everywhere in the phase space
 - Fermi motion of b quark inside B meson
 - shape functions $f(k_+)$ (SF) describe non-perturbative effects
- **Problems:**
 - need to parameterize SF
 - need to measure parameters
 - sub-leading shape functions
- parameters can be measured from the moments of the photon energy spectrum in $b \rightarrow s\gamma$ decays

BELLE: (exponential SF)

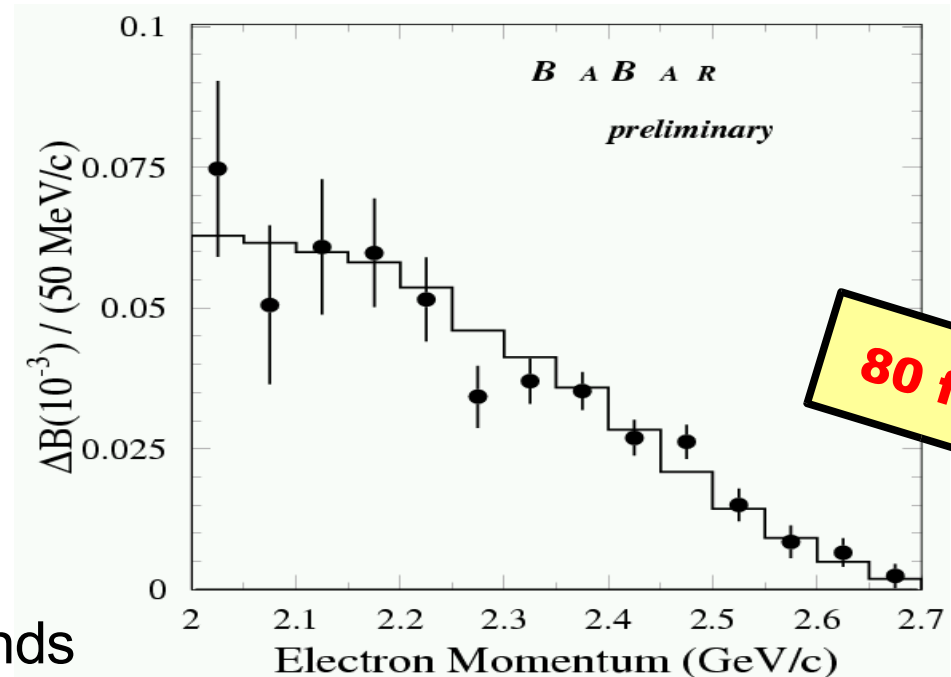
- $\Lambda^{\text{SF}} = 0.66 \text{ GeV}$
- $\lambda_1^{\text{SF}} = 0.40 \text{ GeV}^2$



$|V_{ub}|$ - Lepton endpoint

hep-ex/0408075

- Electrons in energy range
 $2.0 < E_l < 2.6 \text{ GeV}$
- cut on
 - event shapes
 - missing momentum
 - subtract continuum using
 - off-peak data
 - on-peak for $E_l > 2.8 \text{ GeV}$
- Fit E_l spectrum with $B\bar{B}$ backgrounds from charm ($D e \nu$, $D^* e \nu$, $D^{**} e \nu$, $D^{(*)} \pi e \nu$, non-resonant, and $X_u e \nu$)

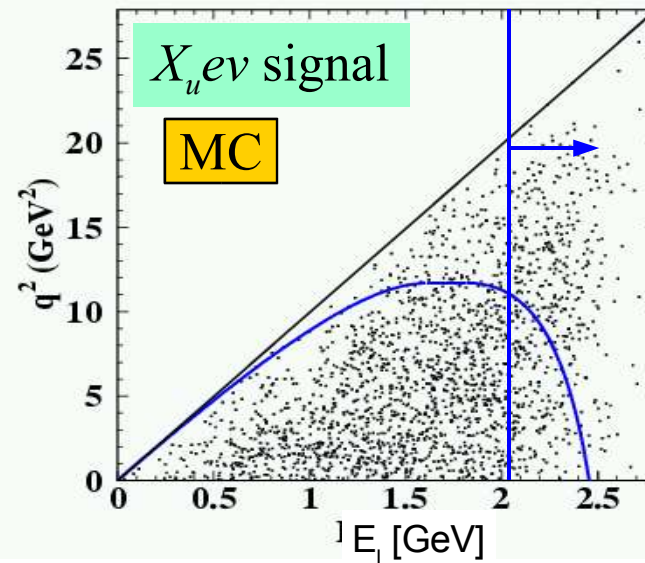
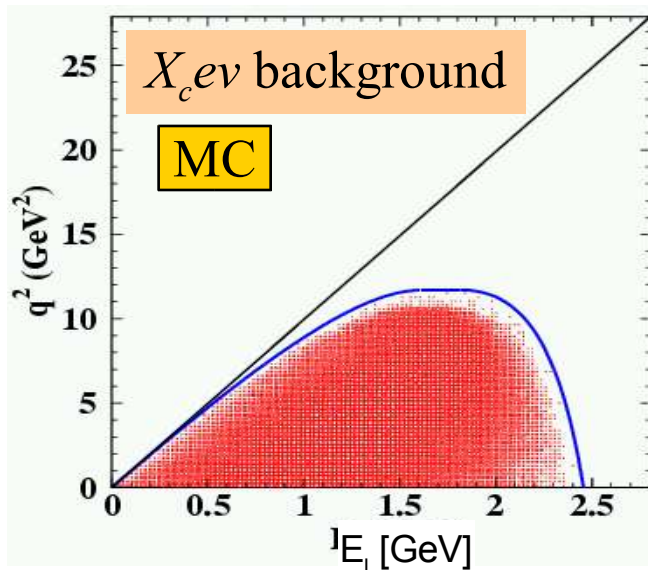


$$\text{partial BF} = (4.85 \pm 0.29_{(\text{stat})} \pm 0.53_{(\text{syst})}) \times 10^{-4}$$

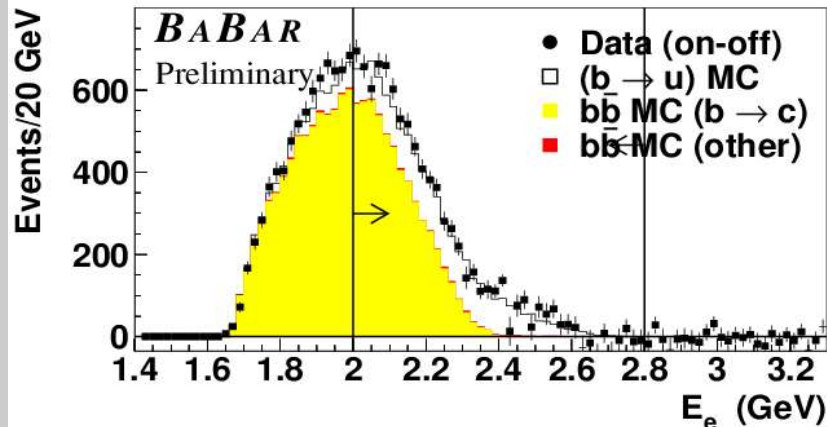
$$|V_{ub}| = (4.40 \pm 0.13_{(\text{stat})} \pm 0.25_{(\text{syst})} \pm 0.38_{(\text{theo})}) \times 10^{-3}$$

$|V_{ub}| - E_1$ versus q^2

$$s_h^{\max} = m_B^2 + q^2 - 2m_B E_e \sqrt{\frac{1 \mp \beta}{1 \pm \beta}} - 2m_B \left(\frac{q^2}{4E_e} \right) \sqrt{\frac{1 \pm \beta}{1 \mp \beta}} \quad \text{with boost } \beta=0.06$$



80 fb⁻¹

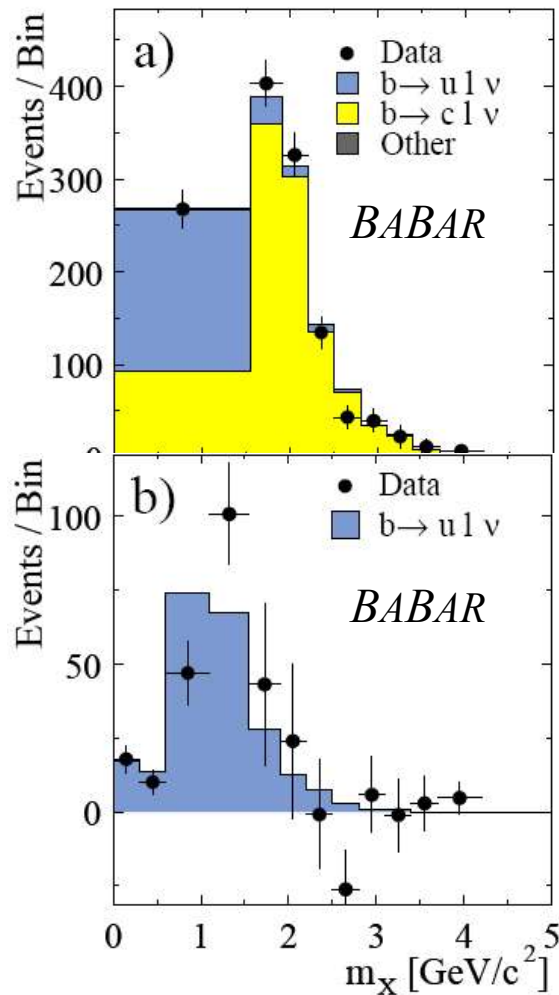


$$B = (2.76 \pm 0.26_{\text{stat}} \pm 0.50_{\text{syst}} \pm 0.21_{\text{SF}}) \times 10^{-3}$$

$$|V_{ub}| = (4.99 \pm 0.48_{\text{exp}} \pm 0.18_{\text{SF}} \pm 0.22_{\text{OPE}}) \times 10^{-3}$$

$|V_{ub}| - m_X$ spectrum

hep-ex/0408068



- Use fully-reconstructed B events
- reconstruct m_X of the recoil system analog to $b \rightarrow cl\nu$
- Background modelling from simulation
- Fit m_X signal+background to data
- Integrate background subtracted m_X -spectrum up to $m_X = 1.55\text{GeV}$
- Extract $|V_{ub}|$ following the **DeFazio-Neubert^[1]** approach

80 fb⁻¹

$$B(B \rightarrow X_u \ell \nu) = (2.81 \pm 0.32_{\text{stat}} \pm 0.31_{\text{syst}} \pm 0.23_{\text{theo}}) \times 10^{-3}$$

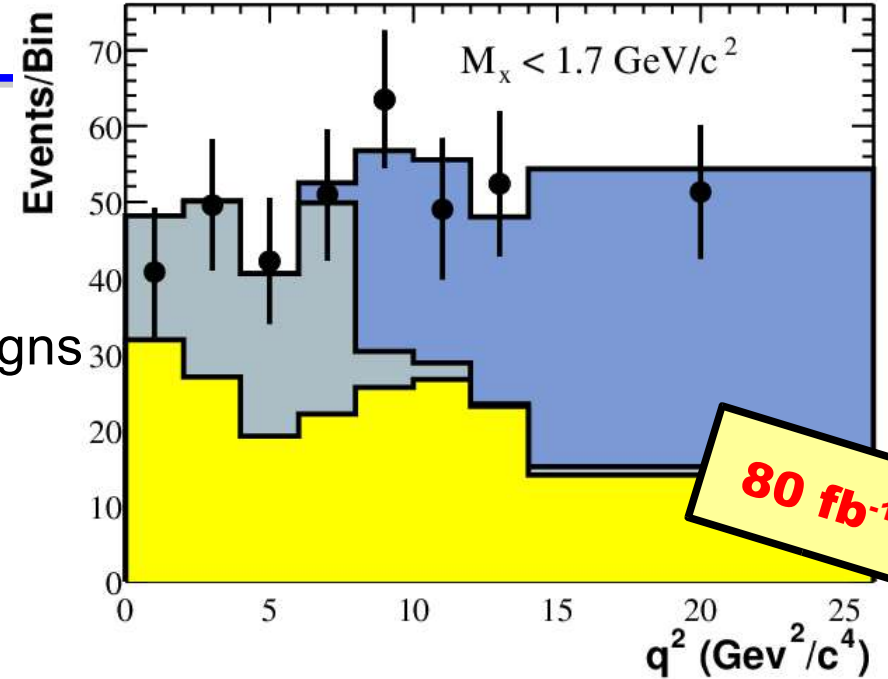
$$|V_{ub}| = (5.22 \pm 0.30_{\text{stat}} \pm 0.31_{\text{syst}} \pm 0.43_{\text{theo}}) \times 10^{-3}$$

[1] DeF. N. JHEP 06, 017 (1999)

$|V_{ub}| - m_X$ versus q^2

hep-ex/0408068

- Ansatz analog to E_l versus q^2
 - this approach has a potentially **smaller theoretic uncertainty**
 - avoids phase space region where SF reigns
- Experimental technique similar to previous analysis (m_X)
 - **but:** need to determine partial **branching fraction as function of q^2**
- Extract $|V_{ub}|$ following the approach of Bauer, Ligeti, Luke
(BLL hep-ph/0111387):



$$|V_{ub}| = \sqrt{\frac{192\pi^3}{\tau_B G_F^2 m_b^5} \frac{\Delta\mathcal{B}(\bar{B} \rightarrow X_u \ell \bar{\nu})}{G}}$$

$$\Delta\mathcal{B} = (0.90 \pm 0.14_{(\text{stat})} \pm 0.14_{(\text{syst})} \pm 0.02_{(\text{theo})}) \times 10^{-3}$$

$$|V_{ub}| = (4.98 \pm 0.4_{(\text{stat})} \pm 0.39_{(\text{syst})} \pm 0.47_{(\text{theo})}) \times 10^{-3}$$

$|V_{ub}|$ - Unfolding the m_X spectrum

hep-ex/0408068

In order to turn the measured m_X spectrum into an observable which can be

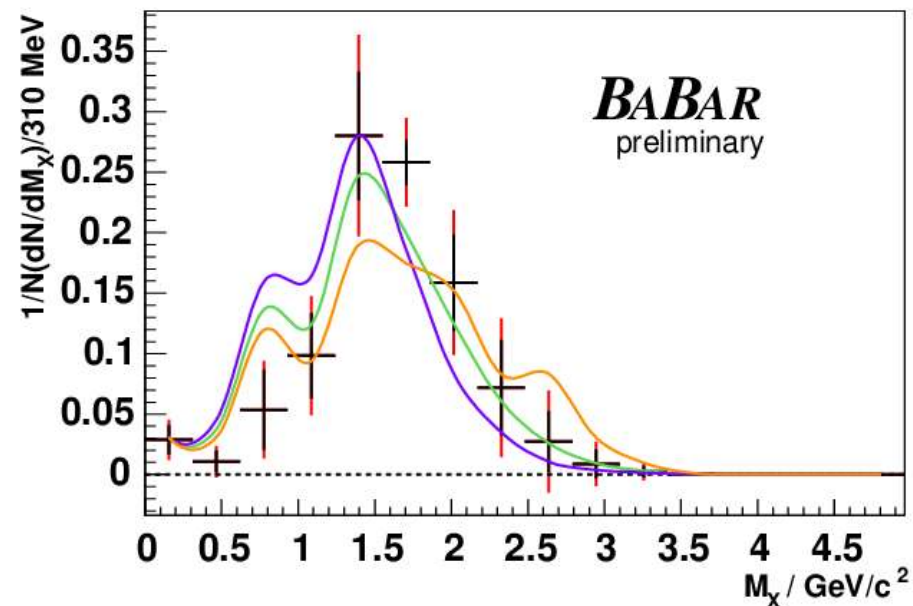
- compared to other experiments, and
- used to extract spectral moments

it is necessary to unfold the detector response.

	M_{max}	value
M_1	<1.86	1.355 ± 0.084
M_2'	<1.86	0.147 ± 0.034
M_1	<5.0	1.584 ± 0.233
M_2'	<5.0	0.270 ± 0.099

With more statistics:

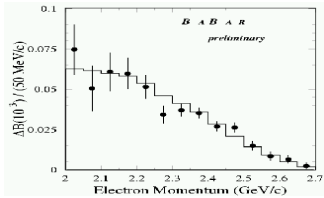
- crosscheck OPE fit results
- check consistency with SF parameterization



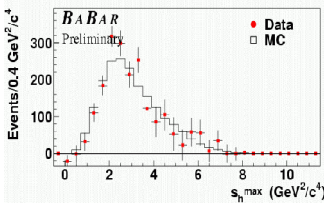
80 fb⁻¹

Summary

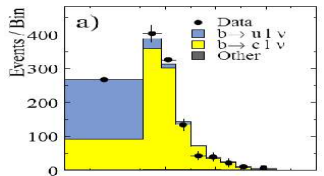
|Vub|



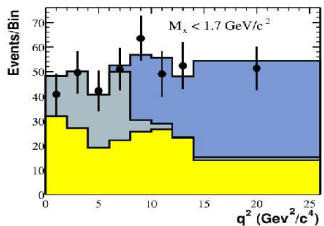
Lepton Endpoint
 $4.40 \pm 0.28_{(\text{exp})} \pm 0.38_{(\text{theo})}$



E_l vs. q^2
 $4.99 \pm 0.48_{(\text{exp})} \pm 0.32_{(\text{theo})}$

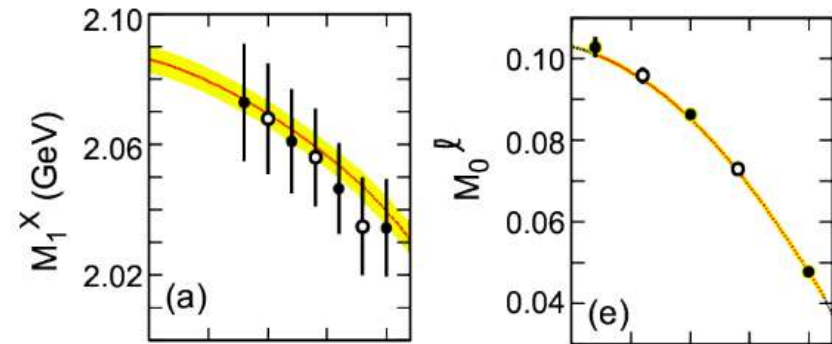


m_X spectrum
 $5.22 \pm 0.42_{(\text{exp})} \pm 0.43_{(\text{theo})}$



m_X vs. q^2
 $4.98 \pm 0.54_{(\text{exp})} \pm 0.47_{(\text{theo})}$

|Vcb|

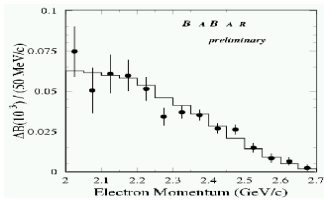


OPE fit to E_l and m_X moments:

$$|Vcb| = (41.4 \pm 0.4_{\text{exp}} \pm 0.4_{\text{HQE}} \pm 0.6_{\text{theo}}) \times 10^{-3}$$

Summary

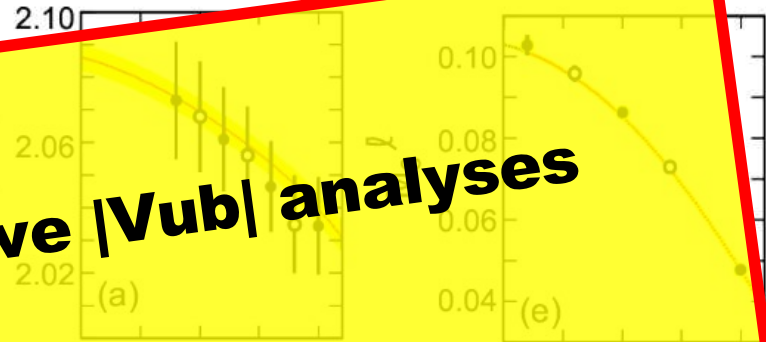
|Vub|



Lepton Endpoint

$$4.40 \pm 0.28_{(exp)} \pm 0.30_{(theo)}$$

|Vcb|

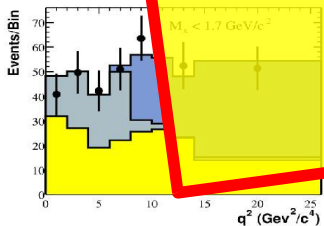
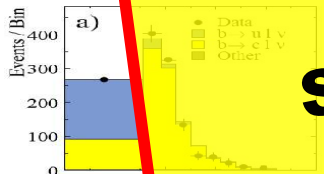
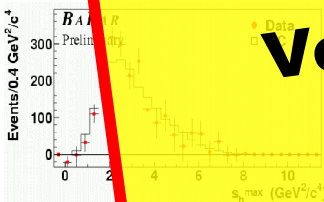


Very soon:

new / updated exclusive |Vub| analyses

Soon:

new results with $b \rightarrow s\gamma$ from BABAR



$$5.22 \pm 0.42_{(exp)} \pm 0.40_{(theo)}$$

$$4.98 \pm 0.54_{(exp)} \pm 0.47_{(theo)}$$

$$|Vcb| = (41.4 \pm 0.4_{exp} \pm 0.4_{HQE} \pm 0.6_{theo}) \times 10^{-3}$$

HQE Fit Results

(kinetic mass scheme, scale $\mu=1$) Gambino & Uraltsev
 hep-ph/0401063 & 0403166

$$|V_{cb}| = (41.4 \pm 0.4_{\text{exp}} \pm 0.4_{\text{HQE}} \pm 0.6_{\text{th}}) \times 10^{-3}$$

$$B_{c\ell\nu} = (10.61 \pm 0.16_{\text{exp}} \pm 0.06_{\text{HQE}})\%$$

$$m_b = (4.61 \pm 0.05_{\text{exp}} \pm 0.04_{\text{HQE}} \pm 0.02_{\alpha_s}) \text{ GeV}$$

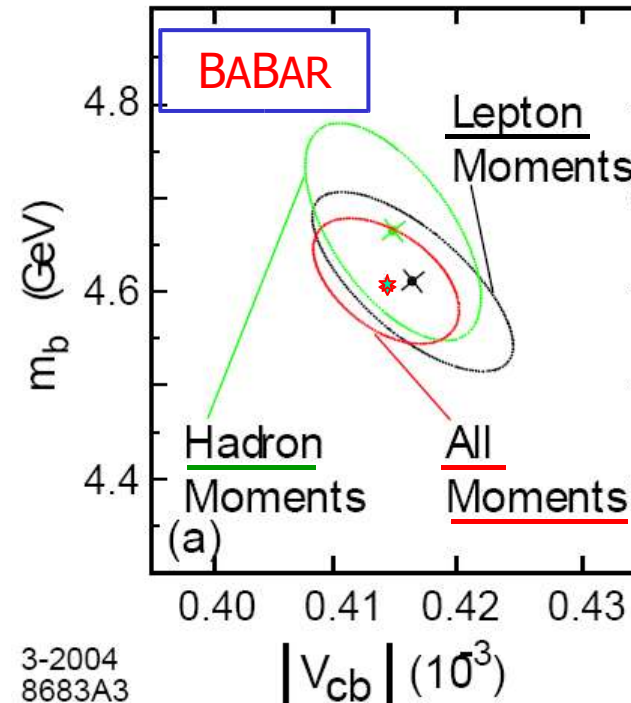
$$m_c = (1.18 \pm 0.07_{\text{exp}} \pm 0.06_{\text{HQE}} \pm 0.02_{\alpha_s}) \text{ GeV}$$

$$\mu_\pi^2 = (0.45 \pm 0.04_{\text{exp}} \pm 0.04_{\text{HQE}} \pm 0.01_{\alpha_s}) \text{ GeV}^2$$

$$\mu_G^2 = (0.27 \pm 0.06_{\text{exp}} \pm 0.03_{\text{HQE}} \pm 0.02_{\alpha_s}) \text{ GeV}^2$$

$$\rho_D^3 = (0.20 \pm 0.02_{\text{exp}} \pm 0.02_{\text{HQE}} \pm 0.00_{\alpha_s}) \text{ GeV}^3$$

$$\rho_{LS}^3 = (-0.09 \pm 0.04_{\text{exp}} \pm 0.07_{\text{HQE}} \pm 0.01_{\alpha_s}) \text{ GeV}^3$$



- ❖ Separate fits to hadron and lepton moments give consistent results
- ❖ μ_G^2 and ρ_{LS}^3 are consistent with B-B* mass splitting and HQ sum rules
- ❖ Considerable improvement in precision for $|V_{cb}|$ ($\pm 2\%$) and $B_{c\ell\nu}$ (1.6%) and quark masses (factor of 6), as well as HQE parameters