Consequences of a Large Top-Quark Chiral Weak-Moment ($\Lambda_+ \sim 53\,\text{GeV}$)

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OUTLINE

- Introduction to $t \to W^+ b$ decay helicity amplitudes
- Theoretical Puzzles.
- Theoretical Patterns: { • Analytic Relations; • tWb-transformations }
- Relation to the observed top quark.

For this talk:

The amplitudes $A_+(\lambda_{W^+}, \lambda_b)$ in the case of an additional $(f_M + f_E)$ coupling of relative strength $\Lambda_+ = \frac{E_W}{2} \sim 53\,\text{GeV}$ will be interpreted as corresponding to the observed top-quark decays.

Orientation:

In t-quark rest frame, due to rotational invariance there are 4 helicity decay amplitudes $A(\lambda_{W+}, \lambda_b)$

$$<\theta,\phi,\lambda_{W+},\lambda_b\,|\,1/2,\lambda_t> = D^{1/2^*}_{\lambda_t,\mu}(\phi,\theta,0)\,A(\lambda_{W+},\lambda_b)\ ,\quad \mu = \lambda_{W+} - \lambda_L$$

So there are 4 moduli and 3 relative phases to be measured.

$$\begin{array}{ccccc}
A(0,1/2) & \longleftarrow & \alpha_0 & \longleftarrow & A(0,-1/2) \\
 & & & & \downarrow \\
 & & & & \beta_L \\
 & & & & \downarrow \\
A(1,1/2) & \longleftarrow & \alpha_1 & \longleftarrow & A(-1,-1/2)
\end{array}$$

R-handed b-quark amplitudes

L-handed b-quark amplitudes

We considered

(V - A) + "Single Additional Lorentz Structures"

$$\bar{u}_b(p)(\Gamma^\mu{}_V + \Gamma^\mu{}_A)u_t(k)$$

$$\Gamma^\mu{}_V = g_V\gamma^\mu + \frac{g_S}{2\Lambda_S}(k+p)^\mu + \frac{f_M}{2\Lambda_M} i\sigma^{\mu\nu}(k-p)_\nu + \cdots$$

$$\Gamma^\mu{}_A = g_A\gamma^\mu\gamma_5 + \frac{g_P}{2\Lambda_P}(k+p)^\mu\gamma_5 + \frac{f_E}{2\Lambda_E} i\sigma^{\mu\nu}(k-p)_\nu\gamma_5 + \cdots$$

We found 2 dynamical-phase-type ambiguities:

(V - A) + (S + P)

(V - A) + (f_M + f_E) and 3 numerical puzzles !!!

Theoretical Puzzles

Table: Numerical Values of Helicity Amplitudes $A(\lambda_{W+}, \lambda_b)$:

In $g_L = 1$ units :

	A(0, -1/2)	A(-1, -1/2)	A(0, 1/2)	A(1, 1/2)
V – A	338	220	- 2.33	- 7.16
S + P	- 338	220	- 24.4	- 7.16
$f_M + f_E$	220	-143	1.52	- 4.67

Divided by Square Root of respective Decay Width

	A(0, -1/2)	A(-1, -1/2)	A(0, 1/2)	A(1, 1/2)
V – A	0.84	0.54	- 0.0058	- 0.018
S + P	- 0.84	0.54	- 0.060	- 0.018
$f_M + f_E$	0.84	- 0.54	0.0058	- 0.018

Important Remarks:

1. **Respective effective-mass scales for 2 dynamical phase-type ambiguities:** $\Lambda_{S+P} = -35\ GeV$ $\quad \underline{\underline{\Lambda_+ \equiv \Lambda_{f_M+f_E} = 53\ GeV}}$ ⟵

2. **In Phys. Rev. D65 , 074033 (2002), the amplitudes are listed explicitly for (V - A) SM case and for (V - A) + (f_M + f_E) (+) case**

3. **Present mass values:**

m_t = **174.3 +/- 5.1 GeV (3% precision, most accurately measured quark mass)**

m_W = **80.434 +/- 0.037**

m_b = **4.6 +/- 0.2 (pole mass)**

1st Puzzle's associated mass relation:

From empirical mass values

$\underline{\underline{y = m_W / m_t}} = 0.461 \pm 0.014$

From 1st puzzle, by expanding $A_+(0, -1/2)$ amplitude in the mass ratio $\underline{\underline{x^2 = (m_b / m_t)^2}} = 7 \times 10^{-4}$

$$1 - \sqrt{2}y - y^2 - \sqrt{2}y^3 = x^2 \left(\frac{2}{1-y^2} - \sqrt{2}y\right) + \cdots$$

$$= 1.89\, x^2 - 0.748\, x^4 + \cdots$$

Only real-valued solution to this cubic equation is

$y = 0.46\,006 \quad (m_b = 0)$

(note 3% error on "empirical y value" due to m_t's 3% precision)

2nd and 3rd numerical puzzles

As explained below, for

$$\Lambda_+ = E_W / 2 = (m_t/4)(1+y^2-x^2)$$

the occurrence of the same magnitudes is an analytic equality, not a numerical one, for the two R-handed b-quark amplitudes

$$A_{NEW} = \frac{A_{gL=1}}{\sqrt{\Gamma}}$$

for the SM and (+) cases .

Theoretical Patterns

Five ~~Four~~ Types of Analytic Relations:

(I)
$$\frac{A_i(0,1/2)}{A_i(-1,-1/2)} = \frac{1}{2}\,\frac{A_i(1,1/2)}{A_i(0,-1/2)}$$

for separately $i = (SM),\ (+)$

(II)
$$\frac{A_+(0,1/2)}{A_+(-1,-1/2)} = \frac{A_{SM}(0,1/2)}{A_{SM}(-1,-1/2)}$$

These 3 equations occur to all orders in the "y" and "x" mass ratios; they do not depend on the value of Λ_+ . The occurrence of 3 equations is also a stronger result than is apparent from the 2nd & 3rd numerical puzzles.

(II) 2nd-Type of Ratio-Relations: (cont'd)

Two sign flip relations:

$$\frac{A_+(0,1/2)}{A_+(-1,-1/2)} = \frac{A_{sm}(0,1/2)}{A_{sm}(-1,-1/2)}$$

$$\frac{A_+(0,1/2)}{A_+(-1,-1/2)} = \frac{1}{2}\,\frac{A_{sm}(1,1/2)}{A_{sm}(0,-1/2)}$$

Two non-sign flip relations:

$$\frac{A_+(1,1/2)}{A_+(0,-1/2)} = \frac{A_{sm}(1,1/2)}{A_{sm}(0,-1/2)}$$

$$\frac{A_+(1,1/2)}{A_+(0,-1/2)} = 2\,\frac{A_{sm}(0,1/2)}{A_{sm}(-1,-1/2)}$$

These lead to two additional tWb-transformations

$$A_+ = \underset{\sim}{P}\, A_{sm}\,, \qquad A_+ = \underset{\sim}{B}\, A_{sm}$$

where

$\underset{\sim}{P}$, $\underset{\sim}{B}$

are explicit 4x4 matrices.

Consequently by determining the effective mass scale $\Lambda\,(m_W/m_t\,,\ m_b/m_t)$ by exact equality for ratio of L-handed b-quark amplitudes

(III)

$$\frac{A_+(0,-1/2)}{A_+(-1,-1/2)} = -\frac{A_{SM}(0,-1/2)}{A_{SM}(-1,-1/2)}$$

Analytic Formula for Λ_+ is

$$\Lambda_+ = \frac{m_t}{4}\left[1 + y^2 - x^2 \right]$$

$= E_W/2$, E_W = energy W^+ in t rest frame

The

$$A_{NEW} = \frac{A_{gL=1}}{\sqrt{\Gamma}}$$

amplitudes are exactly equal in magnitude between the SM and the (+) cases to all orders in the two mass ratios, “y”&”x”.

This S-matrix “locking mechanism” supports the interpretation that the 2nd & 3rd numerical puzzles arise due to a large chiral weak-moment of the t-quark. (a transition moment)

(an anomalous “moment” is less radical than a new EW coupling)

(III) Remarks

(1) Equivalently, $\mathcal{N}_+ = Ew/2$ follows from any of

$$\frac{A_+(0,-1/2)}{A_+(-1,-1/2)} = -\frac{1}{2}\,\frac{A_{sm}(1,1/2)}{A_{sm}(0,1/2)}$$

$$\frac{A_+(0,1/2)}{A_+(1,1/2)} = -\,\frac{A_{sm}(0,1/2)}{A_{sm}(1,1/2)}$$

$$\frac{A_+(0,1/2)}{A_+(1,1/2)} = -\frac{1}{2}\,\frac{A_{sm}(-1,-1/2)}{A_{sm}(0,-1/2)}$$

(2) Alternatively, $\underbrace{\mathcal{N}_+ = Ew/2 \quad \sim 53\,\text{GeV}}_{\text{A New Weak Scale}}$ can be

characterized by postulating a <u>tWb-transformation</u>

$$\underline{A_+ = \underset{\sim}{M}\, A_{sm}}$$

$$= v \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} A_{sm}(0,-1/2) \\ A_{sm}(-1,-1/2) \\ A_{sm}(0,1/2) \\ A_{sm}(1,1/2) \end{bmatrix}$$

(3) Effective $t \to W^+ b$ coupling is

$$\frac{1}{2}\Gamma^\mu = g_L\left(\gamma^\mu P_L + i\,\sigma^{\mu\nu} v_\nu P_R\right)$$

$$\boxed{= g_L P_R\left(\gamma^\mu + i\,\sigma^{\mu\nu} v_\nu\right)}$$

$$P_{L,R} = \frac{1}{2}\left(1 \mp \gamma_5\right)$$

v_ν = W-boson's relativistic 4-velocity

Assuming (III), the 4th type of analytic relation is

(IV) $$\boxed{\begin{array}{c} A_+(0,-1/2) = \alpha \; A_{sm}(-1,-1/2) \\ \alpha = 1 + \left(x \equiv \frac{m_b}{m_t}\right) \text{ corrections} \end{array}}$$

<u>Neglecting corrections to $\alpha = 1$</u>

- This implies above $\frac{m_W}{m_t}$ mass relation.
- This is equivalent to velocity formula in t-quark rest frame

$$\left.\begin{array}{l} \sqrt{2} = v\sqrt{\dfrac{1+v}{1-v}} \\ v = \text{velocity of W-boson} \\ \quad = 0.6506\ldots \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \text{There are} \\ \text{corrections} \\ x = \frac{m_b}{m_t} \text{ to} \\ \alpha = 1 \end{array}\right.$$

- This is equivalent to postulating a <u>2nd tWb-transformation:</u>

$$A_+ = \underset{\sim}{P}\, A_{sm}$$

$$\underset{\sim}{P} \equiv v \left[\begin{array}{cc|cc} 0 & 1/v & \multicolumn{2}{c}{\multirow{2}{*}{0}} \\ -v & 0 & & \\ \hline \multicolumn{2}{c|}{\multirow{2}{*}{0}} & 0 & -v/2 \\ & & 2/v & 0 \end{array}\right]$$

So far, we have not directly related $\lambda_b = -1/2$ and $\lambda_b = 1/2$ amplitudes.

Assuming (III) and (IV), we find:

(V)

$$A_+ = \underset{\sim}{B}\, A_{SM} \qquad \text{(3rd twb-transformation)}$$

$$\underset{\sim}{B} \equiv \begin{bmatrix} 0 & 0 & 0 & -\kappa_1 \\ 0 & 0 & 2\kappa_1 & 0 \\ 0 & \frac{v^2}{2\kappa_1} & 0 & 0 \\ -\frac{v^2}{\kappa_1} & 0 & 0 & 0 \end{bmatrix}$$

$$\underline{\kappa_1 = v^{-8}} + \left(x \equiv \frac{m_b}{m_t}\right) \text{ corrections}$$

$$\underline{= 31.152} + \ldots.$$

<u>gives</u>

$$m_b = \frac{m_t}{\kappa_1}\left[1 - \frac{vy}{\sqrt{2}}\right]$$

$$= 4.407 \ldots \text{ GeV}$$

Relation to Observed $t \to W^+ b$ decay

Remarks and Model Building:

Model dependent interpretations and assumptions are needed to relate above analytic realizations to the observed $t\bar{t}$ production.

Let t denote SM quark
T denote (+) quark with $\Lambda_+ = 53$ GeV chiral weak-moment

(I) Model: t or T is the observed top quark:

- **Measure sign of** ($|\eta_L| = 0.5$ kinematic limit)

$$\eta_L = \frac{1}{\Gamma} | A(-1,-1/2) | | A(0,-1/2) | \cos\beta_L$$
$$= \pm 0.46\,(SM, +) \quad \left(\begin{array}{c}\text{opposite}\\ \text{signs}\end{array}\right)$$

$A(0,-\frac{1}{2})$
$\updownarrow$ β_L = relative phase
$A(-1,-\frac{1}{2})$

- **By single top-production, measure partial decay width**

$$\Gamma_{SM} = 1.55\ GeV \qquad (t \to W^+ b)$$

$$\Gamma_+ = v^2\,\Gamma_{SM} = 0.66\ GeV$$

(56% change in life-time if dominant mode)

- Measure closely associated helicity parameter

Prime

$$\eta_L' \equiv \frac{1}{\Gamma} |A(-1,-\tfrac{1}{2})| \, |A(0,-\tfrac{1}{2})| \sin\beta_L$$

In absence of $T_{\text{Final State}}$ - violation,

$$\eta_L' = 0.$$

- Obtain evidence for, or against

$$t \to W^+ s$$

and/or

$$t \to W^+ d$$

decay channels at the expected relative rates of the CKM paradigm versus the observed $t \to W^+ b$ mode.

"Thank you very much!"

$\equiv$ "tante grazie"