Running of α_{QED} in small-angle Bhabha scattering at LEP

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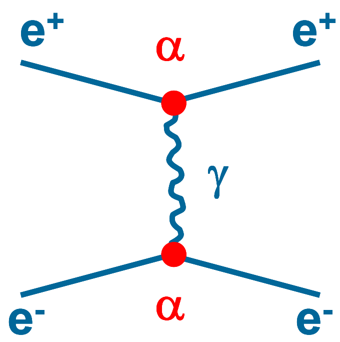
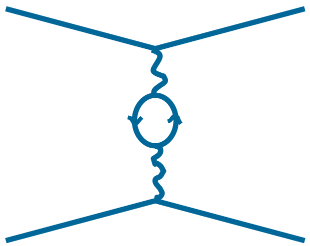
- **Introduction**
- **Small-angle Bhabha scattering:**
  - virtues
  - new OPAL analysis (PR407)
  - crucial experimental issues
  - theoretical uncertainties
  - results
- **Existing measurements (s, t channel)**
  - comparison with L3 result
- **Conclusions**



Les Rencontres de Physique de la Vallee d'Aoste, La Thuile, 27.2-5.3 2005

Introduction



QED $\subset$ SM are Quantum Field Theories

Renormalization $\rightarrow$ **Running Coupling Constants**

QED: photon propagator $\rightarrow$ Vacuum polarization $\rightarrow$ charge screening

Define the **effective QED coupling** as: $\alpha(q^2) = \frac{\alpha_0}{1-\Delta\alpha(q^2)}$

where $\alpha_0 = \alpha(q^2 = 0) \cong 1/137.036$ is the fine structure constant, experimentally known to better than 4×10^{-9}

$\Delta\alpha = \Delta\alpha_{\mathrm{lep}} + \Delta\alpha_{\mathrm{had}}$ is the contribution of vacuum polarization on the photon propagator, due to fermion loops

In the approximation of light fermions $m_f << M_W, \sqrt{s}$
the leading contribution is:

$$\Delta\alpha(s) = \frac{\alpha}{3\pi}\sum_f Q_f^2 (N_c)_f \left(\ln\frac{s}{m_f^2} - \frac{5}{3} \right)$$

The Leptonic contributions are calculable to very high precision

The Quark contributions involve quark masses and hadronic physics at low momentum scales, not calculable with only perturbative QCD.

$\Delta\alpha_{had}$

Optical Theorem,
Dispersion Relations

$$R_{had} = \frac{\sigma\left(e^+e^- \xrightarrow{\gamma} hadrons\right)}{\sigma\left(e^+e^- \xrightarrow{\gamma} \mu^+\mu^-\right)}$$

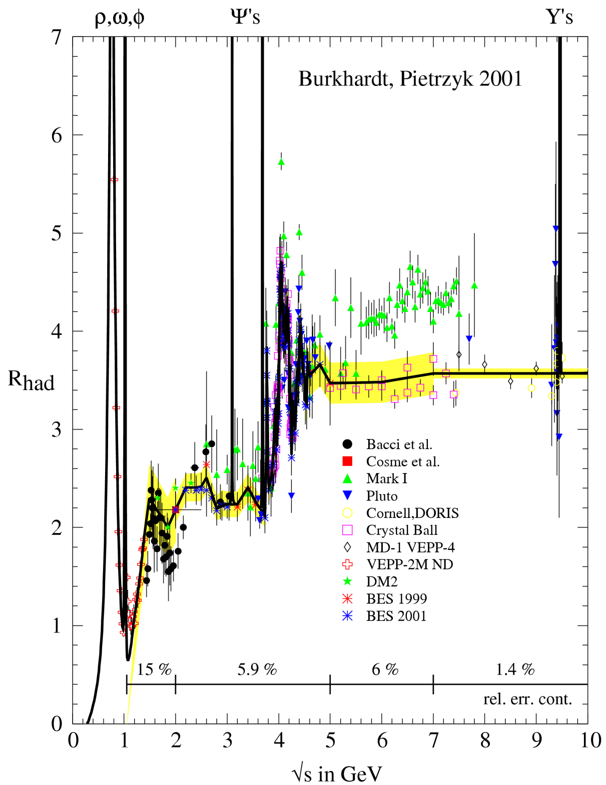
$$\Delta\alpha^{(5)}_{had}\left(q^2\right) = -\frac{\alpha\, q^2}{3\pi}\mathrm{Re}\int_{4m_\pi^2}^{\infty} ds \frac{R_{had}(s)}{s\left(s-q^2-i\varepsilon\right)}$$

- Classic approach: parameterization of measured $\sigma(e^+e^-\rightarrow$hadrons) at low energies plus pQCD above resonances

Alternative theory-driven approaches:

- pQCD applied above ≈2 GeV
- pQCD in the space-like domain (via Adler function) where $\Delta\alpha$ is smooth

error on $\Delta\alpha^{(5)}_{had}(m_Z^2)$ dominated by experimental errors in the energy range 1-5 GeV
One of the dominant uncertainties in the EW fits constraining the Higgs mass



H.Burkhardt, B.Pietrzyk, Phys. Lett. B 513 (2001) 46

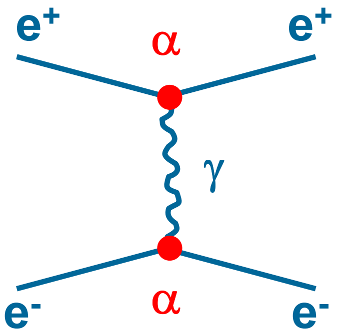
$\Delta\alpha^{(5)}_{had}(m_Z^2) = 0.02761\pm0.0036$

popular parameterization, for $s>10^2$ GeV^2 or $s<0$

$$\Delta\alpha_{had}(s) \cong A + B\ln\left(1 + C|s|\right)$$

Small-angle Bhabha scattering

an almost **pure QED** process. Differential cross section can be written as:



$$\frac{d\sigma}{dt} = \frac{d\sigma^{(0)}}{dt}\left[\frac{\alpha(t)}{\alpha_0}\right]^2 (1+\varepsilon)(1+\delta_\gamma) + \delta_Z$$

Born term for t-channel single γ exchange

$$\frac{d\sigma^{(0)}}{dt} = \frac{4\pi\alpha_0^2}{t^2}$$

$\alpha_0 \cong 1/137.036$

$$\left(\frac{1}{1-\Delta\alpha(t)}\right)^2$$

Effective coupling factorized

Photonic radiative corrections

s-channel γ exchange correction

Z interference correction

$$\delta_Z\ ,\ \delta_\gamma \ll \varepsilon$$

experimentally: high data statistics, very high purity

This process and method advocated by Arbuzov et al., Eur.Phys.J.C 34(2004)267

Small-angle Bhabha scattering

BHLUMI MC (S.Jadach et al.) calculates the photonic radiative corrections up to ***O*****(α²*L*²)** where ***L* = *ln* (|t| / m$_e$²) – *1*** is the **Large Logarithm**

Higher order terms partially included through YFS exponentiation

Many existing calculations have been widely cross-checked with BHLUMI to decrease the theoretical error on the determination of Luminosity at LEP, reduced down to 0.054% (0.040% due to Vacuum Polarization)

Size of the photonic radiative corrections (w.r.t. Born = 1*)*

Canonical coefficients					
		$\theta_{min} = 30$ mrad		$\theta_{min} = 60$ mrad	
		LEP1	LEP2	LEP1	LEP2
$\mathcal{O}(\alpha L)$	$\frac{\alpha}{\pi}4L$	137×10^{-3}	152×10^{-3}	150×10^{-3}	165×10^{-3}
$\mathcal{O}(\alpha)$	$2\frac{1}{2}\frac{\alpha}{\pi}$	2.3×10^{-3}	2.3×10^{-3}	2.3×10^{-3}	2.3×10^{-3}
$\mathcal{O}(\alpha^2L^2)$	$\frac{1}{2}\left(\frac{\alpha}{\pi}4L\right)^2$	9.4×10^{-3}	11×10^{-3}	11×10^{-3}	14×10^{-3}
$\mathcal{O}(\alpha^2L)$	$\frac{\alpha}{\pi}\left(\frac{\alpha}{\pi}4L\right)$	0.31×10^{-3}	0.35×10^{-3}	0.35×10^{-3}	0.38×10^{-3}
$\mathcal{O}(\alpha^3L^3)$	$\frac{1}{3!}\left(\frac{\alpha}{\pi}4L\right)^3$	0.42×10^{-3}	0.58×10^{-3}	0.57×10^{-3}	0.74×10^{-3}

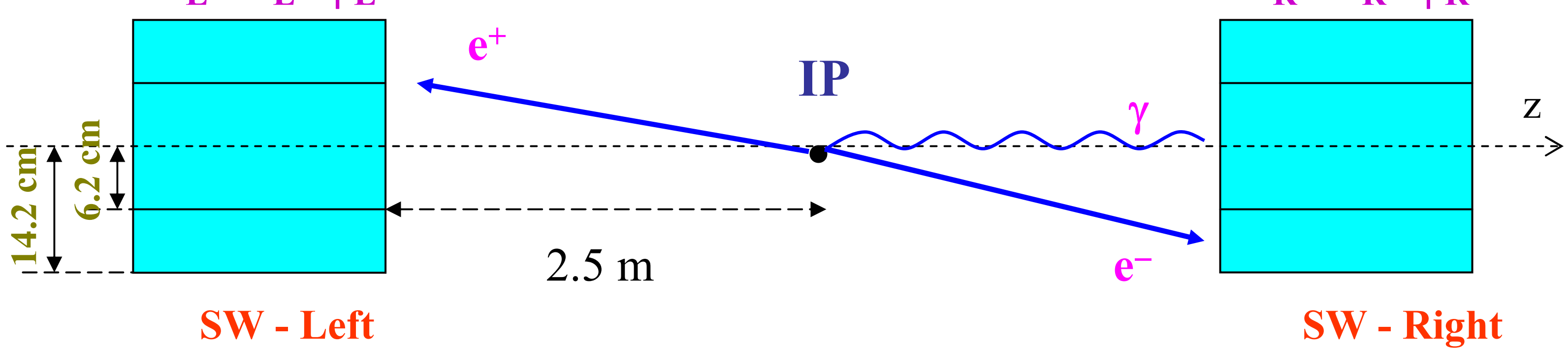
First incomplete terms

O**(α²*L*)**

O**(α³*L*³)**

Small-angle Bhabha scattering in OPAL

$$e^+e^- \rightarrow e^+e^- \qquad \sqrt{s} \approx 91.2\,\text{GeV}$$



2 cylindrical calorimeters encircling the beam pipe at ± 2.5 m from the Interaction Point

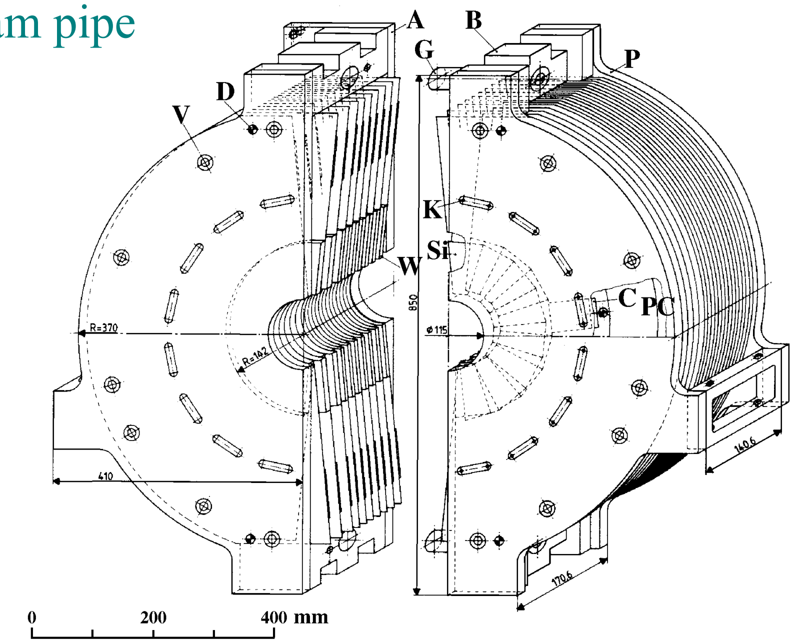
19 Silicon layers

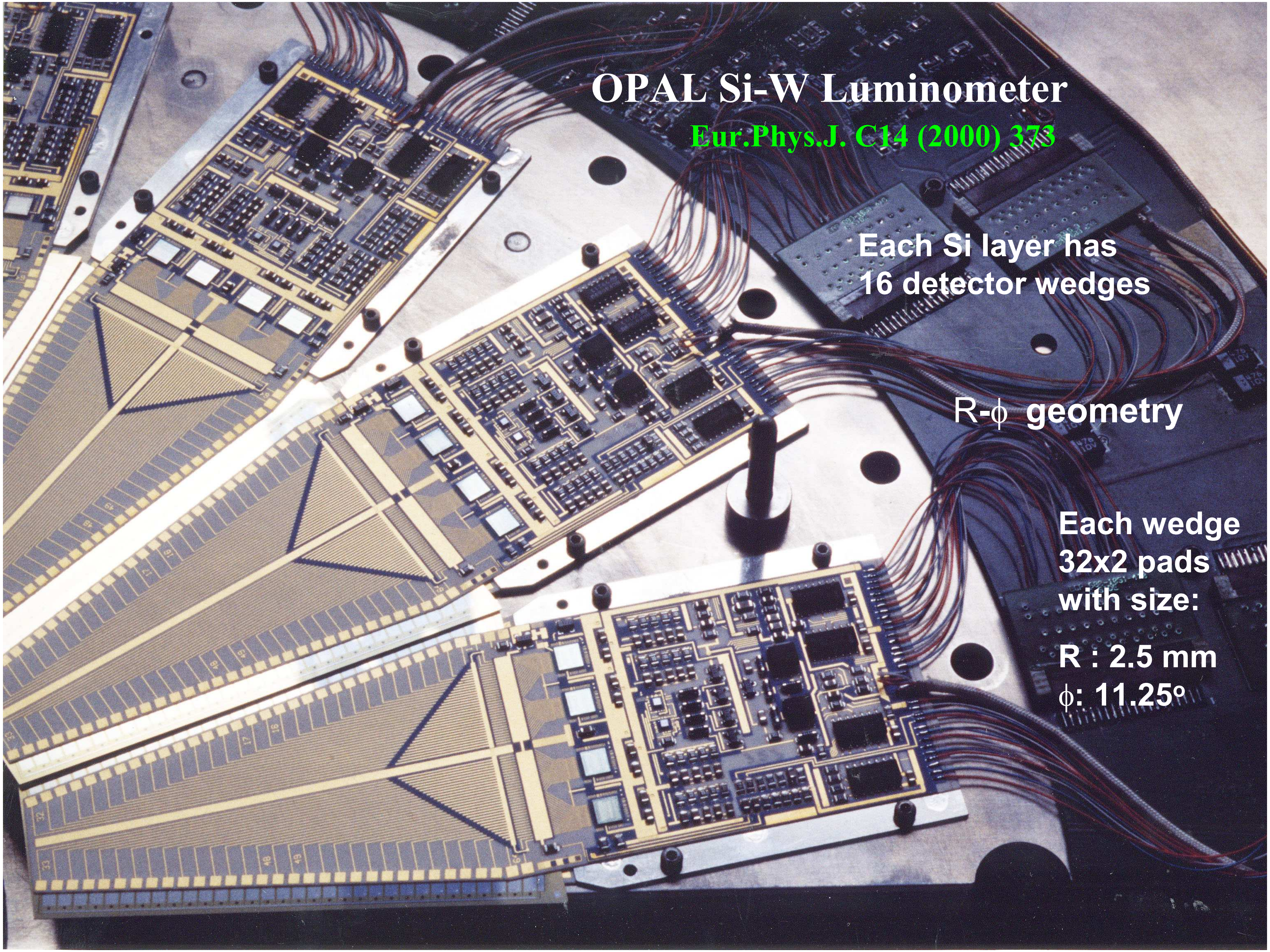
18 Tungsten layers

Total Depth 22 X_0
(14 cm)

Each detector layer divided into 16 overlapping wedges

Sensitive radius: **6.2 – 14.2 cm**, corresponding to **scattering angle** of **25 – 58 mrad** from the beam line




OPAL Si-W Luminometer
Eur.Phys.J. C14 (2000) 373
Each Si layer has
16 detector wedges
R-ϕ geometry
Each wedge
32x2 pads
with size:
R : 2.5 mm
ϕ: 11.25°

Event selection similar to the Luminosity selection

The event sample is dominated by two cluster configurations with almost full energy back-to-back e^+ and e^-

Isolation cuts

6.7 cm < R_R, R_L < 13.7 cm

E_R, E_L > 0.5 E_{beam}

$(E_R + E_L)/2$ > 0.75 E_{beam}

$|\Delta\Phi|$ < 200 mrad

$|\Delta R|$ < 2.5 cm

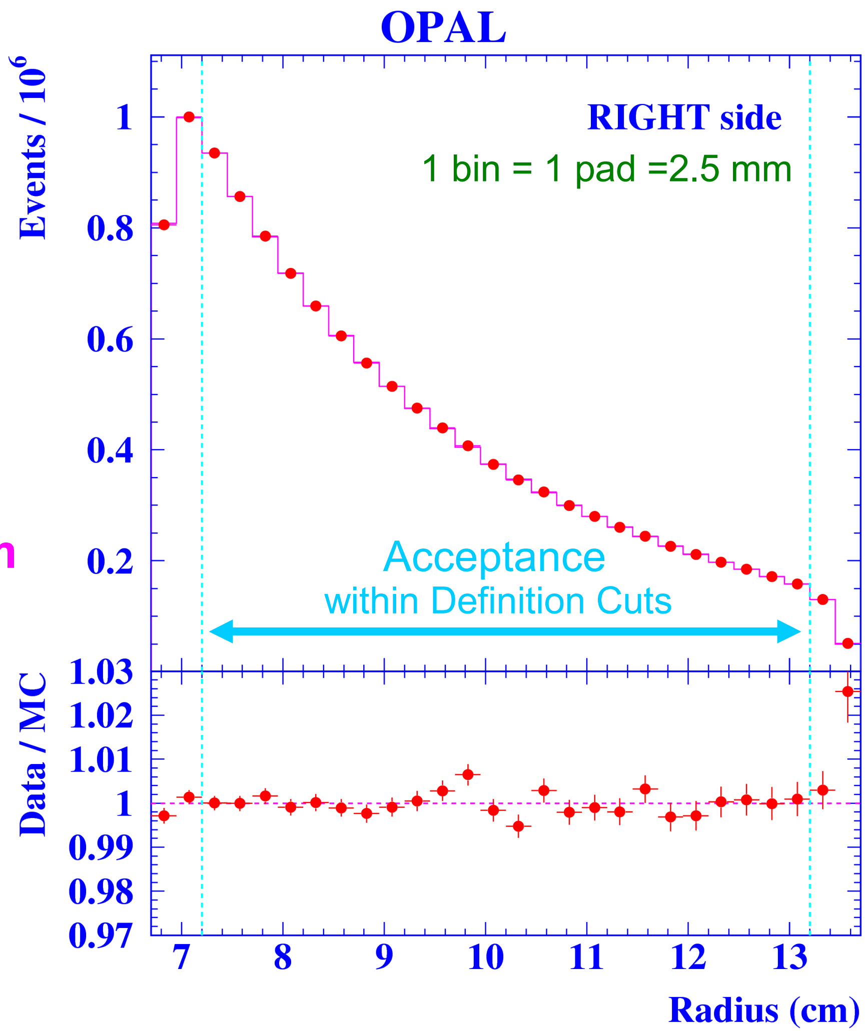
Definition cuts: 7.2 < R < 13.2 cm (at RIGHT or LEFT side),
corresponding to **$2 \leq -t \leq 6$ GeV^2**

$$t = -s\frac{1-\cos\theta}{2} \approx -\frac{s\,\theta^2}{4}$$

$$\tan\theta = \frac{R}{z}$$

$\sqrt{s} \approx 91.1$ GeV

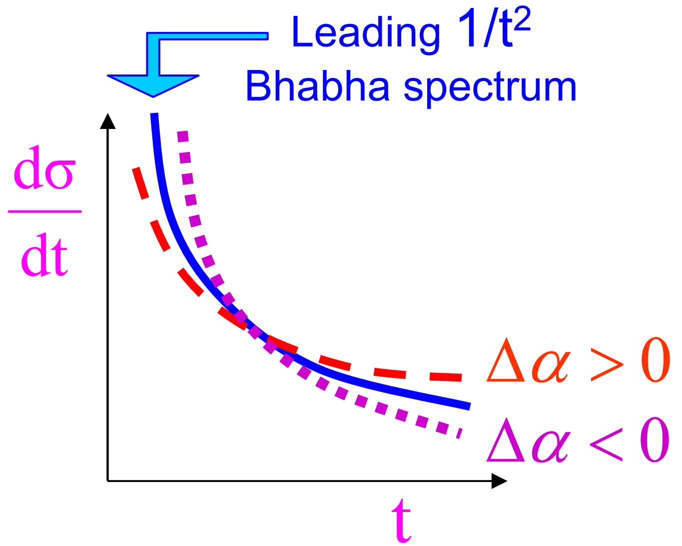
z = 246.0225 cm



Analysis method

We compare the Radial distribution of the data (R$\rightarrow\theta\rightarrow$t) with the theoretical predictions of the BHLUMI MC

The small-angle Bhabha process is used to determine the Luminosity: we cannot make an absolute measurement of α(t), but look at its variation over the t range.



Fit the Ratio f of **data** and **MC with α(t) = α_0**

$$f(t) = \frac{N_{data}(t)}{N^0_{MC}(t)} = a + b \cdot \ln\left(\frac{t}{t_0}\right) \qquad t_0 = -3.3\,\mathrm{GeV}^2$$

We measure the **effective slope** b **of the Bhabha t-spectrum**

b is related to the variation of the coupling by:

$$\Delta\alpha(t_2) - \Delta\alpha(t_1) \cong \frac{b}{2} \cdot \ln\left(\frac{t_2}{t_1}\right)$$

$$t_1 = -1.81\,\mathrm{GeV}^2\ ,\ t_2 = -6.07\,\mathrm{GeV}^2$$

Radial reconstruction

Radial coordinate reconstruction is key to the current measurement

$\mathbf{R \rightarrow \theta \rightarrow t}$

Radial biases as small as **70 μm** in the centre of the radial acceptance could mimic the expected running of α. Similarly would do a uniform metrology error of **0.5 mm** at all radii.

Two complementary strategies used:

- *Unanchored coordinate*: the reconstruction determines a radial coordinate R of incident showering particles in the Right and Left Si-W calorimeters. This is smooth, continuous, and uses a large number of pads throughout the depth of the detector, from many Si layers. It is projected onto a reference layer which is the Si layer at depth of 7 X_0, close to the average longitudinal shower maximum.

- *Anchored coordinate*: the residual bias on the reconstructed R is estimated and corrected by the **anchoring** procedure, which uses the inherent pad structure of the detector. It relies on the fact that, on average, the pad with maximum signal in any particular layer will contain the shower axis (sharp shower core). A correction is applied at each pad boundary in a chosen layer of the detector.

Radial coordinate anchoring

Plot the transition from one pad to the other: *Pad Boundary Images*

For any chosen pad boundary in any chosen Si layer, look at the Probability that the pad with the largest signal in that layer is above the boundary, as a function of the distance of the reconstructed shower from the nominal boundary position.

Fit parameters:

R_{off} is the observed offset

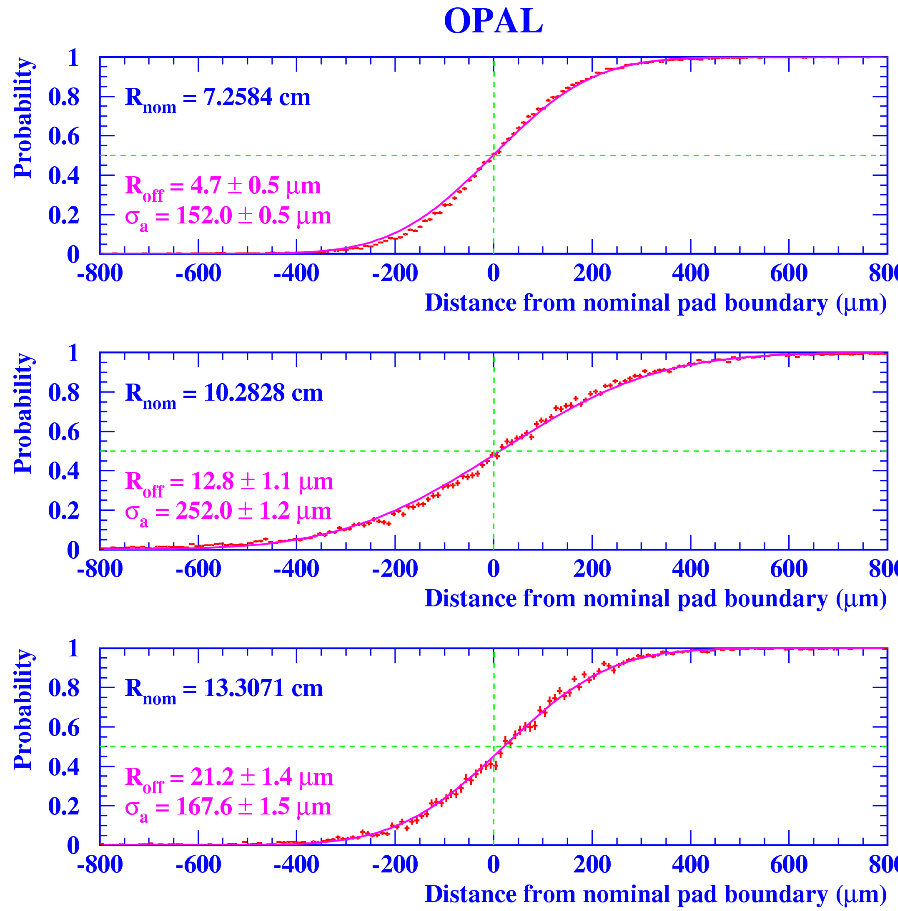
σ_a is the transition width

Total Net Bias δR (*anchor*) in the reconstructed Radius:

$$\delta R = R_{off} + \delta R_{R\varphi} + \delta R_{res}$$

geometric bias (due to $R\phi$ pads), depending on σ_a, determined at a testbeam

small correction for the resolution flow



Anchors

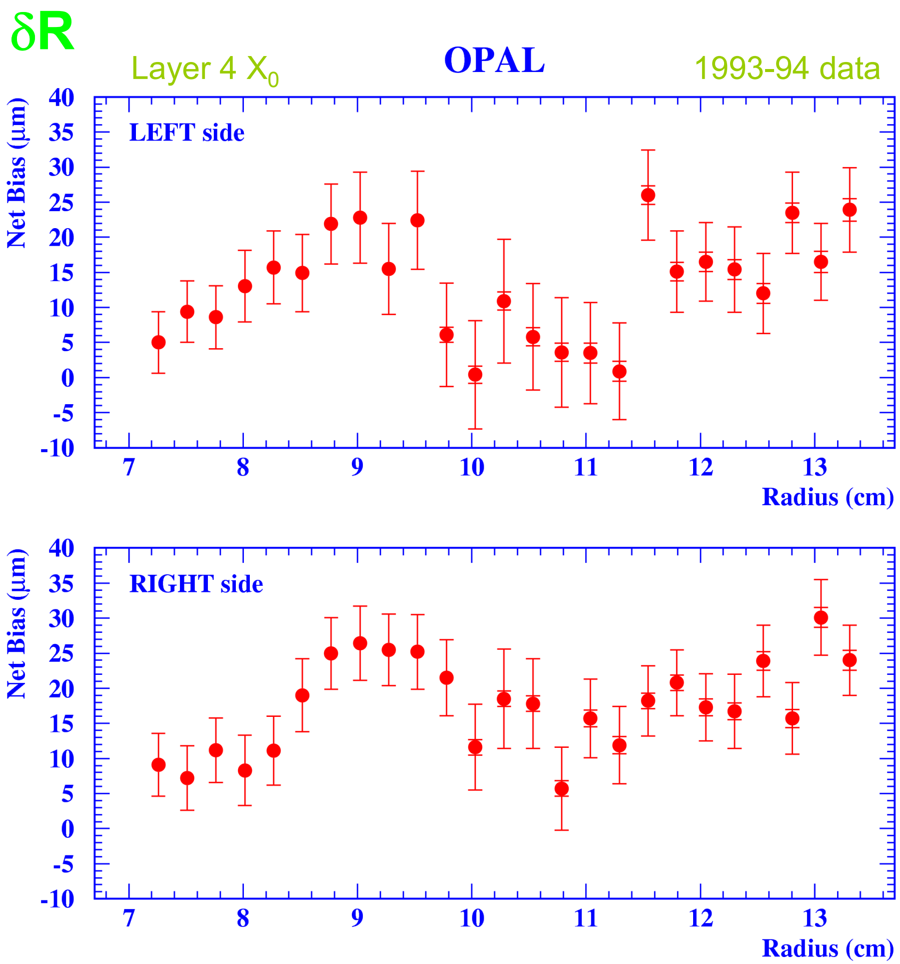
Residual Bias on Radius below 30 μm

Convert anchors to bin-by-bin acceptance corrections:

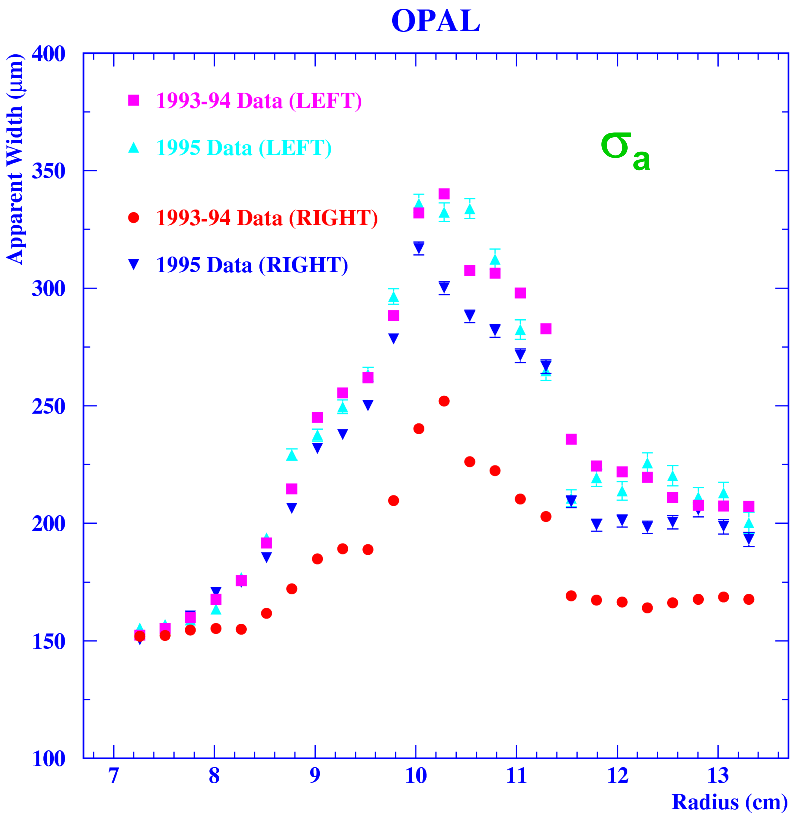
$$\frac{\delta A}{A} = c_{inn}\delta R_{inn} - c_{out}\delta R_{out}$$

for bin boundaries [R_{inn}, R_{out}]

smaller than 1.0% for one-pad-wide bins

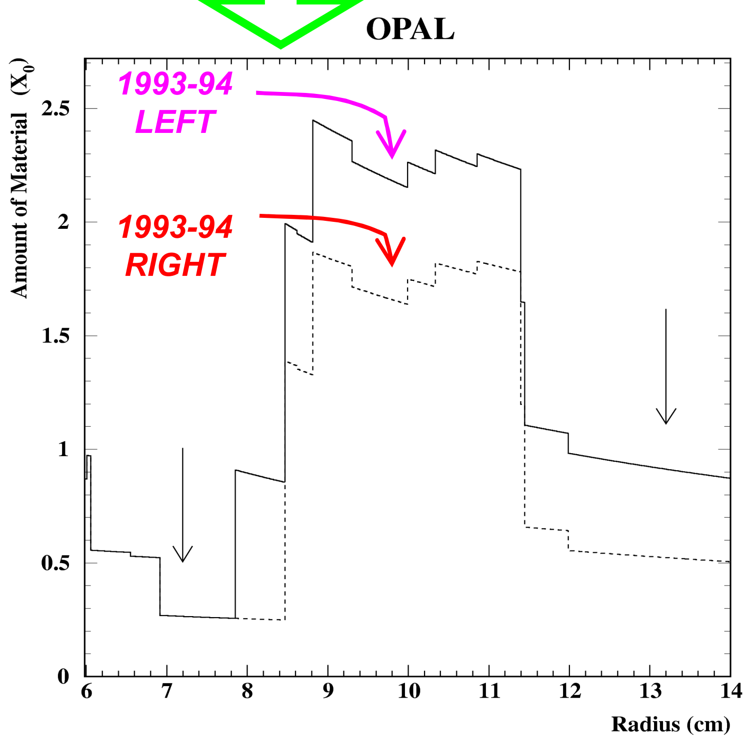


Widths → Radial Resolution



Transition width σ_a of the pad boundary images is related to the radial resolution

depends on the amount of **preshowering material**

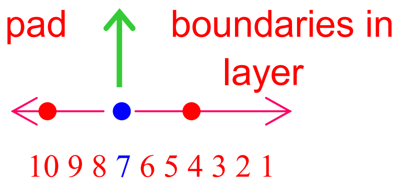


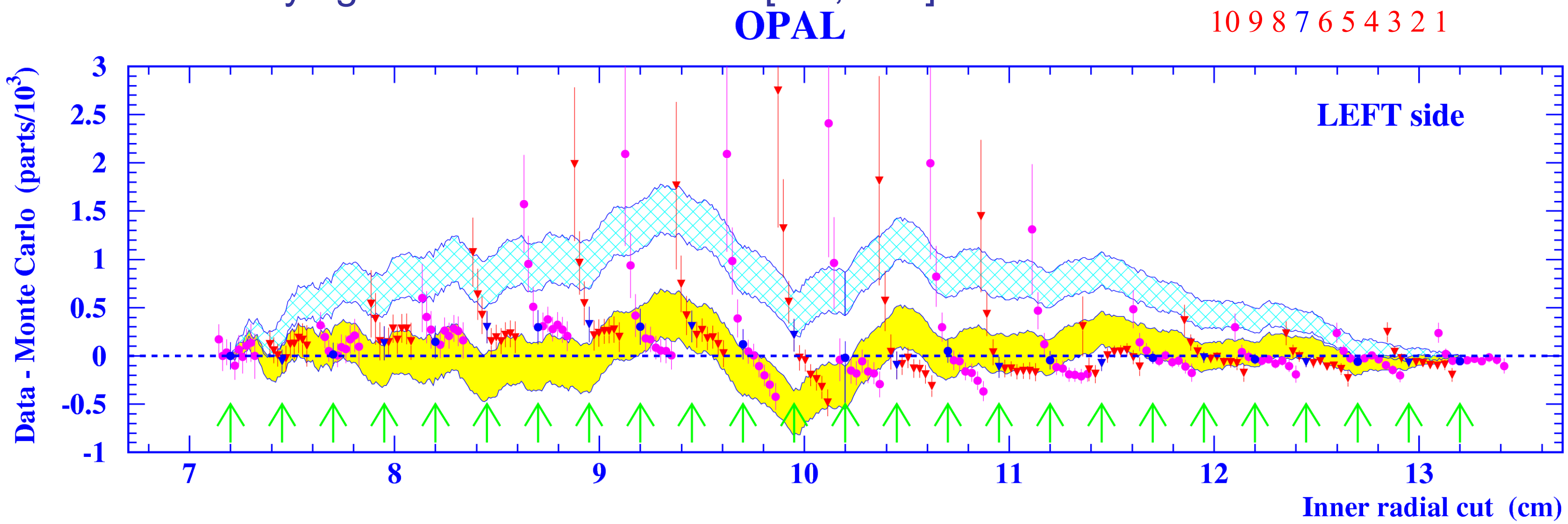
About 2 X_0 covering the middle portion of the SiW calorimeters due to cables and beam pipe structures.

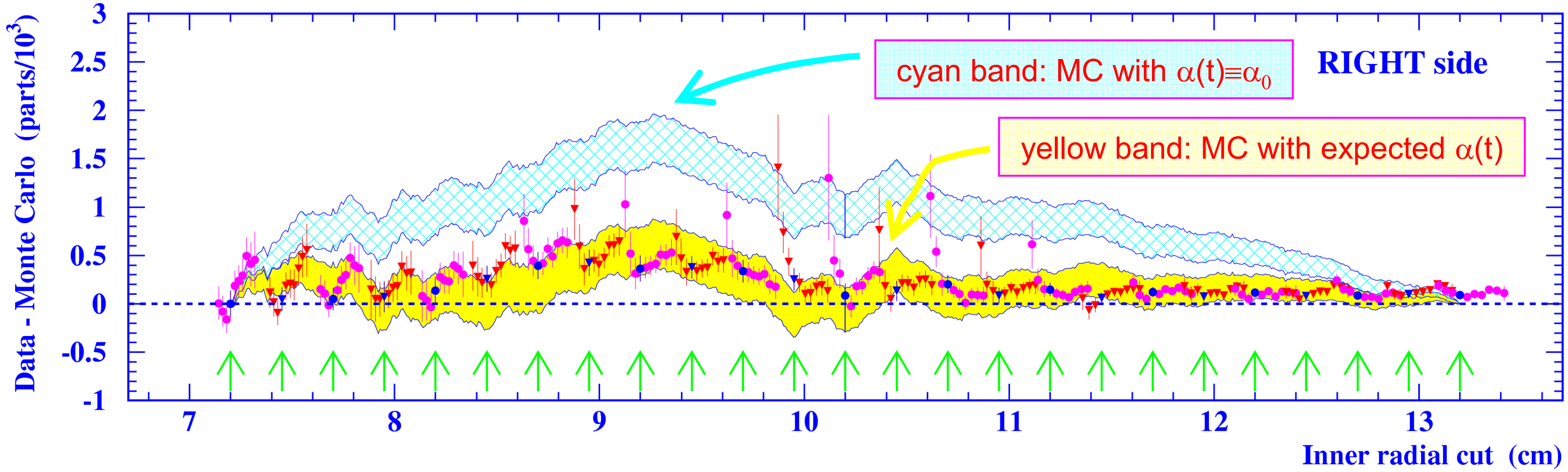
Study R coordinate in Data – MC

Represent graphically the main experimental challenge

Check number of accepted events in data – MC
while varying the inner radial cut in [7.2,13.2] cm







Fit results

Si layers from $1X_0$ to $6X_0$ are safe for anchoring → choose layer $4X_0$

Preshowering Material L-R asymmetric → choose Right side (cleaner than Left)

9 LEP1 data (and MC) subsamples to account for year, centre-of mass energy and running conditions

LEP2 data not included due to narrower acceptance (extra shields for synchrotron radiation) and worse dead-material distribution

Small corrections for: irreducible background from $e^+e^- \to \gamma\gamma$ (-18×10^{-5}) and for Z interference at off-peak energies ($\pm 14 \times 10^{-5}$)

Dataset	$\sqrt{s}$ (GeV)	Number of events	slope b ($\times 10^{-5}$)
93 −2	89.4510	879549	$662 \pm 326 \pm 89$
93 pk	91.2228	894206	$670 \pm 324 \pm 92$
93 +2	93.0362	852106	$640 \pm 332 \pm 89$
94 a	91.2354	885606	$559 \pm 326 \pm 86$
94 b	91.2170	4069876	$936 \pm 152 \pm 71$
94 c	91.2436	288813	$62 \pm 570 \pm 122$
95 −2	89.4416	890248	$839 \pm 325 \pm 124$
95 pk	91.2860	581111	$727 \pm 402 \pm 126$
95 +2	92.9720	885837	$156 \pm 325 \pm 128$
Average	91.2208	10227352	$726 \pm 96 \pm 70$
χ^2/d.o.f. (stat.)			6.9/8
χ^2/d.o.f. (stat.+syst.)			6.5/8

9 subsamples consistent

Statistical errors dominant

Most important systematic errors due to anchoring and preshowering material

Measured slope b $\cong 2 \frac{\delta \Delta\alpha}{\delta \ln t}$

7.6 σ (stat.)

6.1 σ (stat.+syst.)

away from zero.

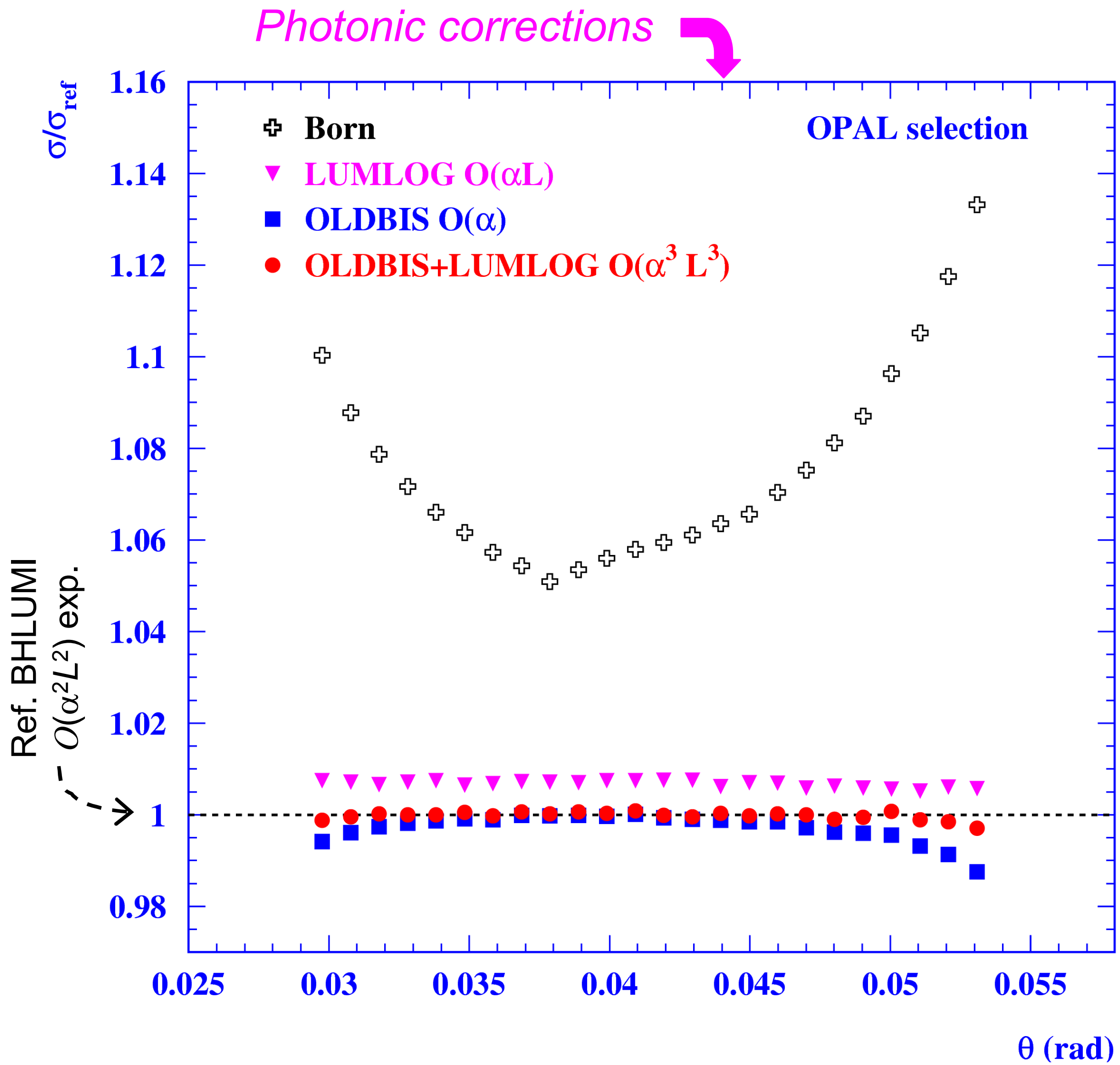
Experimental Systematic errors

Uncertainty	93 −2	93 pk	93 +2	94 a	94 b	94 c	95 −2	95 pk	95 +2
M.C. Statistics									
uncorrelated	56.	56.	56.	56.	28.	103.	56.	56.	56.
correlated	0.	0.	0.	0.	0.	0.	0.	0.	0.
Anchoring									
uncorrelated	10.	10.	10.	10.	10.	10.	29.	29.	29.
correlated	44.	44.	44.	44.	44.	44.	44.	44.	44.
Preshowering Material									
uncorrelated	0.	0.	0.	0.	0.	0.	81.	81.	81.
correlated	30.	30.	30.	30.	30.	30.	30.	30.	30.
Radial Resolution									
uncorrelated	0.	0.	0.	0.	0.	0.	0.	0.	0.
correlated	15.	15.	15.	15.	15.	15.	25.	25.	25.
Acollinearity Bias									
uncorrelated	0.	0.	0.	0.	0.	0.	0.	0.	0.
correlated	9.	9.	9.	9.	9.	9.	9.	9.	9.
Radial Metrology									
uncorrelated	0.	0.	0.	0.	0.	0.	0.	0.	0.
correlated	12.	12.	12.	12.	12.	12.	12.	12.	12.
Radial Thermal									
uncorrelated	0.	0.	0.	0.	0.	0.	0.	0.	0.
correlated	3.	3.	3.	4.	4.	4.	12.	12.	12.
Beam Parameters									
uncorrelated	19.	31.	20.	8.	5.	12.	12.	25.	33.
correlated	7.	7.	7.	7.	5.	9.	8.	8.	8.
Energy									
uncorrelated	0.	0.	0.	0.	0.	0.	0.	0.	0.
correlated	27.	27.	27.	27.	27.	27.	27.	27.	27.
Background									
uncorrelated	0.	0.	0.	0.	0.	0.	0.	0.	0.
correlated	16.	12.	7.	4.	2.	5.	4.	2.	2.
Sum									
uncorrelated	60.	65.	61.	58.	30.	104.	104.	106.	108.
correlated	66.	65.	64.	64.	64.	65.	68.	68.	68.
Total Systematic Error	89.	92.	89.	86.	71.	122.	124.	126.	128.

→ **Dominant**

Error correlations within 10% of the total experimental errors (stat.+syst.)

Theoretical Uncertainties



Reliable determination of $\alpha(t)$ requires precise knowledge of radiative corrections

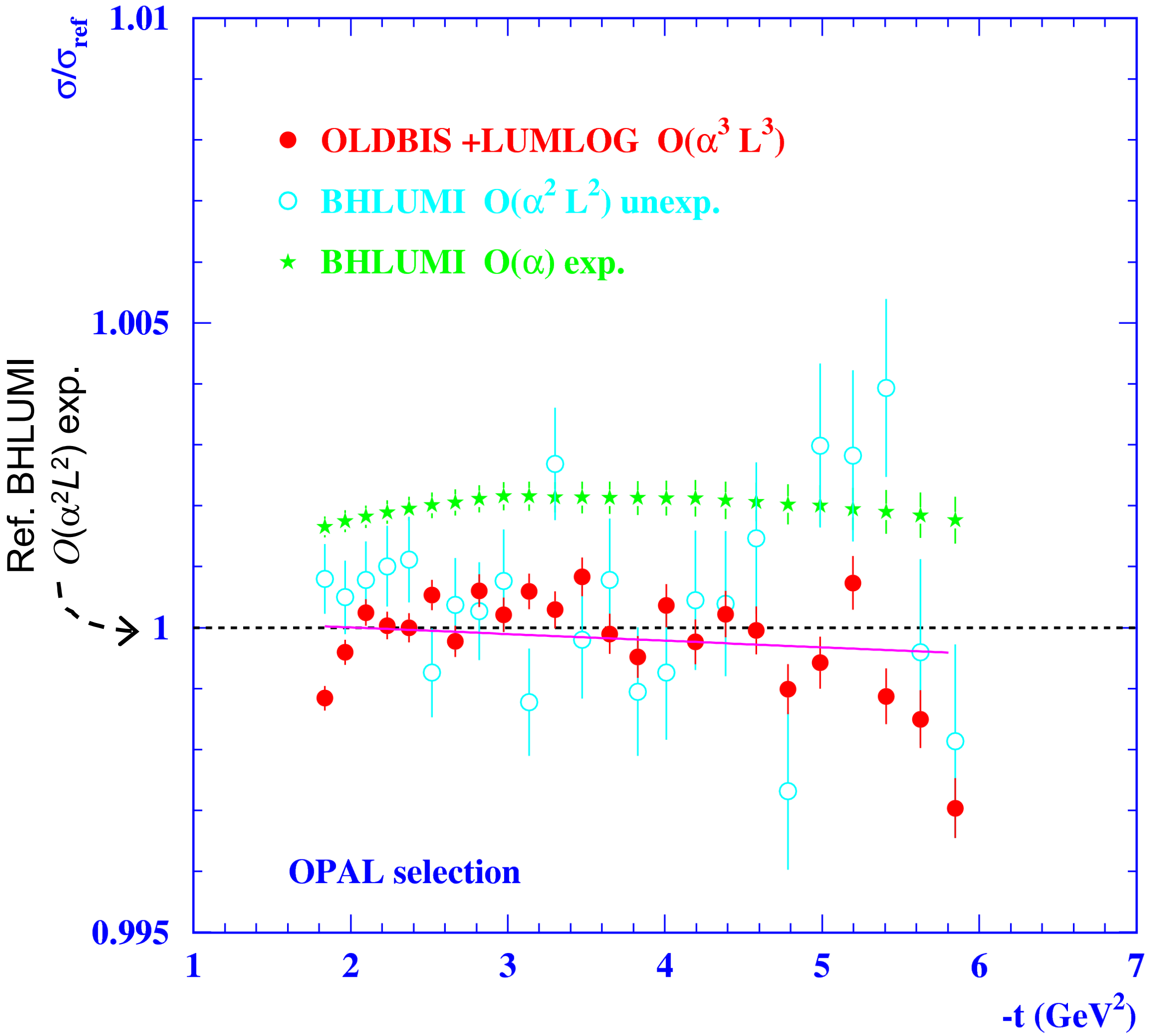
Reference BHLUMI is $O(\alpha^2 L^2)$ exponentiated:

compare with Born, $O(\alpha L)$, $O(\alpha)$, and $O(\alpha^3 L^3)$

vacuum polarization, Z-interference and s-channel switched off

Theoretical Uncertainties

Photonic corrections



Compare the ref. BHLUMI calculation with alternatives differing in the matrix element or in technical aspects

only slope differences count

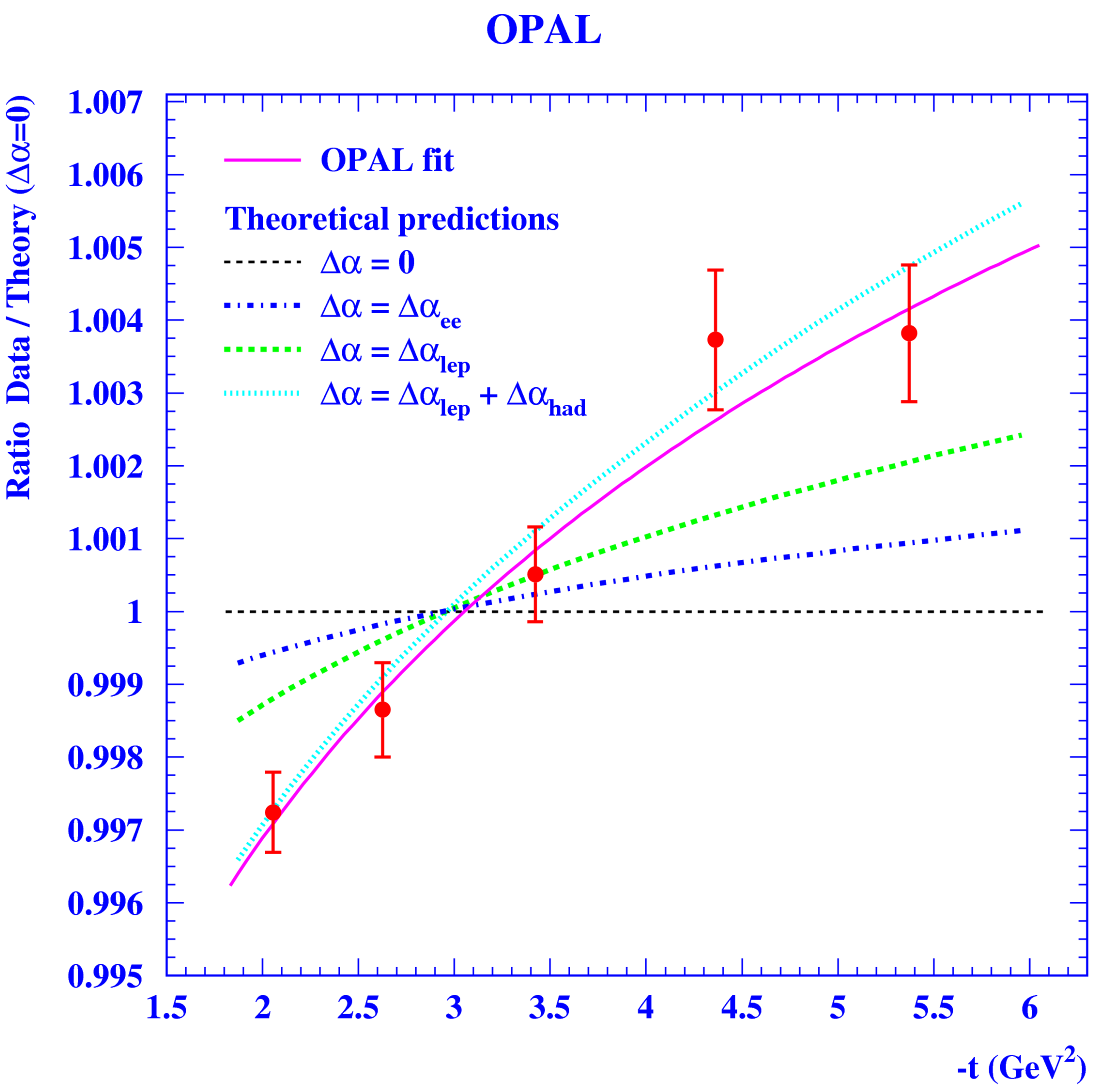
Photonic corrections: the combination of the two independent MCs OLDBIS+LUMLOG allows to assess also the technical precision

Full list

Error source	δb $(\times 10^{-5})$
Photonic corrections	38.
Z interference	30.
Light pairs	11.
Total	50.

summed in quadrature with the experimental errors

Results



$\Delta\chi^2 = 0.8$

$\chi^2/d.o.f. = 1.9/3$ **OPAL fit**

$\Delta\chi^2 = 18$

$\Delta\chi^2 = 37$

$\Delta\chi^2 = 60$

} incompatible

OPAL fit

$$f(t) = \frac{N_{data}(t)}{N^0_{MC}(t)} = a + b \cdot \ln\left(\frac{t}{t_0}\right)$$

$$b = (726 \pm 96 \pm 70) \times 10^{-5}$$

$$\cong 2\frac{\delta\,\Delta\alpha}{\delta\ \ln t}$$

Results

Slope b = $(726 \pm 96 \pm 70 \pm 50) \times 10^{-5}$ $\qquad b \cong 2\dfrac{\delta\,\Delta\alpha}{\delta \ln t}$

Significance: 5.6 σ including all errors for the **total running**

$$\Delta\alpha(-6.07\ \mathrm{GeV}^2) - \Delta\alpha(-1.81\ \mathrm{GeV}^2) = (440 \pm 58 \pm 43 \pm 30) \times 10^{-5}$$

SM : 460×10^{-5} using the Burkhardt-Pietrzyk parameterization

Most significant direct observation of the running of α_{QED} ever achieved

contributions to the slope b in our t range are predicted to be in the proportion: **e : μ : hadron ≈ 1 : 1 : 2.5**

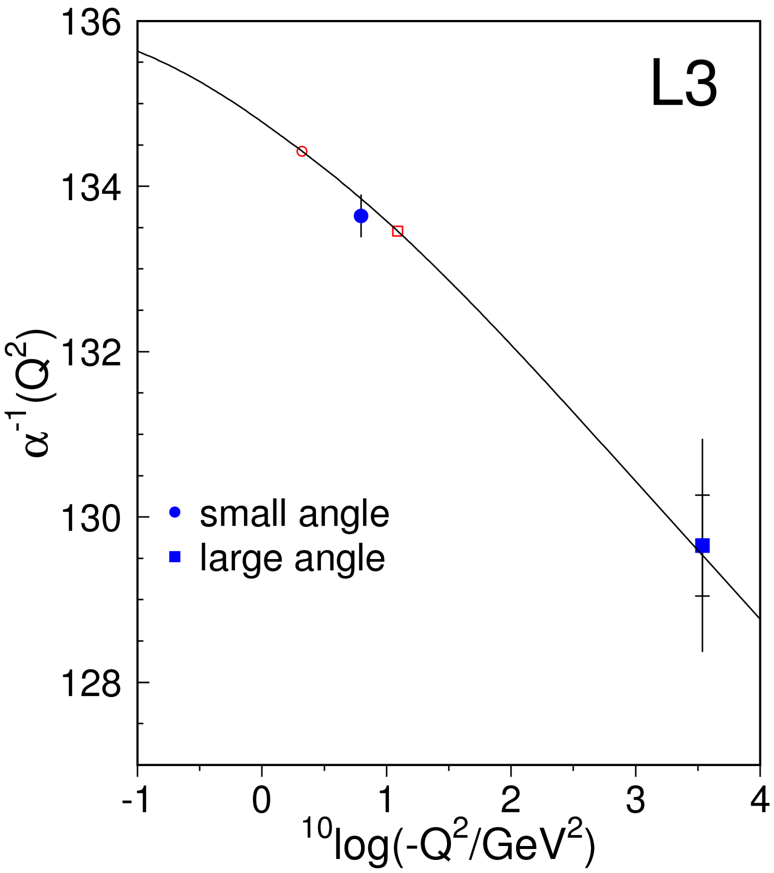
subtracting the precisely calculable leptonic contribution: $\delta(\Delta\alpha_{lep}) = 202 \times 10^{-5}$

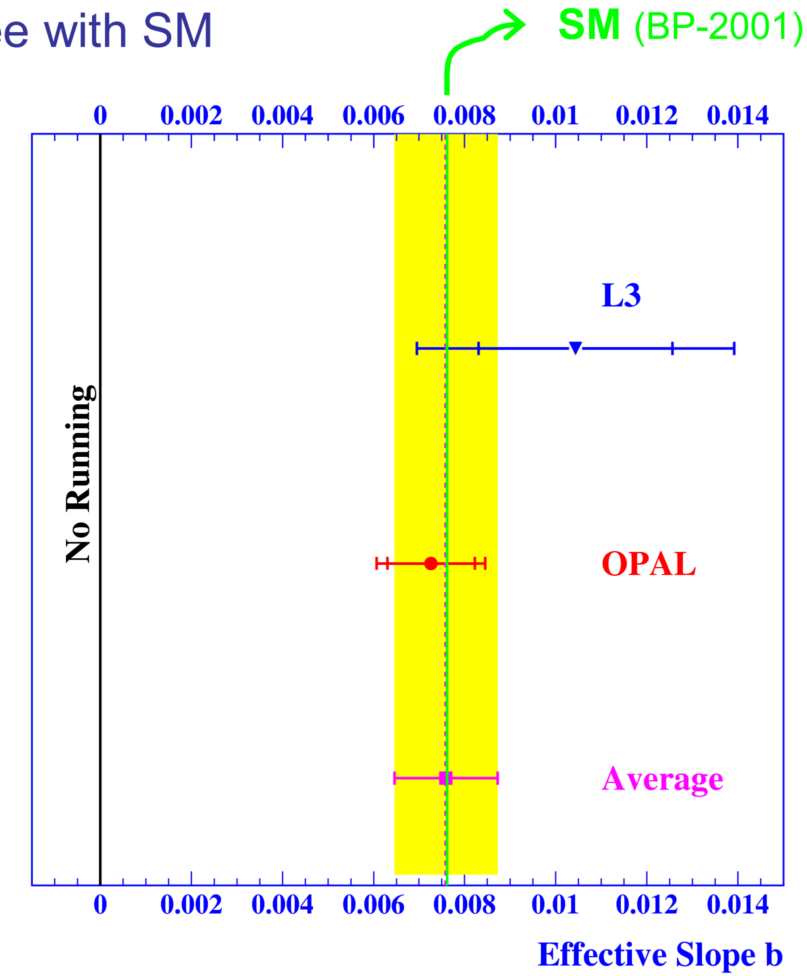
$$\Delta\alpha_{had}(-6.07\ \mathrm{GeV}^2) - \Delta\alpha_{had}(-1.81\ \mathrm{GeV}^2) = (237 \pm 58 \pm 43 \pm 30) \times 10^{-5}$$

Hadronic contribution to the running: First Direct Experimental evidence with **Significance** of **3.0 σ** including all errors

L3 and OPAL

both agree with SM



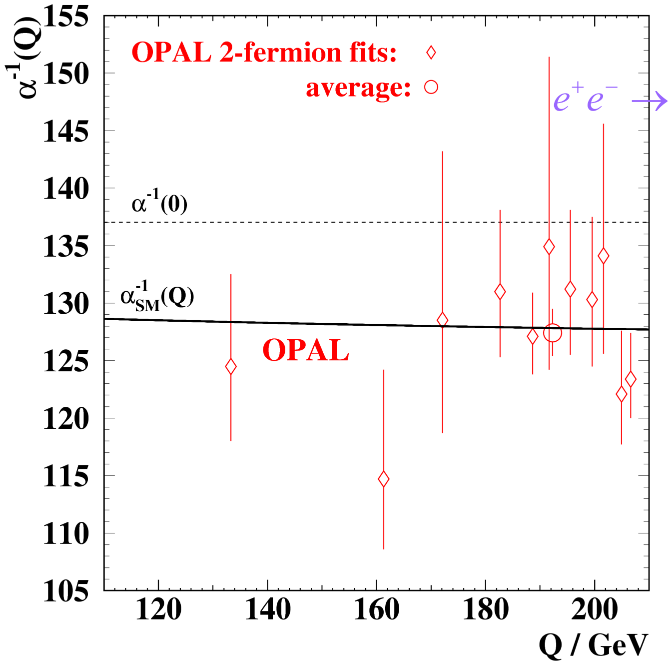


L3: 2 results with 3 σ significance

small-angle: significance of the observed running is 6 σ (dominated by OPAL)

Other Direct experimental observations (s-channel)

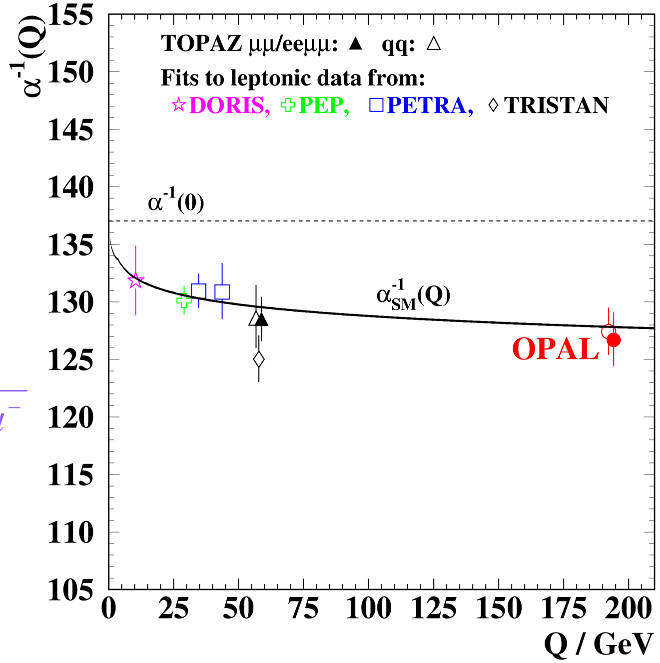
γ exchange dominates BUT full EW theory is needed



$$e^+e^- \rightarrow \frac{\mu^+\mu^-, \tau^+\tau^-}{hadrons}$$

TOPAZ

$$\frac{e^+e^- \rightarrow \mu^+\mu^-}{e^+e^- \rightarrow e^+e^-\mu^+\mu^-}$$



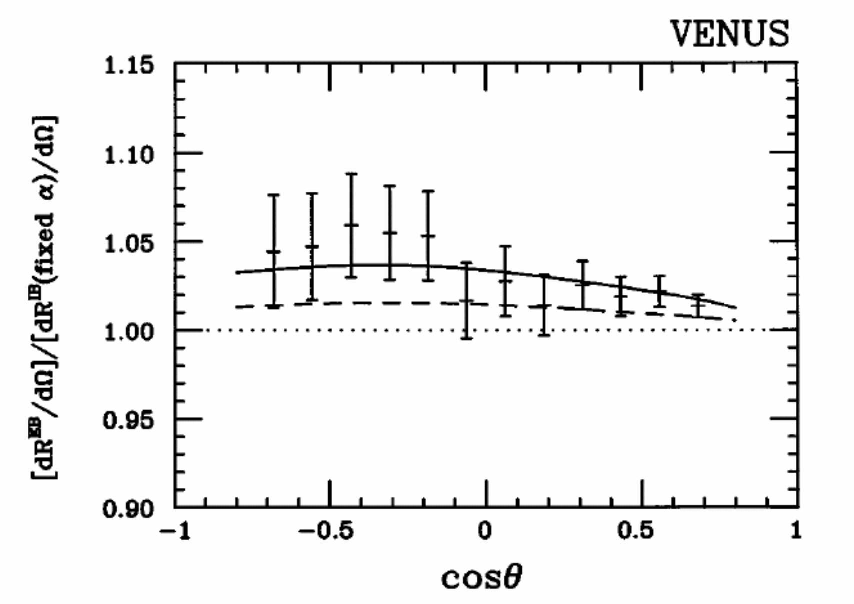
OPAL $\alpha^{-1}\left(\left\langle\sqrt{s}\right\rangle = 193.2\ \mathrm{GeV}\right) = 126.7^{+2.4}_{-2.3}$

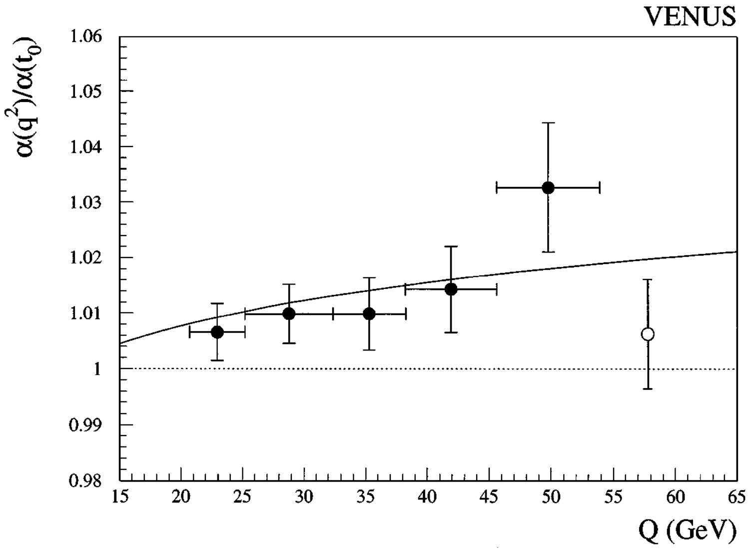
TOPAZ $\alpha^{-1}\left(\sqrt{s} = 57.77\ \mathrm{GeV}\right) = 128.5 \pm 1.8 \pm 0.7$ ⟶ (Theor.Unc. may be underestimated)

Significance: 4.3 - 4.4 σ w.r.t. the no-running hypothesis BUT despite the large change in c.m.s. energy from TOPAZ to OPAL there is no sensitivity to the running of α_{QED} between the measurements.

Other Direct experimental observations (t-channel)

Large angle Bhabha:
s-channel γ exchange and **Z** interference both important





VENUS: $10^2 \leq -t \leq 54^2$ GeV^2 and s-channel determined from $e^+e^- \to \mu^+\mu^-$
Claimed Significance ≈ 4 σ but Theor.Unc. ≈ 0.5% → 2.0% could reduce it

L3 (LEP2 data): $12.25 \leq -t \leq 3434$ GeV^2 $\quad \delta\alpha^{-1} = 3.80 \pm 0.61 \pm 1.14$

Significance ≈ 3 σ dominated by Theor.Unc. ≈ 0.5 - 2.0 %

Conclusions

- New **OPAL** result (*PR407*): scale dependence of the effective QED coupling measured from the angular spectrum of small-angle Bhabha scattering for negative momentum transfers $1.8 \leq -t \leq 6.1$ GeV^2
  - theoretically almost ideal situation (precise calculations, t-channel dominance, almost pure QED, Z interference very small)
  - experimentally challenging BUT: large statistics, excellent purity, precise detector
- Effective slope $b \cong 2\ \delta\Delta\alpha / \delta\ln t$ measured, good agreement with SM predictions

$$\Delta\alpha(-6.07\ \mathrm{GeV}^2) - \Delta\alpha(-1.81\ \mathrm{GeV}^2) = (440 \pm 58 \pm 43 \pm 30) \times 10^{-5}$$

- Strongest direct evidence for the running of α_{QED} ever achieved in a single experiment, with significance above 5 σ
- First clear experimental evidence for the hadronic contribution to the running with significance of 3 σ
- Can Theory use this kind of t-channel measurements for $\alpha_{QED}(m_Z^2)$?