La Thuile, 28 February '05

Models of Neutrino Masses & Mixings

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Some recent work by our group

G.A., F. Feruglio, I. Masina, hep-ph/0210342 (Addendum: v2 in Nov. '03), hep-ph/0402155.

Reviews:

G.A., F. Feruglio, New J.Phys.6:106,2004 [hep-ph/0405048]; G.A., hep-ph/0410101; F. Feruglio, hep-ph/0410131

The current experimental situation is still unclear

- LSND: true or false?
- what is the absolute scale of ν masses?
- ...

Different classes of models are still possible:

If LSND true

sterile ν(s)??

CPT violat'n??

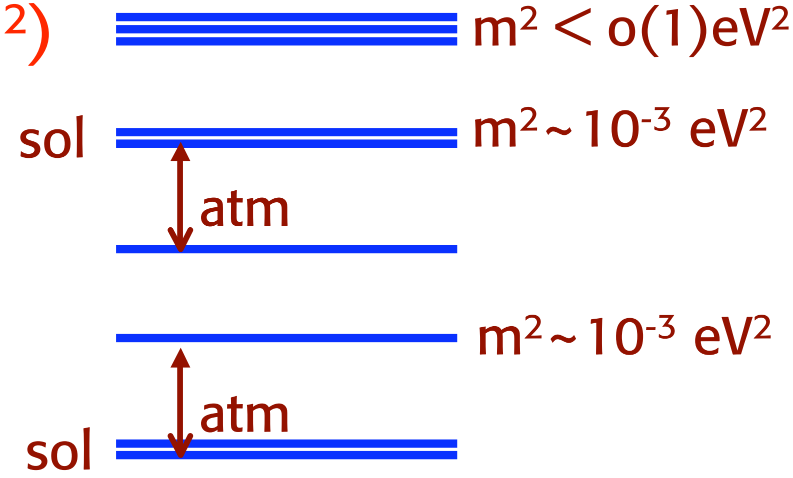
- "3-1" or "3-n"

$\nu_{sterile}$

If LSND false → 3 light ν's are OK

We assume this case here

- Degenerate ($m^2 >> \Delta m^2$)
- Inverse hierarchy
- Normal hierarchy



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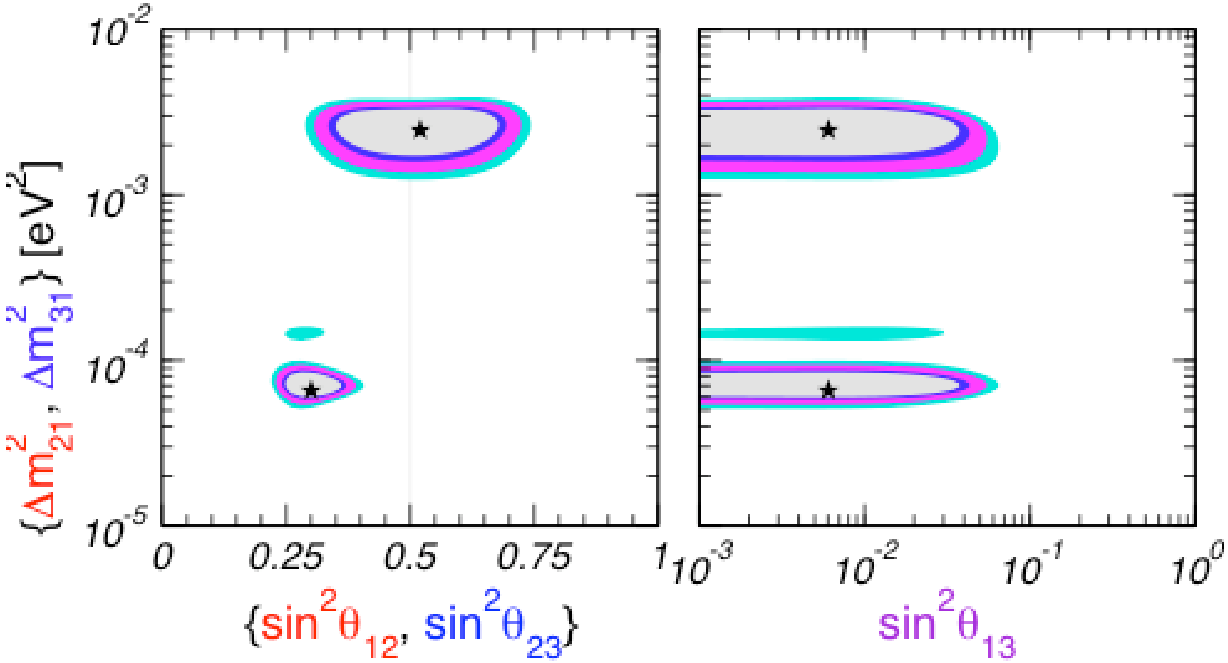
Neutrino oscillation parameters

Maltoni et al '04

parameter	best fit	2σ	3σ	5σ
$\Delta m^2_{21}\ [10^{-5}\text{eV}^2]$	6.9	6.0–8.4	5.4–9.5	2.1–28
$\Delta m^2_{31}\ [10^{-3}\text{eV}^2]$	2.6	1.8–3.3	1.4–3.7	0.77–4.8
$\sin^2\theta_{12}$	0.30	0.25–0.36	0.23–0.39	0.17–0.48
$\sin^2\theta_{23}$	0.52	0.36–0.67	0.31–0.72	0.22–0.81
$\sin^2\theta_{13}$	0.006	$\lesssim 0.035$	$\lesssim 0.054$	$\lesssim 0.11$



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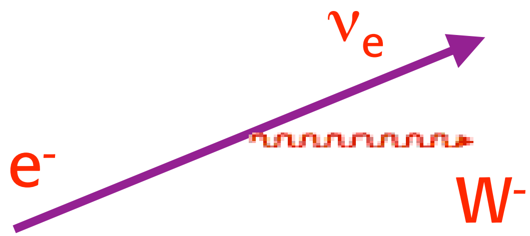
3-ν Models

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

flavour mass



$U = U_{P\text{-}MNS}$

Pontecorvo
Maki, Nakagawa, Sakata

In basis where e^-, μ^-, τ^- are diagonal:

δ: CP violation

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim$$

s = solar: large

CHOOZ: $|s_{13}| < \sim 0.2$

$$\sim \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ \ldots & \ldots & c_{13}s_{23} \\ \ldots & \ldots & c_{13}c_{23} \end{bmatrix}$$

atm.: ~ max

$$U \cong \begin{bmatrix} c & -s & 0 \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(some signs are conventional)

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$$m_\nu \sim U^* \begin{bmatrix} e^{i\phi_1} m_1 & 0 & 0 \\ 0 & e^{i\phi_2} m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} U^+$$

$L^T m_\nu L$

In general 9 parameters:
3 masses, 3 angles,
3 phases

For $s_{13} \sim 0$:

$0\nu\beta\beta$

$$m_\nu \sim \begin{bmatrix} m_1c^2+m_2s^2 & (m_1-m_2)cs/\sqrt{2} & (m_1-m_2)cs/\sqrt{2} \\ \dots & (m_1s^2+m_2c^2+m_3)/2 & (m_1s^2+m_2c^2-m_3)/2 \\ \dots & \dots & (m_1s^2+m_2c^2+m_3)/2 \end{bmatrix}$$

Note:
- m_ν is symmetric
- phases included in m_i

Relation between masses and frequencies:

$P(\nu_e \leftrightarrow \nu_\mu) = P(\nu_e \leftrightarrow \nu_\tau) = 1/2 \sin^2 2\theta_{12} \cdot \sin^2\Delta_{sun}$

$P(\nu_\mu \leftrightarrow \nu_\tau) = \sin^2\Delta_{atm} - 1/4 \sin^2 2\theta_{12} \cdot \sin^2\Delta_{sun}$

$$\Delta_{sun} = \frac{m_2^2 - m_1^2}{4E} L \quad ; \qquad \Delta_{atm} = \frac{m_3^2 - m_{1,2}^2}{4E} L$$

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In our def.: $\Delta_{sun} > 0$, $\Delta_{atm} >$ or < 0

ν oscillations measure Δm^2. What is m^2?

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$; $\Delta m^2_{sun} \sim 8\ 10^{-5}\ eV^2$

End-point tritium β decay (Mainz, Troitsk)

- Direct limits

$m_{"\nu e"} < 2.2$ eV

$m_{"\nu\mu"} < 170$ KeV

$m_{"\nu\tau"} < 18.2$ MeV

$m_{ee} = |\Sigma\, U_{ei}^2\, m_i|$

- $0\nu\beta\beta$ $m_{ee} < 0.2 - 0.5 - ?$ eV (nucl. matrix elmnts)

Evidence of signal?   Klapdor-Kleingrothaus

- Cosmology $\Omega_\nu\, h^2 \sim \Sigma_i m_i\, /94 eV$ $(h^2 \sim 1/2)$

$\Sigma_i m_i < 0.7\text{-}1.8\text{-}?$ eV (dep. on priors)

WMAP, 2dFGRS...

→ Any ν mass $< 0.23\text{-}0.6\text{-}?$ eV

Why ν's so much lighter than quarks and leptons?
Because ν's are Majorana particles: $m_\nu \sim m^2/M$

Lahav

Neutrino mass from Cosmology

Data	Authors	$M_\nu = \Sigma m_i$ 95%cl
2dFGRS	Elgaroy et al. 02	< 1.8 eV
WMAP+2dF+…	Spergel et al. 03	< 0.7 eV
WMAP+2dF	Hannestad 03	< 1.0 eV
SDSS+WMAP	Tegmark et al. 04	< 1.7 eV
WMAP+2dF+ SDSS	Crotty et al. 04	< 1.0 eV

By itself CMB (WMAP, ACBAR) do not fix M_ν
Only in combination with galaxy power spectrum (2dFGRS, SDSS) become sensitive.

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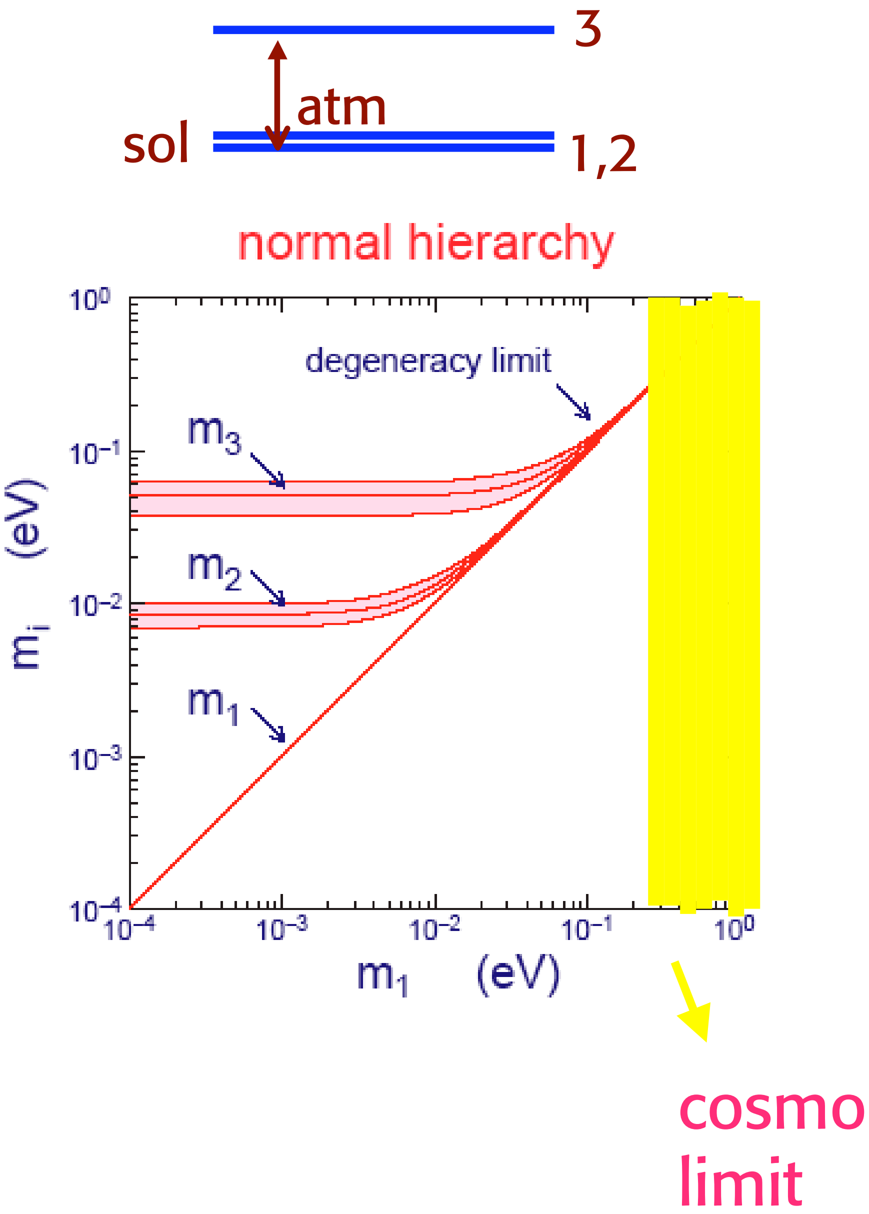

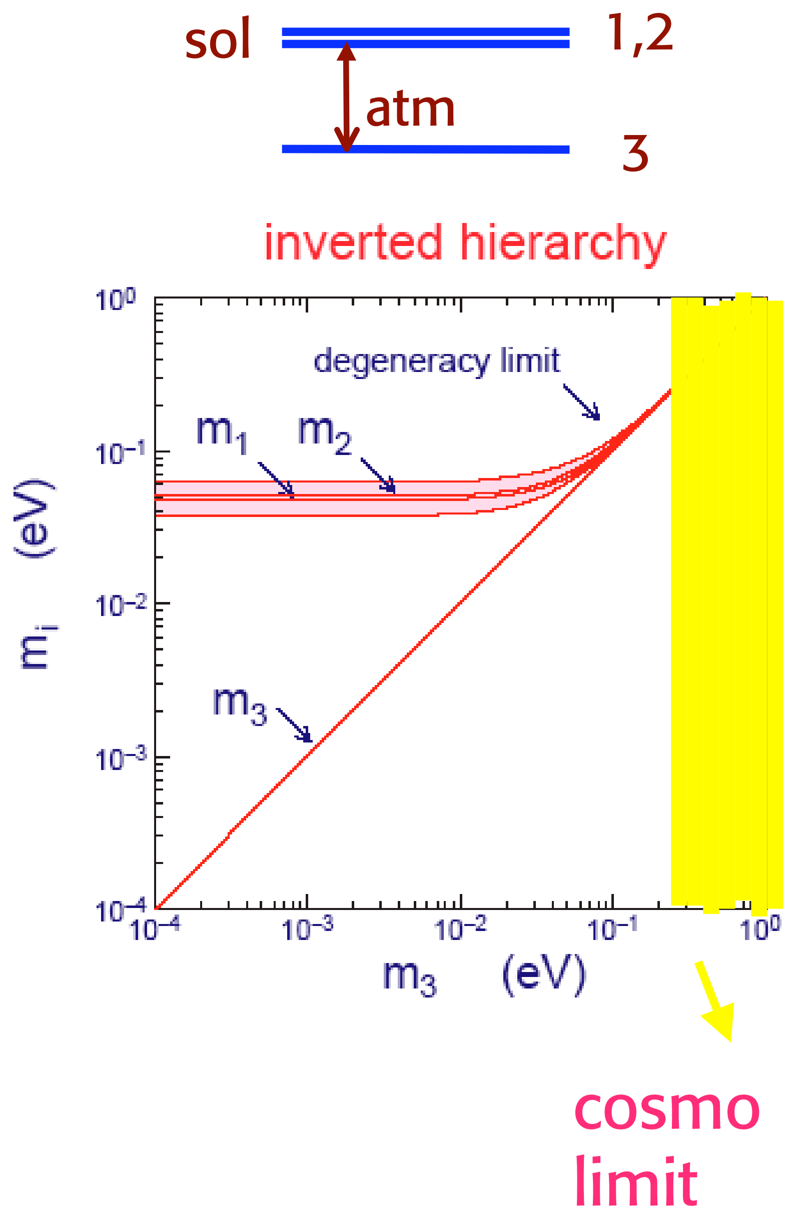

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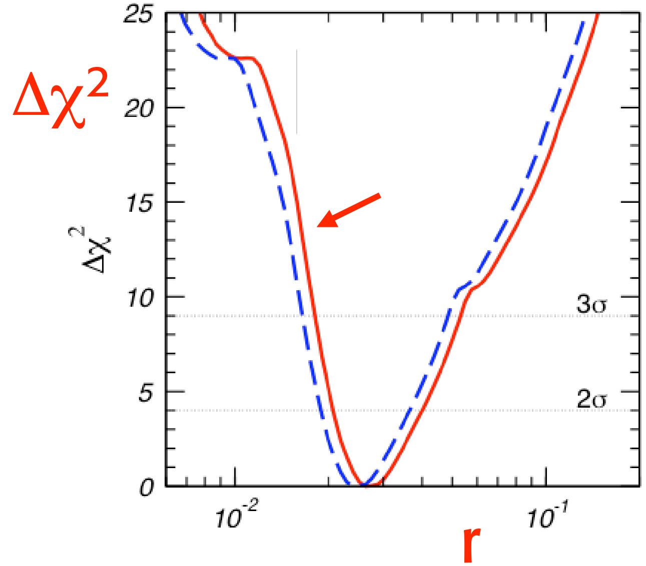
Only moderate degeneracy allowed

After KamLAND, SNO and WMAP not too much hierarchy is needed for ν masses:

$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/35$



Precisely at 3σ: $0.018 < r < 0.053$

or

$m_{heaviest} < 1 - 0.6$ eV
$m_{next} > \sim 8\ 10^{-3}$ eV

For a hierarchical spectrum: $\frac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$

Comparable to: $\lambda_C \approx 0.22$ or $\sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$

Suggests the same "hierarchy" parameters for q, l, ν

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→ e.g. θ_{13} not too small!

$0\nu\beta\beta$ can tell degenerate, inverted or normal hierarchy

$$|m_{ee}| = c_{13}^2\,[m_1 c_{12}^2 + e^{i\alpha} m_2 s_{12}^2] + m_3 e^{i\beta} s_{13}^2$$

LA: ~0.3-1

Degenerate: $\sim |m|\,|c_{12}^2 + e^{i\alpha} s_{12}^2|$

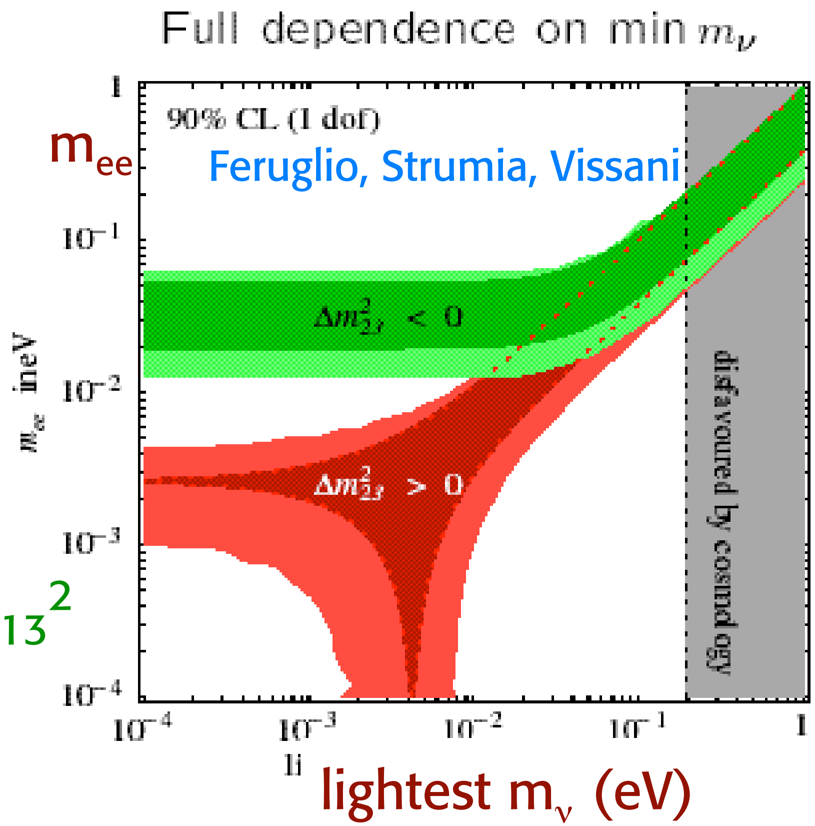
$|m_{ee}| \sim |m|\,(0.3\ \text{-}1) < 0.23\text{-}1$ eV

IH: $\sim (\Delta m^2_{atm})^{1/2} |c_{12}^2 + e^{i\alpha} s_{12}^2|$

$|m_{ee}| \sim (1.6\text{-}5)\ 10^{-2}$ eV

NH: $\sim (\Delta m^2_{sol})^{1/2} s_{12}^2 + (\Delta m^2_{atm})^{1/2} e^{i\beta} s_{13}^2$

$|m_{ee}| \sim$ (few) 10^{-3} eV



Present exp. limit: $m_{ee} < 0.3\text{-}0.5$ eV
(and a hint of signal????? Klapdor Kleingrothaus)

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- Still large space for non maximal 23 mixing

3-σ interval $0.31 < \sin^2\theta_{23} < 0.72$

Maximal θ_{23} theoretically hard

- θ_{13} not necessarily too small
  probably accessible to exp.

$\sin\theta_{13} \sim 1/2 \sin\theta_{12}$
not excluded!

Very small θ_{13} theoretically hard

Normal models: θ_{23} large but not maximal,
θ_{13} not too small (θ_{13} of order λ_C or λ_C^2)
Exceptional models: θ_{23} maximal and/or θ_{13} very small
or also: all mixing from the charged lepton sector....

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$U = U_e^+ U_\nu$

Degenerate ν's

$m^2 >> \Delta m^2$

- Apriori compatible with hot dark matter ($m \sim 1\text{-}2$ eV)
  → was considered by many
- Limits on m_{ee} from $0\nu\beta\beta$ then imply large mixing also for solar oscillations: (Vissani; Georgi,Glashow)

$m_{ee} < 0.3\text{-}0.5$ eV (Exp)

$$m_{ee} = c^2_{13}(m_1 c^2_{12} + m_2 s^2_{12}) + s^2_{13} m_3 \sim m_1 c^2_{12} + m_2 s^2_{12}$$

If $|m_1| \sim |m_2| \sim |m_2| \sim 1\text{-}2$ eV → $m_1 = -m_2$ and $c^2_{12} \sim s^2_{12}$

LA solution: $\sin^2\theta \sim 0.3$ → $\cos^2\theta - \sin^2\theta \sim 0.4$

a moderate suppression factor!

Trusting WMAP&2dF: $|m| < 0.23$ eV, only a moderate degeneracy is allowed: for LA, $m/(\Delta m^2_{atm})^{1/2} < 5$, $m/(\Delta m^2_{sol})^{1/2} < 30$.

Less constraints from $0\nu\beta\beta$ (both $m_1 = \pm m_2$ allowed)

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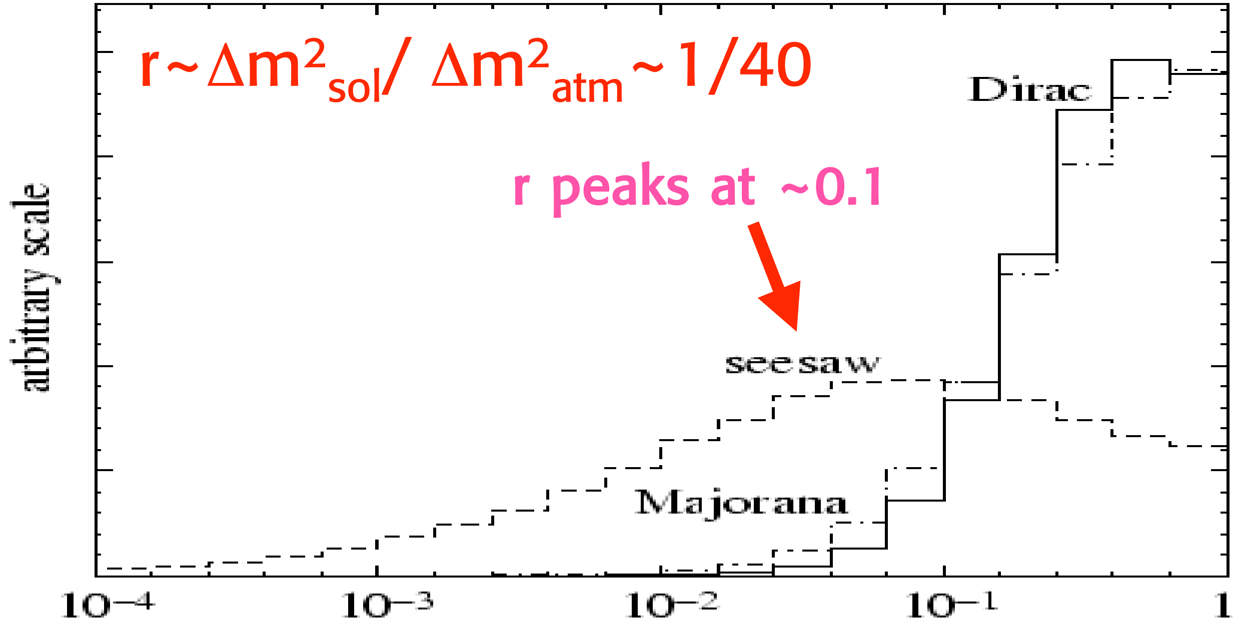
Recall: leptogenesis prefers $|m| < 0.1$ eV

Anarchy (or accidental hierarchy): No structure in the leptonic sector

Hall, Murayama, Weiner

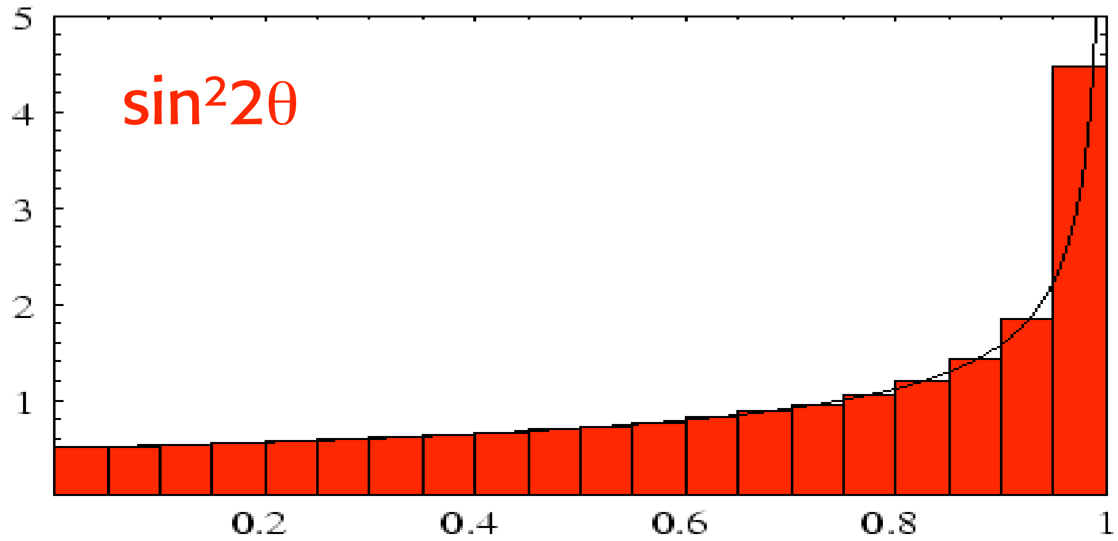
See-Saw:
$m_\nu \sim m^2/M$
produces hierarchy
from random m,M

could fit the data



But: all mixing angles
should be large →

marginal: predicts
θ_{13} near bound



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Semianarchy: no structure in 23

Consider a matrix like $m_\nu \sim \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}$

Note: $\theta_{13} \sim \lambda$
$\theta_{23} \sim 1$

with coeff.s of o(1) and det23~o(1)
[λ~1 corresponds to anarchy]

After 23 and 13 rotations $m_\nu \sim \begin{pmatrix} \lambda^2 & \lambda & 0 \\ \lambda & \eta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Normally two masses are of o(1) and $\theta_{12} \sim \lambda$
But if, accidentally, $\eta \sim \lambda$, then the solar angle is also large.

The advantage over anarchy is that θ_{13} is small, but
the hierarchy $m^2_3 >> m^2_2$ is accidental

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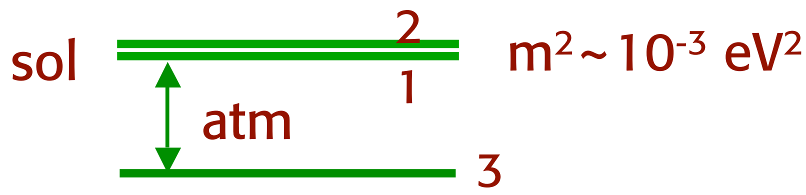
Ramond et al, Buchmuller et al

Inverted Hierarchy

Zee, Joshipura et al;
Mohapatra et al; Jarlskog et al;
Frampton,Glashow; Barbieri et al
Xing; Giunti, Tanimoto.......



An interesting model:

An exact U(1) L_e-L_μ-L_τ symmetry for m_ν predicts:

(a good 1st approximation)

$$m_\nu = U m_{\nu diag} U^T = m \begin{bmatrix} 0 & a & -b \\ a & 0 & 0 \\ -b & 0 & 0 \end{bmatrix} \quad \text{with} \quad m_{\nu diag} = \begin{bmatrix} m & 0 & 0 \\ 0 & -m & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- $\theta_{13} = 0$
- $\theta_{12} = \pi/4$ ← θ_{sun} maximal!
- $\sin^2\theta_{23} = b^2$ ← θ_{atm} generic

Can arise from see-saw or dim-5 $L^T H H^T L$

- 1-2 degeneracy stable under rad. corr.'s

1^{st} approximation

$$m_{\nu diag} = \begin{bmatrix} m & 0 & 0 \\ 0 & -m & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad m_\nu = U m_{\nu diag} U^T = m \begin{bmatrix} 0 & a & -b \\ a & 0 & 0 \\ -b & 0 & 0 \end{bmatrix}$$

• Data? This texture prefers θ_{sol} closer to maximal than θ_{atm}
i.e $\theta_{sol} - \pi/4$ small for $(\Delta m^2_{sol}/\Delta m^2_{atm})_{LA} \sim 1/40$

In fact: 12-> $\begin{bmatrix} 0 & a \\ a & 0 \end{bmatrix} \rightarrow$ Pseudodirac θ_{12} maximal   23-> $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \rightarrow \theta_{23} \sim o(1)$

With perturbations: $\begin{bmatrix} 0 & a & -b \\ a & 0 & 0 \\ -b & 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} \delta & 1 & 1 \\ 1 & \eta & \eta \\ 1 & \eta & \eta \end{bmatrix}$ (modulo o(1) coeff.s)

one gets $1 - tg^2\,\theta_{12} \sim o(\delta + \eta) \sim (\Delta m^2_{sol}/\Delta m^2_{atm})_{LA}$

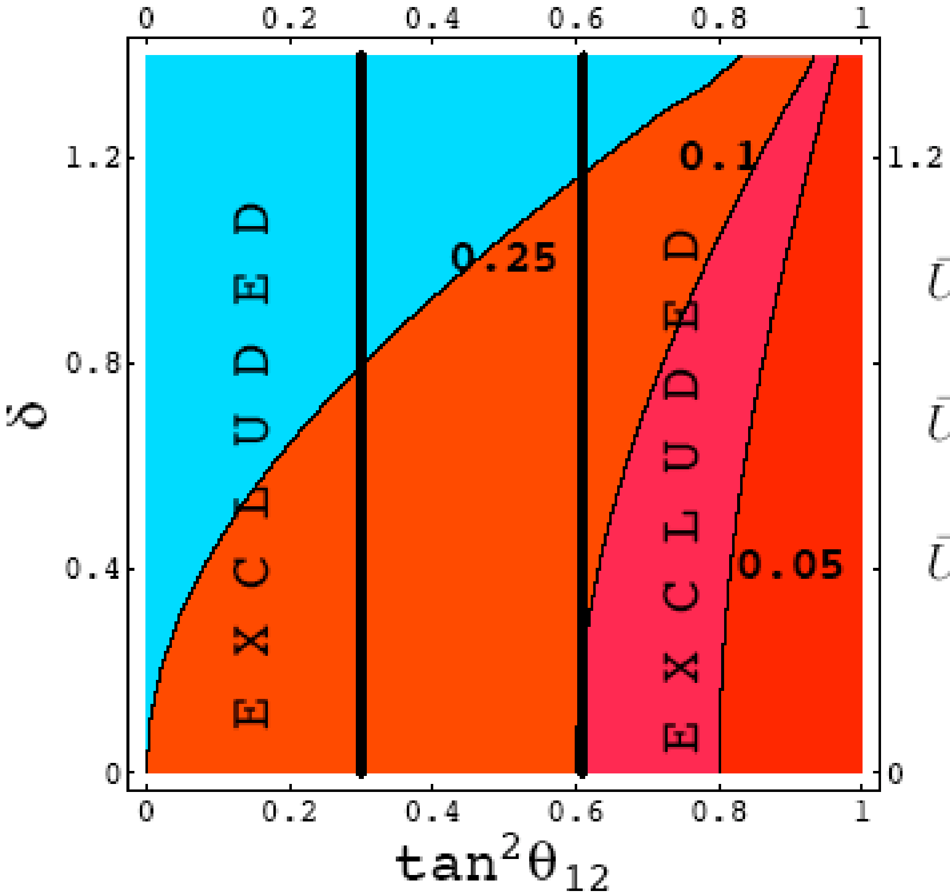
Exp. (3σ): 0.39-0.70   0.024-0.060

- In principle one can use the charged lepton mixing to go away from θ_{12} maximal.
  In practice constraints from θ_{13} small ($\delta\theta_{12} \sim \theta_{13}$)

Frampton et al; GA, Feruglio, Masina '04

For the corrections from the charged lepton sector, typically $|\sin\theta_{13}| \sim (1 - \tan^2\theta_{12})/4\cos\delta \sim 0.15$

GA, Feruglio, Masina '04



$$\bar{U}_{12} = -\frac{e^{-i(\alpha_1+\alpha_2)}}{\sqrt{2}} + \frac{s^e_{12}e^{-i\alpha_2} + s^e_{13}e^{i\delta_e}}{2}$$

$$\bar{U}_{13} = \frac{s^e_{12}e^{-i\alpha_2} - s^e_{13}e^{i\delta_e}}{\sqrt{2}}$$

$$\bar{U}_{23} = -e^{-i\alpha_2}\frac{1 + s^e_{23}e^{i\alpha_2}}{\sqrt{2}}$$

Corr.'s from s^e_{12}, s^e_{13} to U_{12} and U_{13} are of first order (2nd order to U_{23})

- In general: more θ_{12} is close to maximal, more is IH likely

For charged lepton masses
L_e-L_μ-L_τ typically implies:

$$m_e m_e^\dagger \sim \begin{bmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{bmatrix}$$

$\bar{L} m_e R$

$m'_e = U^+ m_e V$

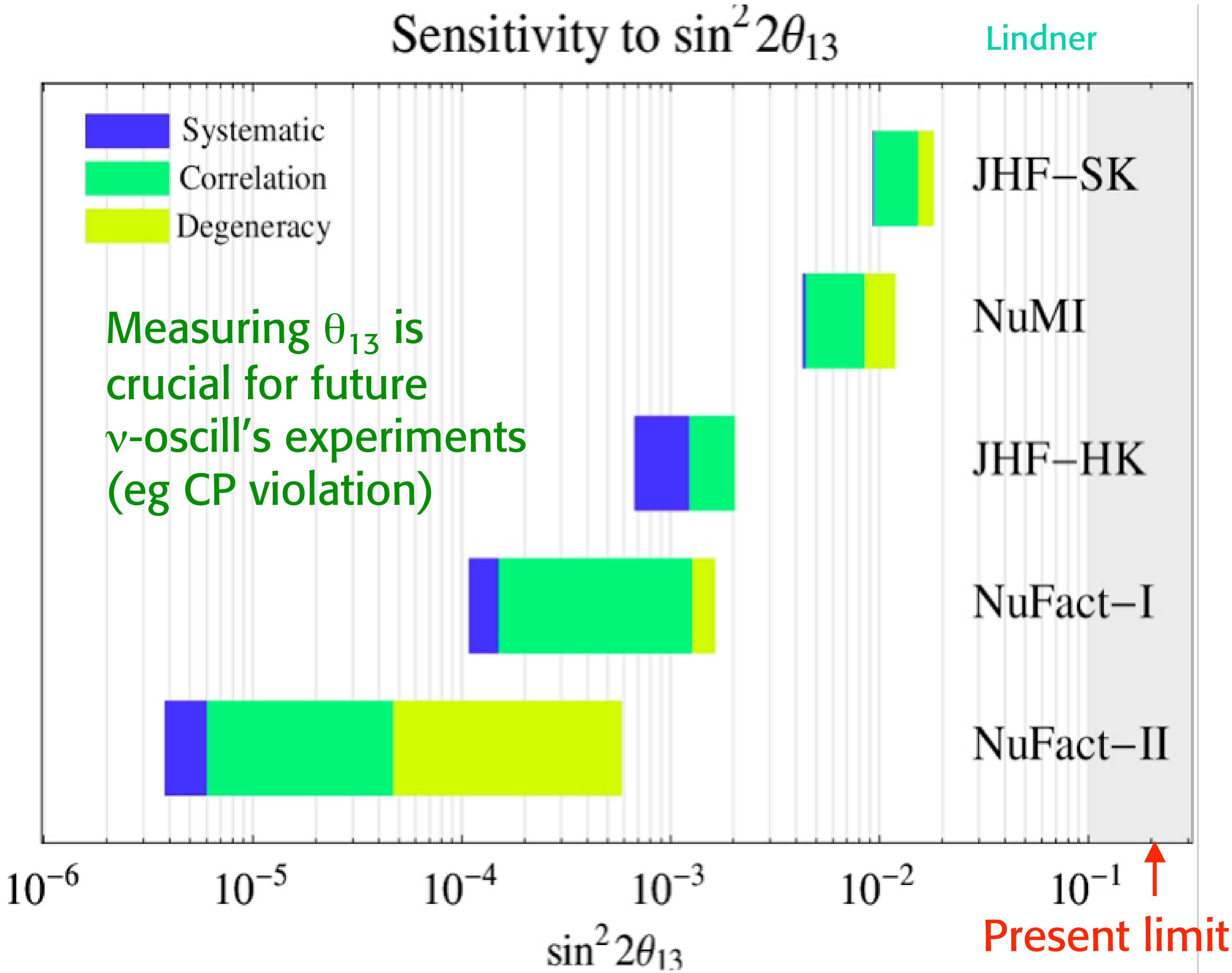
$m'_e m'^+_e = U^+ m_e m_e^+ U$

or $m_e m_e^+$ transforms as $\bar{L}\, L$

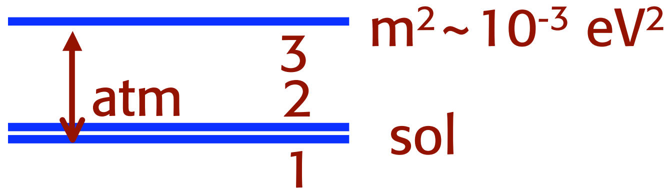
After diagonalisation of charged leptons θ_{23} remains large, while modifications to θ_{13} and θ_{12} are small.

In conclusion IH is viable but prefers θ_{12} close to maximal, and given the exp. value of θ_{12}, needs θ_{13} near its upper bound

[Both anarchy and IH point to θ_{13} near bound]


Sensitivity to $\sin^2 2\theta_{13}$
Lindner
Systematic
Correlation
Degeneracy
JHF−SK
NuMI
JHF−HK
NuFact−I
NuFact−II
Measuring θ_{13} is crucial for future ν-oscill's experiments (eg CP violation)
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
$\sin^2 2\theta_{13}$
Present limit

Normal Hierarchy



- Assume 3 widely split light neutrinos.
- For u, d and l^- Dirac matrices the 3rd generation eigenvalue is dominant.
- May be this is also true for $m_{\nu D}$: diag $m_{\nu D} \sim (0,0,m_{D3})$.

  (but not at all necessary!)
- Assume see-saw is dominant: $m_\nu \sim m^T_D M^{-1} m_D$

  See-saw quadratic in m_D: tends to enhance hierarchy
- Maximally constraining: GUT's relate q, l^-, ν masses!

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- A crucial point: in the 2-3 sector we need both large m_3-m_2 splitting and large mixing.

$$m_3 \sim (\Delta m^2_{atm})^{1/2} \sim 5\ 10^{-2}\ \mathrm{eV}$$
$$m_2 \sim (\Delta m^2_{sol})^{1/2} \sim 8\ 10^{-3}\ \mathrm{eV}$$

- The "theorem" that large Δm_{32} implies small mixing (pert. th.: $\theta_{ij} \sim 1/|E_i - E_j|$) is not true in general: all we need is (sub)det[23]~0

- Example: $m_{23} \sim \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$

Det = 0; Eigenvl's: 0, $1+x^2$
Mixing: $\sin^2 2\theta = 4x^2/(1+x^2)^2$

So all we need are natural mechanisms for det[23]=0

For x~1
large splitting
and large mixing!

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Examples of mechanisms for Det[23]~0

see-saw $m_\nu \sim m^T_D M^{-1} m_D$

1) A ν_R is lightest and coupled to μ and τ

King; Allanach; Barbieri et al......

$$M \sim \begin{bmatrix} \varepsilon & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow M^{-1} \sim \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix}$$

$$m_\nu \sim \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} \approx 1/\varepsilon \begin{bmatrix} a^2 & ac \\ ac & c^2 \end{bmatrix}$$

2) M generic but m_D "lopsided"

$$m_D \sim \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix}$$

Albright, Barr; GA, Feruglio, ...

$$m_\nu \sim \begin{bmatrix} 0 & x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix} = c \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$$

Caution: if 0 -> 0(ε), det23=0 could be spoiled by suitable $1/\varepsilon$ terms in M^{-1}

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An important property of SU(5)

Left-handed quarks have small mixings (V_{CKM}),
but right-handed quarks can have large mixings (unknown).

In SU(5): LH for d quarks ⟷ RH for l^- leptons

$\bar{5}$ → $m_d \sim \bar{d}_R d_L$ ← 10

10 → $m_e \sim \bar{e}_R e_L$ ← $\bar{5}$

$\bar{5}$: $(\bar{d},\bar{d},\bar{d}, \nu, e^-)$ — the three $\bar{d}$ are R; ν, e^- are L

$$m_d = m_e^T$$

cannot be exact, but approx.

Most "lopsided" models are based on this fact. In these models large atmospheric mixing arises (at least in part) from the charged lepton sector.

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- Hierarchical ν's and see-saw dominance

$$L^T m_\nu L \to m_\nu \sim m_D^2/M$$

allow to relate q, l, ν masses and mixings in GUT models.
For dominance of dim-5 operators -> less constraints

$$\lambda^2/M\,(LH)(LH) \to m_\nu \sim \lambda^2 v^2/M$$

- The correct pattern of masses and mixings, also including ν's, is obtained in simple models based on

$SU(5)xU(1)_{flavour}$

Ramond et al; GA, Feruglio+Masina; Buchmuller et al; King et al; Yanagida et al, Berezhiani et al; Lola et al.......

- SO(10) models could be more predictive, as are non abelian flavour symmetries, eg O(3)

Albright, Barr; Babu et al; Buccella et al; Barbieri et al; Raby et al; King, Ross

- The non trivial pattern of fermion masses and mixing demands a flavour structure (symmetry)

- (SUSY) $SU(5)XU(1)_F$ models offer a minimal description of flavour symmetry

- A flexible enough framework used to realize and compare models with anarchy or hierarchy (direct or inverse) in ν sector, with see-saw dominance or not.

- On this basis we found that there is still a significant preference for hierarchy vs anarchy

G.A., F. Feruglio, I. Masina, hep-ph/0210342 (v2 Nov '03)

Previous related work: Haba,Murayama; Hirsch,King; Vissani; Rosenfeld,Rosner; Antonelli et al….

Hierarchy for masses and mixings via horizontal U(1) charges.

Froggatt, Nielsen '79

Principle:

A generic mass term

$\bar{R}_1 m_{12} L_2 H$

is forbidden by U(1)

if $q_1+q_2+q_H$ not 0

q_1, q_2, q_H:
U(1) charges of
$\bar{R}_1$, L_2, H

U(1) broken by vev of "flavon" field θ with U(1) charge q_θ= -1.
The coupling is allowed: if vev θ = w, and w/M=λ we get:

Δ_{charge}

$\bar{R}_1 m_{12} L_2 H \, (\theta/M)^{q1+q2+qH}$ $\quad m_{12} \rightarrow m_{12}\, \lambda^{q1+q2+qH}$

Hierarchy: More Δ_{charge} -> more suppression (λ small)

One can have more flavons (λ, λ', ...)
with different charges (>0 or <0)etc -> many versions

With suitable charge assignments all relevant patterns can be obtained

1st fam. 2nd 3rd

Ψ_{10}: (5, 3, 0)
Ψ_5: (2, 0, 0)
Ψ_1: (1,-1, 0)

Equal 2,3 ch. for lopsided

Recall: u~ 10 10
d=e^T~ $\bar{5}$ 10
ν_D~$\bar{5}$ 1;M_{RR}~ 1 1

Model	Ψ_{10}	$\Psi_{\bar{5}}$	Ψ_1	(H_u, H_d)
Anarchical (A)	(3,2,0)	(0,0,0)	(0,0,0)	(0,0)
Semi-Anarchical (SA)	(2,1,0)	(1,0,0)	(2,1,0)	(0,0)
Hierarchical (H_I)	(6,4,0)	(2,0,0)	(1,-1,0)	(0,0)
Hierarchical (H_{II})	(5,3,0)	(2,0,0)	(1,-1,0)	(0,0)
Inversely Hierarchical (IH_I)	(3,2,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)
Inversely Hierarchical (IH_{II})	(6,4,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)

No structure for leptons → Anarchical (A)

No automatic det23 = 0 → Semi-Anarchical (SA)

all charges positive

Automatic det23 = 0 → Hierarchical

not all charges positive

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All entries are a given power of λ times a free o(1) coefficient

$$m_u \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix}$$

In a statistical approach we generate these coeff.s as random complex numbers $\rho e^{i\phi}$ with $\phi = [0, 2\pi]$ and $\rho = [0.5,2]$ (default) or [0.8,1.2], or [0.95,1.05] or [0,1] (real numbers also considered for comparison)

For each model we evaluate the success rate (over many trials) for falling in the exp. allowed window:

(boundaries $\sim 3\sigma$ limits)

Maltoni et al, hep-ph/0309130

$r \sim \Delta m^2_{sol} / \Delta m^2_{atm}$

$0.018 < r < 0.053$

$|U_{e3}| < 0.23$

$0.30 < \tan^2\theta_{12} < 0.64$

$0.45 < \tan^2\theta_{23} < 2.57$

for each model the λ, λ' values are optimised

The optimised values of λ are of the order of λ_C or a bit larger (moderate hierarchy)

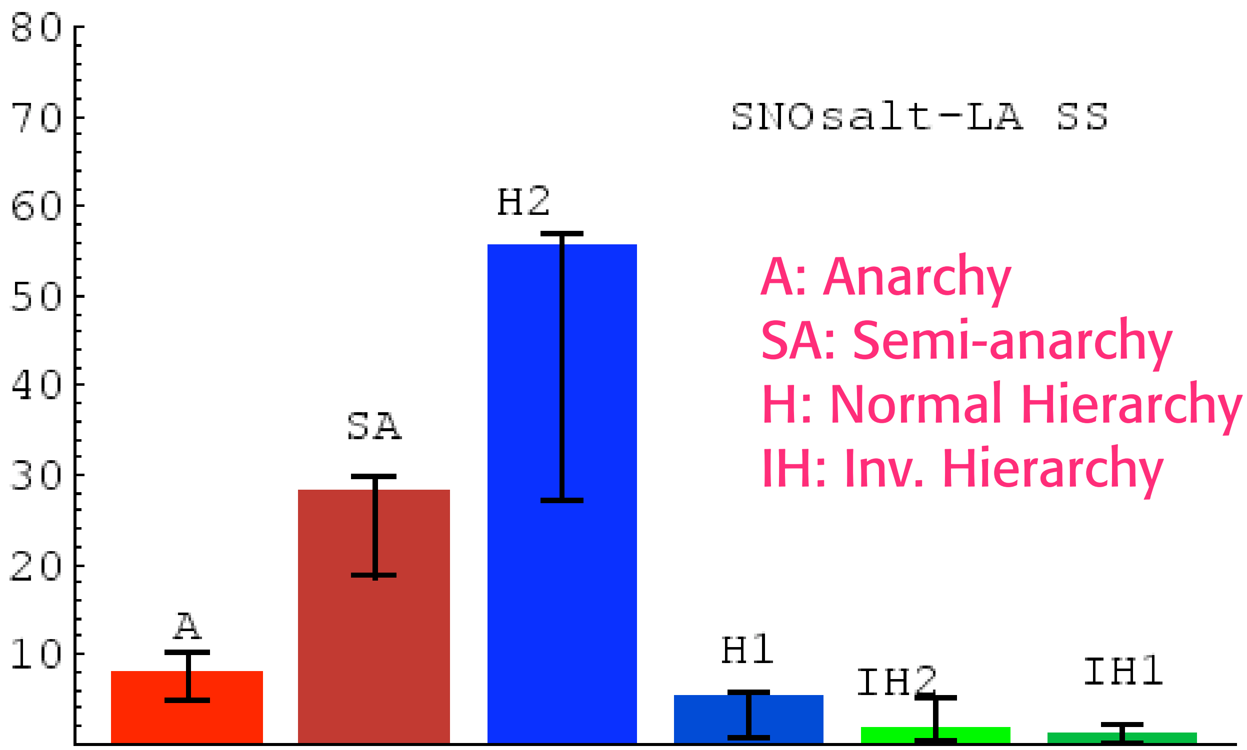
model	$\lambda(=\lambda')$
A_{SS}	0.2
SA_{SS}	0.25
$H_{(SS,II)}$	0.35
$H_{(SS,I)}$	0.45
$IH_{(SS,II)}$	0.45
$IH_{(SS,I)}$	0.25

Results with see-saw dominance (updated in Nov. '03):

1 or 2 refer to models with 1 or 2 flavons of opposite ch.

With charges of both signs and 1 flavon some entries are zero

Scale: Σrates=100



A: Anarchy
SA: Semi-anarchy
H: Normal Hierarchy
IH: Inv. Hierarchy

Errors are linear comb. of stat. and syst. errors (varying the extraction procedure: interval of ρ, real or complex)

H2 is better than SA, better than A, better than IH

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Example: Normal Hierarchy

G.A., Feruglio, Masina

1st fam. → 2nd → 3rd

$q(10)$: (5, 3, 0)
$q(\bar{5})$: (2, 0, 0)
$q(1)$: (1,-1, 0)

Note: not all charges positive
--> det23 suppression

$q(H) = 0,\ q(\bar{H}) = 0$
$q(\theta) = -1,\ q(\theta') = +1$

In first approx., with $\langle\theta\rangle/M \sim \lambda \sim \lambda' \sim 0.35 \sim o(\lambda_C)$

$10_i 10_j$

$$m_u \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix},$$

$10_i \bar{5}_j$

$$m_d = m_e^T \sim v_d \begin{bmatrix} \lambda^7 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^2 & 1 & 1 \end{bmatrix}$$

"lopsided"

$\bar{5}_i 1_j$

$$m_{\nu D} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda' & 1 \\ \lambda & \lambda' & 1 \end{bmatrix},$$

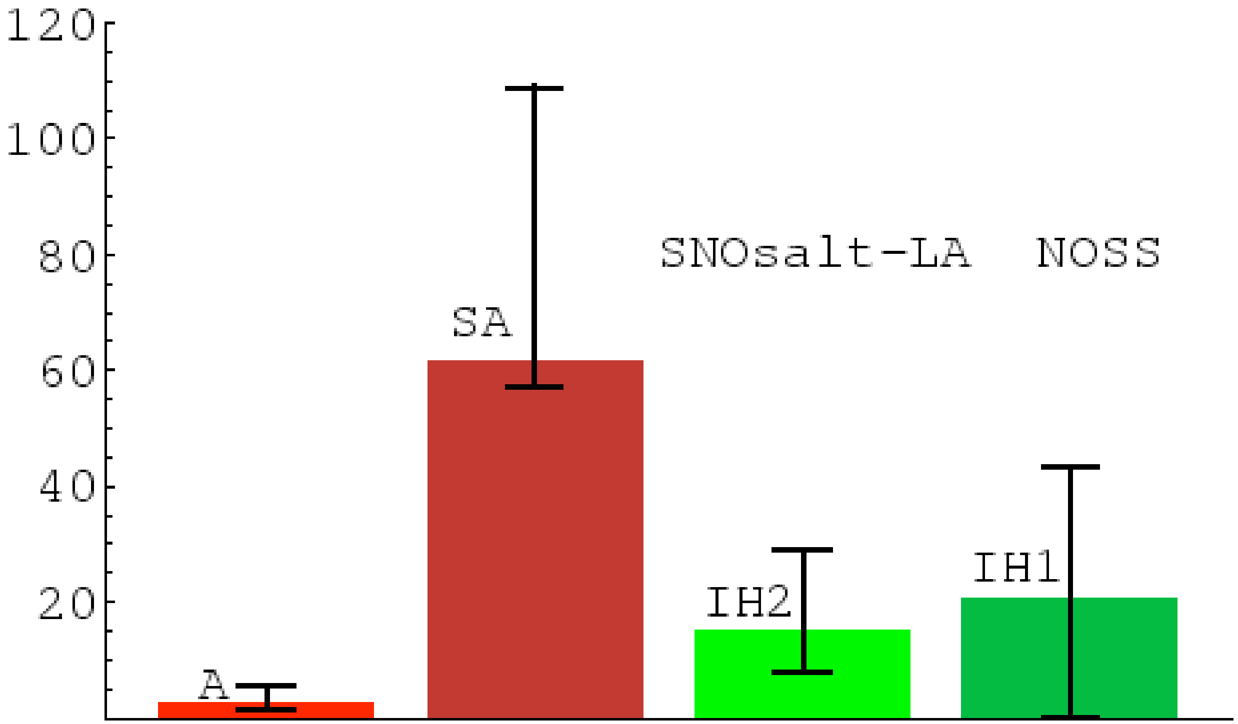
$1_i 1_j$

$$M_{RR} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix}$$

Note: coeffs. O(1) omitted, only orders of magnitude predicted

With no see-saw (m_ν generated directly from $L^T m_\nu L \sim \bar{5}\,\bar{5}$) IH is better than A

[With no-see-saw H coincide with SA]



Note: we always include the effect of diagonalising charged leptons

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What if θ_{23} is really maximal?

Would be challenging!

All existing models invoke peculiar symmetries (non abelian or discrete are crucial) Early models: Barbieri et al, Wetterich....

A set of recent models are based on obtaining, in the basis of (nearly) diagonal charged leptons →

Grimus, Lavoura..., Ma,....

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

This predicts $\theta_{13}=0$ and θ_{23} max.

Imposing a 2-3 perm. symmetry on $L^T m_\nu L$ does not work, because $\overline{R}$ L then produces a charged lepton mixing that spoils θ_{23} max.

In some models, discrete broken symmetries are used to make charged leptons and Dirac neutrino masses diagonal, while the perm. symmetry is in the Majorana RR matrix

Grimus, Lavoura

An interesting particular case

Harrison, Perkins, Scott

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

A simple mixing matrix compatible with all present data

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

In the basis of diagonal ch. leptons:

$m_\nu = U\mathrm{diag}(m_1, m_2, m_3)U^T$

$$m_\nu = \frac{m_3}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{m_2}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{m_1}{6}\begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

Eigenvectors: $m_3 \to \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\quad m_2 \to \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad m_1 \to \frac{1}{\sqrt{6}}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

Same as in Fritzsch models but with 1 and 3 interchanged, so that here θ_{23} is maximal while $\sin^2 2\theta_{12} = 8/9$

Models based on the A4 discrete symmetry (even perm. of 1234) are the best but contrived

Ma..., also: GA, Feruglio (to appear soon)

Can ν mixings arise only from the charged lepton sector?

G.A., Feruglio, Masina '04

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \Longrightarrow U = U_e^+ U_\nu$$

flavour mass

diag of ch leptons

$m_\nu = U^* \, m_\nu^{diag} U^+$

$m_e = V_e m_e^{diag} U_e^+$

$$\begin{cases} \bar{R} m_e L \\ L_{diag} = U_e L \\ R_{diag} = V_e R \end{cases}$$

Assume that, in the lagrangian basis where all symmetries are specified, we have: $U_\nu \sim 1$. Then: $U \sim U_e^+ \sim$

$$\begin{bmatrix} c & -s & 0 \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(small effects like s_{13} can be thought to arise from U_ν- 1. Phases dropped for simplicity)

Given $m_e^{diag} \sim m_\tau diag[0,\eta,1]$ (with $\eta = m_\mu/m_\tau$) we obtain:

$$m_e = V_e m_e^{diag} U \sim V_e m_\tau \begin{bmatrix} 0 & 0 & 0 \\ \frac{s\eta}{\sqrt{2}} & -\frac{c\eta}{\sqrt{2}} & \frac{\eta}{\sqrt{2}} \\ -\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

For $V_e \sim 1$ this is a generalisation of lopsided (s large) but with $det_{12}=0$

Independent of V_e:

$$m_e^+ m_e \sim U^+ (m_e^{diag})^2 U \sim m_\tau^2 \, \frac{1+\eta^2}{2} \cdot \begin{bmatrix} s^2 & -cs & -s(1-2\eta^2) \\ -cs & c^2 & c(1-2\eta^2) \\ -s(1-2\eta^2) & c(1-2\eta^2) & 1 \end{bmatrix}$$

- all matrix elements of same order (because s is large) "democratic" (hierarchy of masses non trivial)
- $s_{13}=0$ (i.e. eigenvector $(c,s,0)^T$) -> first two columns proportional

Note: in minimal SU(5) models $m_e = m_d^T$. This implies $V_e = U_d$

Quark mixings are small: $V_{CKM} = U_u^+ U_d$

Two possibilities:

- Both U_u and U_d nearly diagonal -> $V_e \sim 1$
- $U_u \sim U_d$ nearly equal and non diagonal

This is the way of democratic models:

$U_u \sim U_d \sim U_e \rightarrow V_e \sim U_e$

$V_e \sim 1$

$$m_e = m_\tau \begin{bmatrix} 0 & 0 & 0 \\ \frac{s\eta}{\sqrt{2}} & -\frac{c\eta}{\sqrt{2}} & \frac{\eta}{\sqrt{2}} \\ -\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$V_e \sim U_e$

$$m_e = m_\tau \frac{1+\eta}{2} \cdot \begin{bmatrix} s^2 & -cs & -s(1-2\eta) \\ -cs & c^2 & c(1-2\eta) \\ -s(1-2\eta) & c(1-2\eta) & 1 \end{bmatrix}$$

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The first two columns are proportional

Our general conclusion:

From the charged lepton sector:

a large s_{23} can easily be produced

example: lopsided models

U_e

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

m_e

but different orders for s_{12} and s_{13} is not simple

Still we have formulated a model where all mixings arise naturally from the charged lepton sector.

A set of U(1) charges garantees that m_ν is diagonal

The spectrum of one family is like in the 27 of E6

charged leptons

$$27 = 1 + 10 + 16 = 1 + (5 + \bar{5}) + (1 + \bar{5} + 10)$$

E6 SO(10) SU(5)

A see-saw mechanism involving the two sets of $\bar{5}$ leeds to the required zero determinant condition in m_e

The model works but requires a complicated setup of charges and flavons.

Note that it borrows the see-saw tricks from the neutrino model building

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Conclusion

We favour:

Normal models: θ_{23} large but not maximal, θ_{13} not too small (θ_{13} of order λ_C or λ_C^2 vs θ_{12}, θ_{23} ~o(1))

- Semi anarchy
- Inverse hierarchy (needs θ_{13} close to present bound)

In particular

- Normal hierarchy with suppressed 23 determinant

Exceptional models: θ_{23} maximal or θ_{13} very small or also: all mixing from the charged lepton sector.... are interesting but rather contrived, not very plausible