RELATIVITY AND $c/\sqrt{3}$

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gr-qc

1. General Relativity: in gravitational field clocks run slowly $\Longrightarrow$ delay of radar echo from inner planets
   I. Shapiro (1964)

2. Ultrarelativistic particle follows the trajectory of photon, so retardation must take place for ultrarelativistic particles as well.

3. Nonrelativistic particles accelerate when they are falling radially onto a gravitating body.

From 2 and 3 it follows that some intermediate velocity v_c should exist which remains constant for a particle falling in gravitational field of the Sun (or another star). Let us find it.

For radial motion $d\theta = d\varphi = 0$, expression for interval has the well known Schwarzschild form:

$$ds^2 = g_{00}dt^2 - g_{rr}dr^2 \equiv d\tau^2 - dl^2 \ ,$$

$$g_{00} = (g_{rr})^{-1} = 1 - \frac{r_g}{r} \ , r_g = 2G_N M$$

local velocity of a particle measured by a local observer at rest:

$$v = \frac{dl}{d\tau} = \left(\frac{g_{rr}}{g_{00}}\right)^{1/2} \frac{dr}{dt} = \frac{1}{g_{00}} \frac{dr}{dt}$$

Coordinate velocity measured by observer at infinity ($g_{00}(\infty) = g_{rr}(\infty) = 1$):

$$\mathrm{V} = \frac{dr}{dt} = g_{00}v \ ,$$

$$t = \int_a^b \frac{dr}{\mathrm{V}}$$

So, V is relevant for radar echo.

For a particle moving in static gravitational field conserved energy can be introduced:

$$E = \frac{m\sqrt{g_{00}}}{\sqrt{1-v^2}} = \frac{m\sqrt{g_{00}}}{\sqrt{1-(\mathrm{v}/g_{00})^2}}$$

from energy conservation $E(r=\infty) = E(r)$ we get:

$$\mathrm{v}^2 = g_{00}^2 - g_{00}^3 + g_{00}^3 \mathrm{v}_\infty^2 = g_{00}^2[1 - g_{00}(1-\mathrm{v}_\infty^2)]\,, \quad (*)$$

and substituting $g_{00} = 1 - \frac{r_g}{r}$ for weak field we get:

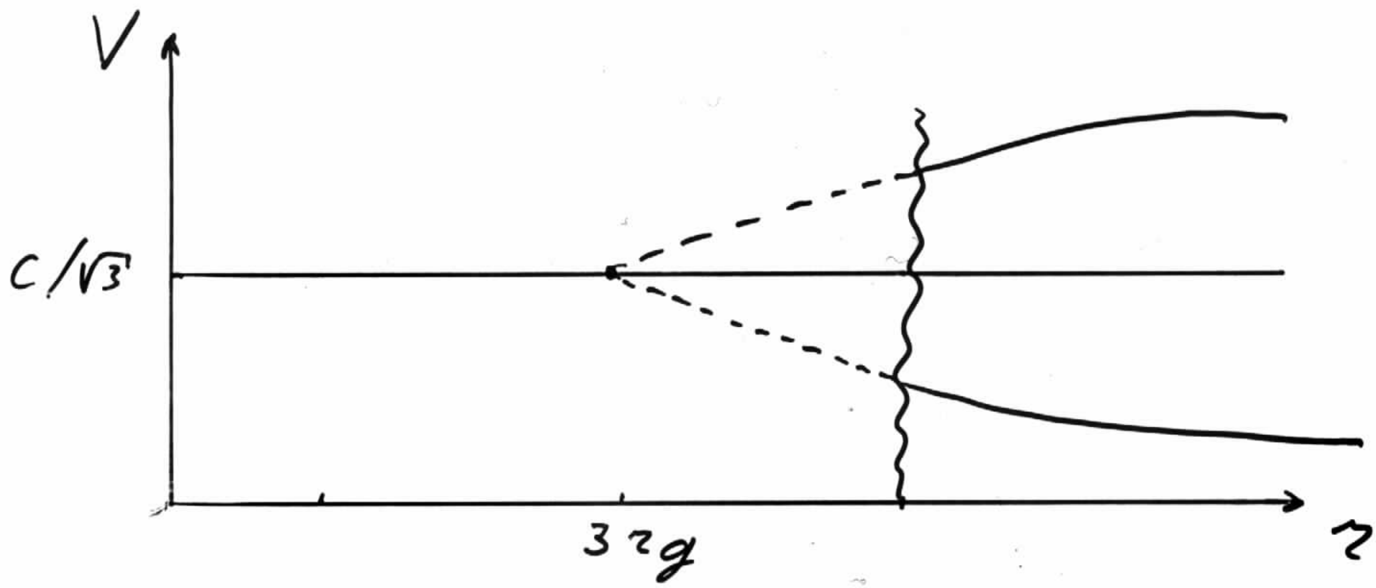
$$\mathrm{v}^2 = \mathrm{v}_\infty^2 + \frac{r_g}{r}(1 - 3\mathrm{v}_\infty^2)$$

For nonrelativistic particle:

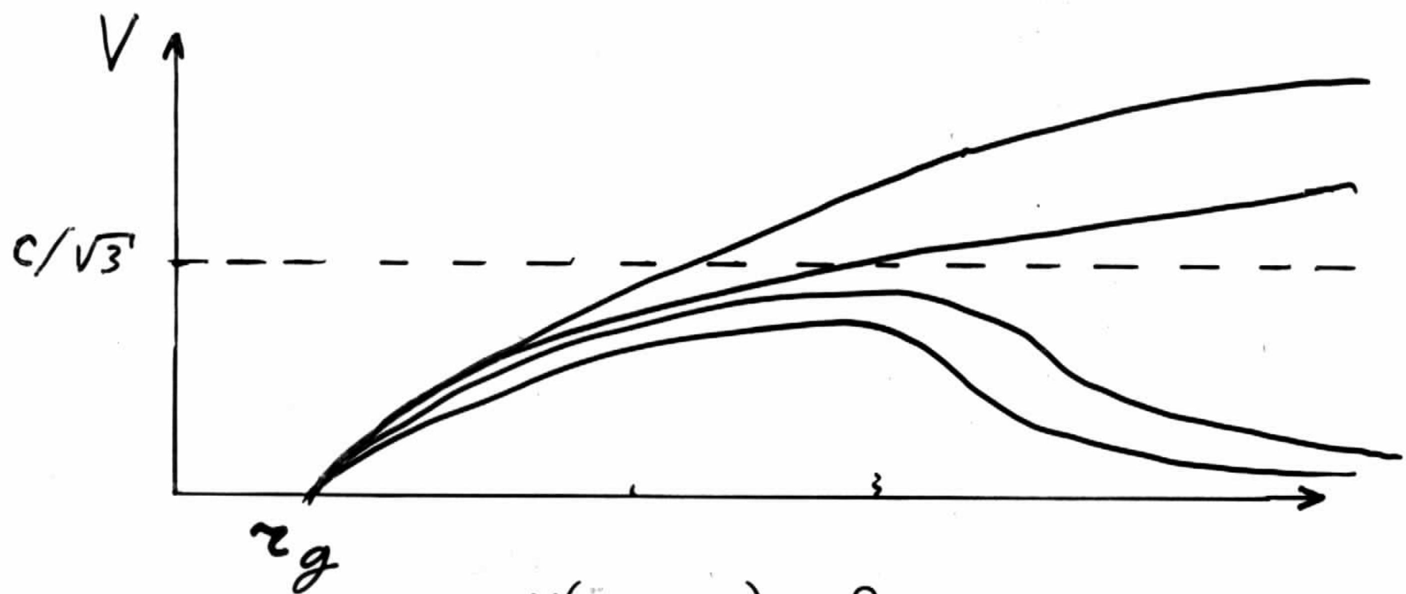
$$\mathrm{v}^2 = \mathrm{v}_\infty^2 + \frac{2MG}{r} \quad \text{-- well -- known expression .}$$

For $\mathrm{v}_\infty = \mathrm{v}_c = 1/\sqrt{3}$ the coordinate velocity of particle does not change, while it grows for $\mathrm{v}_\infty < 1/\sqrt{3}$ and diminishes for $\mathrm{v}_\infty > 1/\sqrt{3}$.

At $r = 3r_g$ coordinate velocities become equal to v_c:



However, for $r = 3r_g$ weak field approximation fails. Disposing the assumption of the weak field we get:



$$v(r = r_g) = 0$$

Non-radial motion: $\theta = \theta_\gamma \frac{1+v_\infty^2}{2v_\infty^2}$; gravity is never ignored.

Ultrarelativistic plasma, equation of state $p = \varepsilon/3$, where p is pressure and ε – energy density. Speed of sound u_s:

$$u_s^2 = c^2 \frac{dp}{d\varepsilon} = \frac{c^2}{3}$$

$u_s = \mathrm{v}_c$: Coincidence, or there is some physical reason? Let us consider n-dimensional space, $n \neq 3$. Equation of state:

$$p = \varepsilon/n\,, \quad u_s^2 = \frac{c^2}{n}\,.$$

[Stress tensor T_{ik} is diagonal and traceless, $T_{00} = \varepsilon$, $T_{ii} = p = \varepsilon/n$.]

Schwarzschild metric in $n+1$ dimensions:

$$ds^2 = \left[1 - (\frac{r_{gn}}{r})^{n-2}\right] dt^2 - \left[1 - (\frac{r_{gn}}{r})^{n-2}\right]^{-1} dr^2\,,$$

Tangherlini, Nuovo Cimento (1963)

$$(r_{gn})^{n-2} = G_n M$$

$$\mathrm{v}^2 = \mathrm{v}_\infty^2 + (\frac{r_{gn}}{r})^{n-2}(1 - 3\mathrm{v}_\infty^2)\,,$$

So $\mathrm{v}_c = c/\sqrt{3}$, where 3 is not a dimension of space – it is due to cubical polynomial in (*).

CONCLUSIONS

The speed of sound in relativistic plasma depends on the dimension of space, while the critical velocity $v_c = c/\sqrt{3}$ is universal.

Critical velocity in weak field was considered by M. Carmeli, Lett. al Nuovo Cimento (1972). Relation with speed of sound was not considered there.