Measurements of the Unitarity Triangle by Belle

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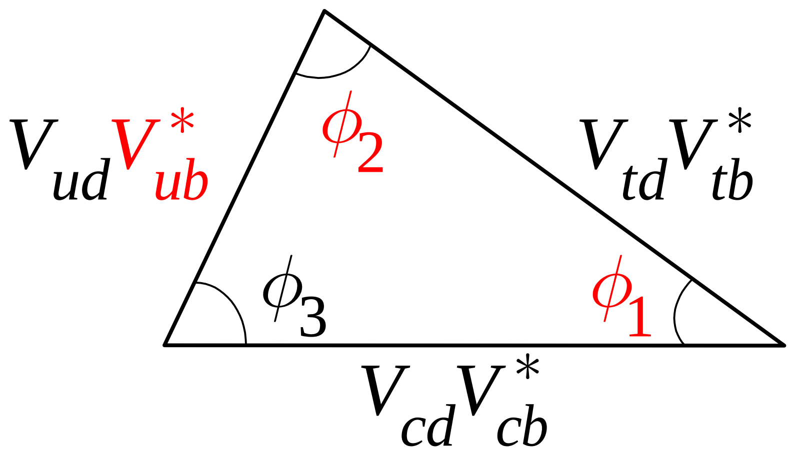
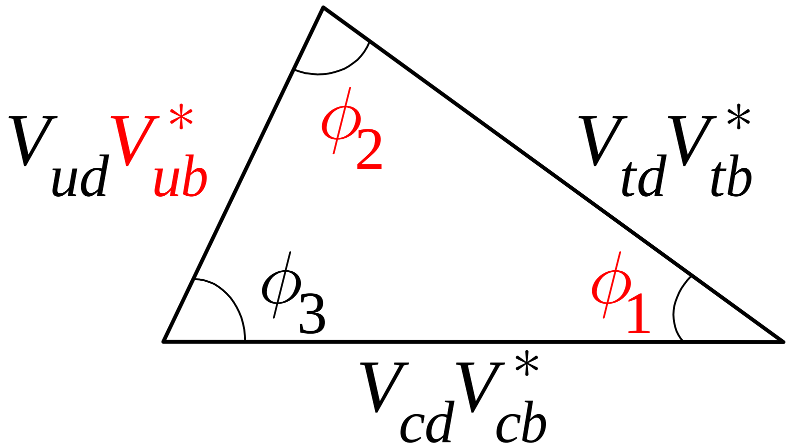
The unitarity triangle



XVII Rencontres de Physique de la Vallee d'Aoste

Today's Contents

- **Introduction**
  - Introduction to the unitarity triangle
- **Interior angle measurements (measurements of time-dependent *CP* violation)**
  - ϕ_2: $B^0 \to \pi^+\pi^-$
  - ϕ_1: $b \to c\bar{c}s$, $b \to s\bar{s}s$
- **Side measurement**
  - $|V_{ub}|$: $B^0 \to D_s^+\pi^-$
- **Conclusions**

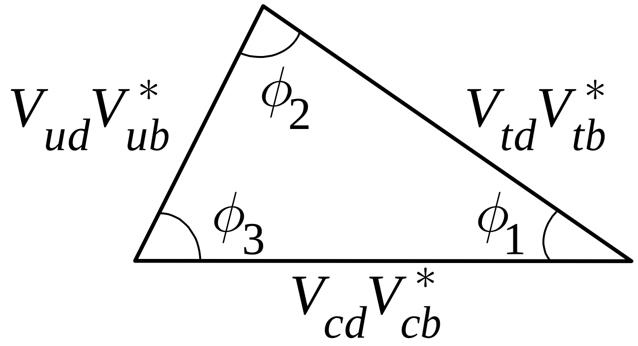


Introduction

Introduction to the Unitarity Triangle

- **What is the unitarity triangle?**
  - It is a triangle formed by elements of quark-mixing-matrix of three generations.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



- **Which part of physics is the triangle related to?**
  - The quark-mixing-matrix of three generations is a key to describe ***CP* violation**.
  - N.B: The three-generation matrix is proposed by Kobayashi and Maskawa in 1973 (**KM-model**).

$$\Gamma(X \to f_{CP}) \neq \Gamma(\overline{X} \to f_{CP})$$

Introduction – Cont'd

- **Predictions of the KM-model**
  - There are $\geq$ 6 quarks.
    - ☑ **Discovery of *c*-, *b*-, and *t*-quarks.**
  - *CP* violation in the *B* system.
    - ☑ **sin2ϕ_1 measurement.**
  - The unitarity triangle is really a triangle.
    - ☐ **Test by angles (ϕ_1, ϕ_2, ϕ_3) and sides (V_{ub} etc.) in *B* decays.**

Motivations for Belle

BELLE

Measurement of the unitarity triangle is an important test of the Standard Model

Time-Dependent CP Violation

- Introduction to time-dependent CP violation
- ϕ_2 measurement with $B^0 \to \pi^+\pi^-$ decay
- ϕ_1 measurement with $b \to c\bar{c}s$, $s\bar{s}s$ transitions

Time-Dependent CP Violation

$$A_{CP}(\Delta t) \equiv \frac{\Gamma(\overline{B}^0 \to f_{CP};\Delta t) - \Gamma(B^0 \to f_{CP};\Delta t)}{\Gamma(\overline{B}^0 \to f_{CP};\Delta t) + \Gamma(B^0 \to f_{CP};\Delta t)}$$
$$= \mathcal{S}\sin(\Delta m_d \Delta t) + \mathcal{A}\cos(\Delta m_d \Delta t)$$

If either $\mathcal{S}$ or $\mathcal{A}$ is non-zero, $B \to f_{CP}$ has CP asymmetry.

Note: Standard Model prediction ($\xi_f \equiv CP$ eigenvalue)

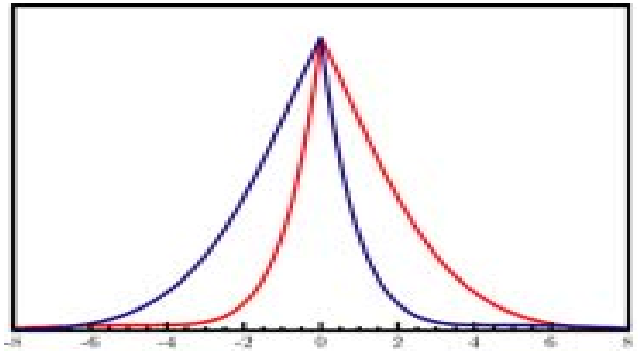
	$b \to u\bar{u}d$	$b \to c\bar{c}s$	$b \to s\bar{s}s$
$\mathcal{S}$	$+\xi_f \sin 2\phi_2$(?)	$-\xi_f \sin 2\phi_1$	$-\xi_f \sin 2\phi_1$(?)
$\mathcal{A}$	0(?)	0	0(?)

$\mathcal{S}$ and $\mathcal{A}$ are related to the interior angle of the unitarity triangle.

Proper-Time Difference: Δt

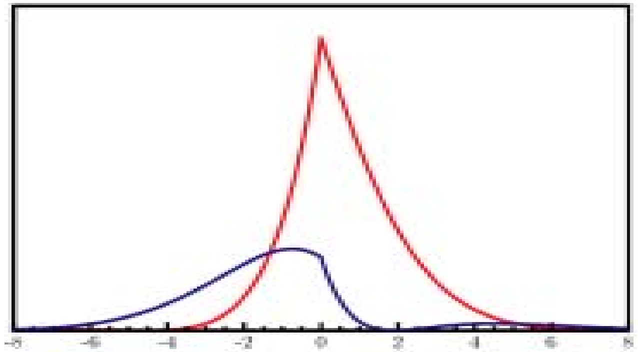
$$f(\overline{B}^0 \to f_{CP};\Delta t) = e^{-\frac{\Delta t}{\tau_{B^0}}}\{1 + [\mathcal{S}\sin(\Delta m_d \Delta t) + \mathcal{A}\cos(\Delta m_d \Delta t)]\}$$

$$f(B^0 \to f_{CP};\Delta t) = e^{-\frac{\Delta t}{\tau_{B^0}}}\{1 - [\mathcal{S}\sin(\Delta m_d \Delta t) + \mathcal{A}\cos(\Delta m_d \Delta t)]\}$$



$\mathcal{S} = 0.8$
$\mathcal{A} = 0.0$

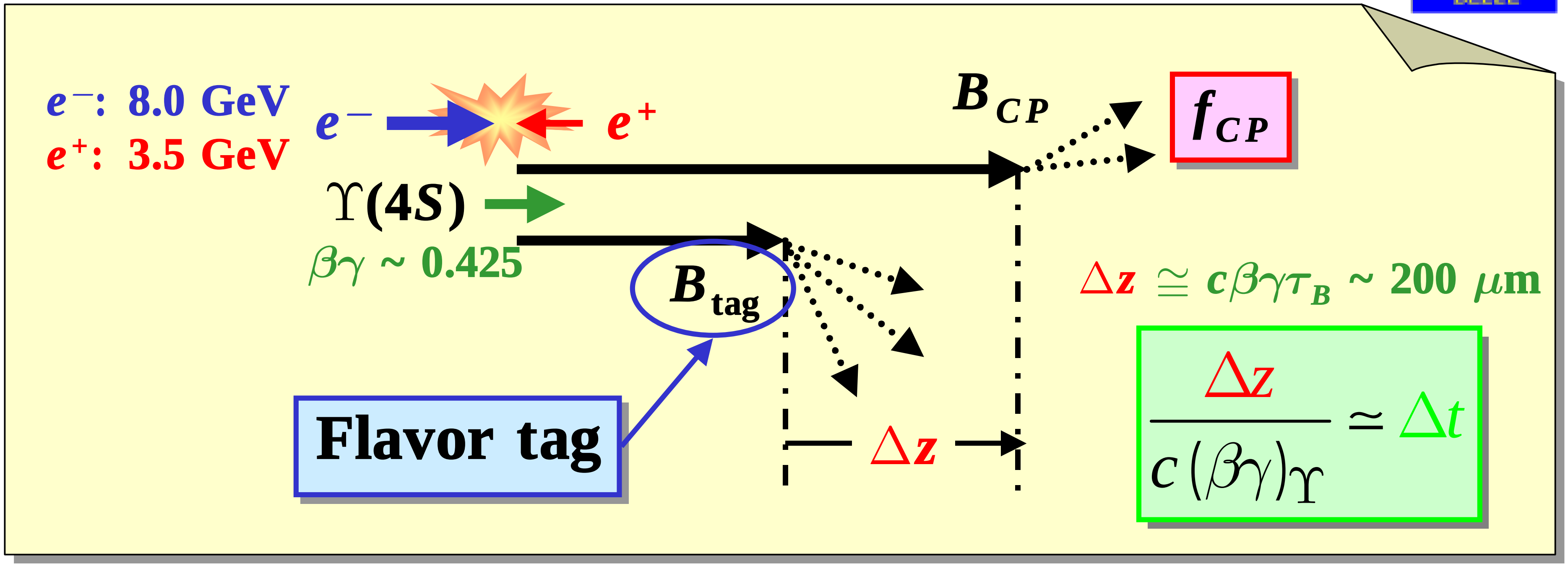
$\tau_B = 1.542$ ps
$\Delta m_d = 0.489$ ps^{-1}



$\mathcal{S} = 0.8$
$\mathcal{A} = 0.6$

From **red**/**blue** asymmetry in the observed Δt distributions, we determine the *CP*-violating parameters, $\mathcal{S}$ and $\mathcal{A}$.

How Can We Measure CP Violation?



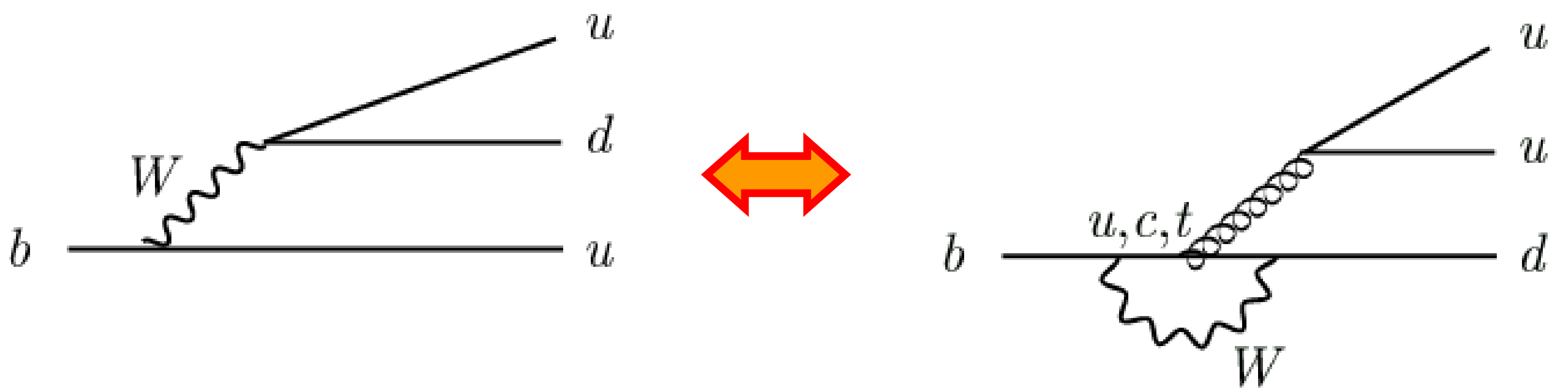
4 steps toward the CP violation measurement

- Reconstruct $B \to f_{CP}$ decays
- Determine flavor of $B_{\rm tag}$
- Measure proper-time difference: Δt
- Evaluate asymmetry from the obtained Δt distributions

ϕ_2 Measurement

- **ϕ_2 can be measured by $b \to u\bar{u}d$ transition**
  - We use $B^0 \to \pi^+\pi^-$ decay for ϕ_2 measurement.

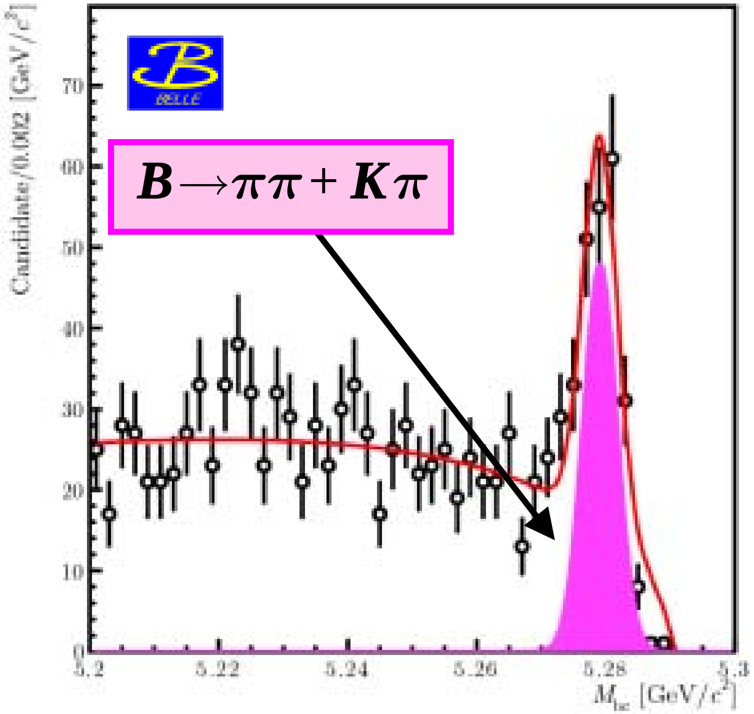
- **"Direct" CP violation**



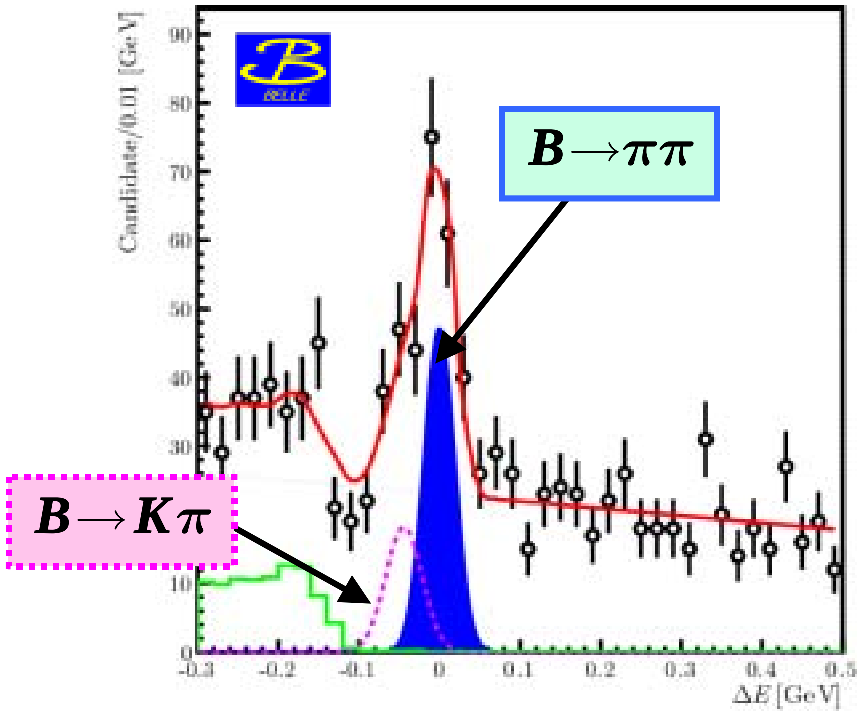
  - Tree and penguin diagrams have amplitudes of the same order with different strong/weak phases. **"Direct" CP violation ($\mathcal{A} \neq 0$) is expected**.
  - In explicit words: $\Gamma(B^0 \to \pi^+\pi^-) \neq \Gamma(\overline{B}^0 \to \pi^+\pi^-)$

$B^0 \to \pi^+\pi^-$ Reconstruction

Mass



*E*nergy difference
from expected value
(0 GeV if signal)

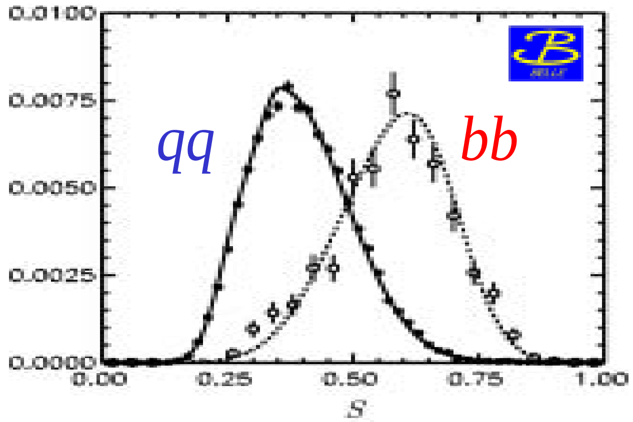


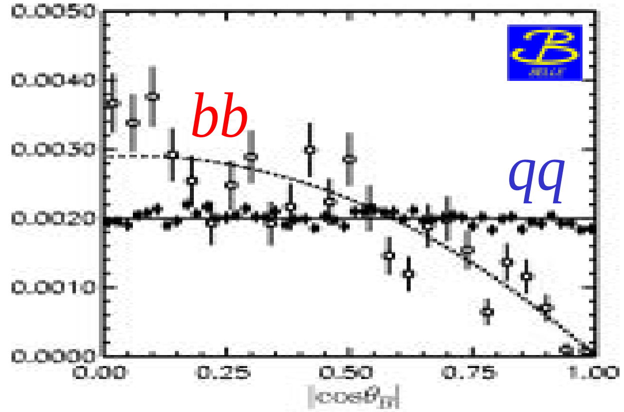
760 $B^0 \to \pi^+\pi^-$ candidates in signal region

Number of expected signal is 163^{+23}_{-24}

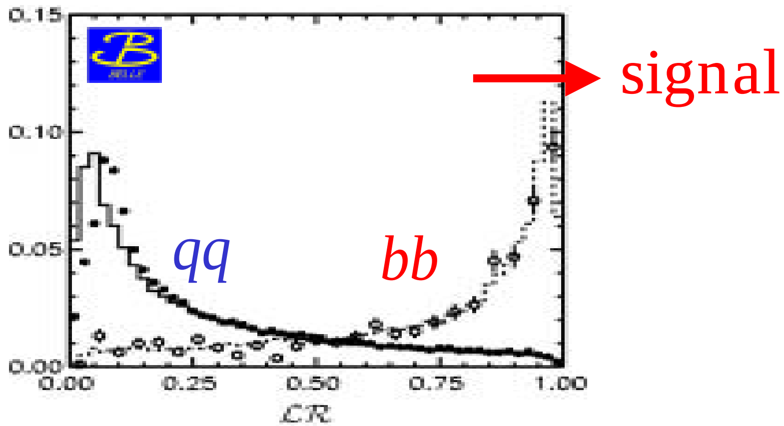
$e^+e^- \to q\bar{q}$ Background Suppression

- **$b\bar{b}$ or $q\bar{q}$ likelihood**
  - Construct $b\bar{b}$ or $q\bar{q}$ likelihood with Fisher discriminant, reconstructed B momentum direction, etc.

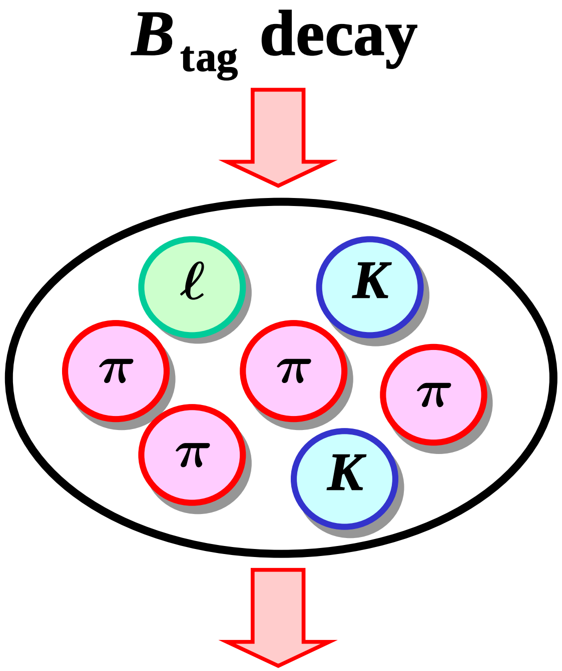




- **Signal selection with likelihood ratio $\mathcal{L}_{bb}/(\mathcal{L}_{bb}+\mathcal{L}_{qq})$**



Flavor Tagging



$$\bar{b} \rightarrow \bar{c}\, \ell^{+}\, \nu$$

$$\bar{c} \rightarrow \bar{s}\, \ell^{-}\, \bar{\nu}$$

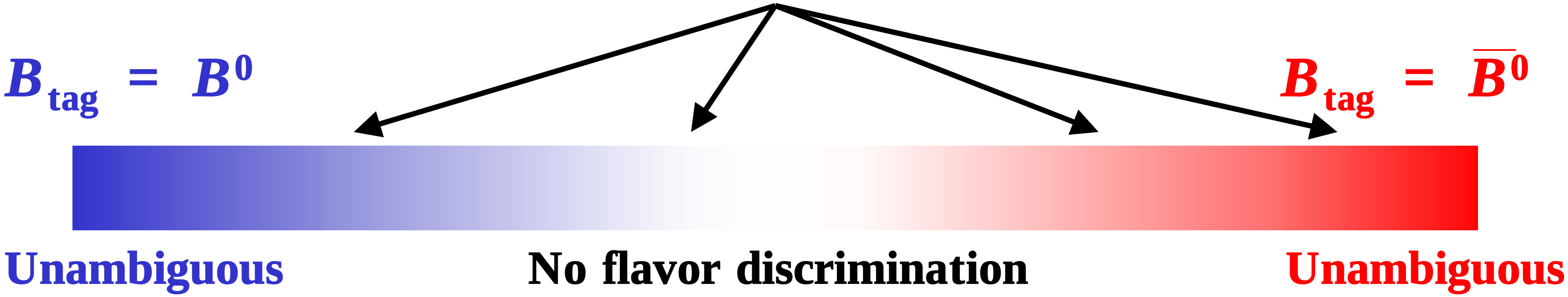
$$B^{0} \rightarrow D^{(*)-}\, \pi^{+},\;\; D^{(*)-}\, \rho^{+}\,,\; etc.$$

$$D^{(*)-} \rightarrow K^{+}\, X$$

$$D^{(*)-} \rightarrow \overline{D}^{\,0}\, \pi^{-}$$

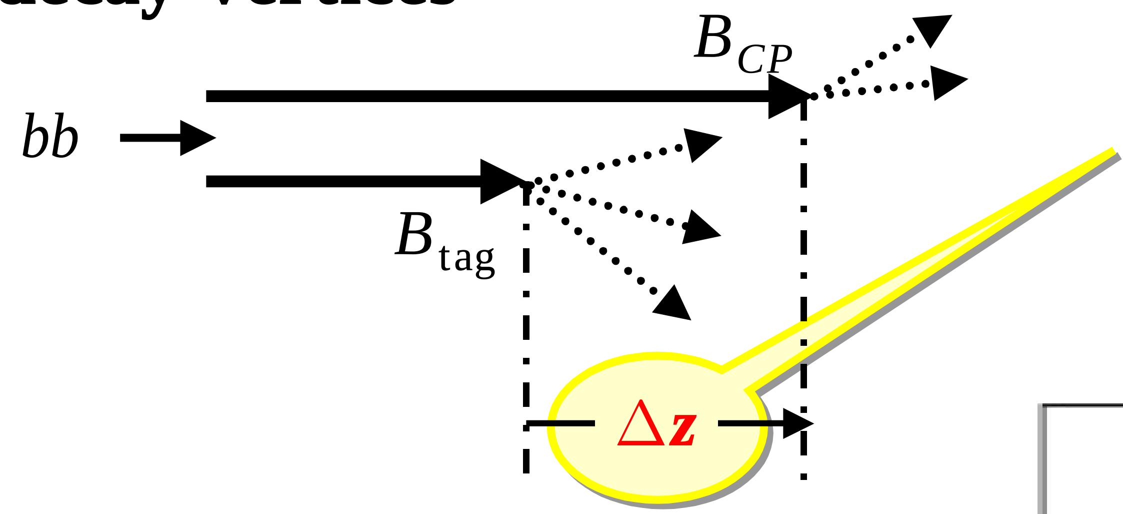
$$\rho^{+} \rightarrow \pi^{+}\, \pi^{0}$$

Examine B_{tag} decay products;
type, charge, momentum, etc.



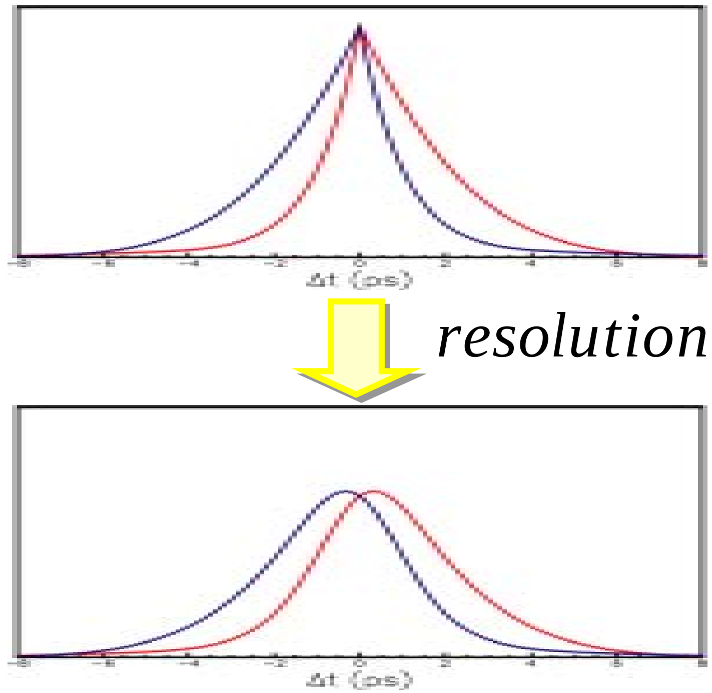
Proper-Time Difference Reconstruction

- **Δt is calculated from distance between two B decay vertices**



$$\frac{\Delta z}{c(\beta\gamma)_{\Upsilon}} \simeq \Delta t$$

- **Detector resolution**
  - Due to limited vertex resolution sharp peaks are dulled.
  - Don’t be surprised if you see round peaks.



Fit for $\mathcal{S}$ and $\mathcal{A}$ Determination

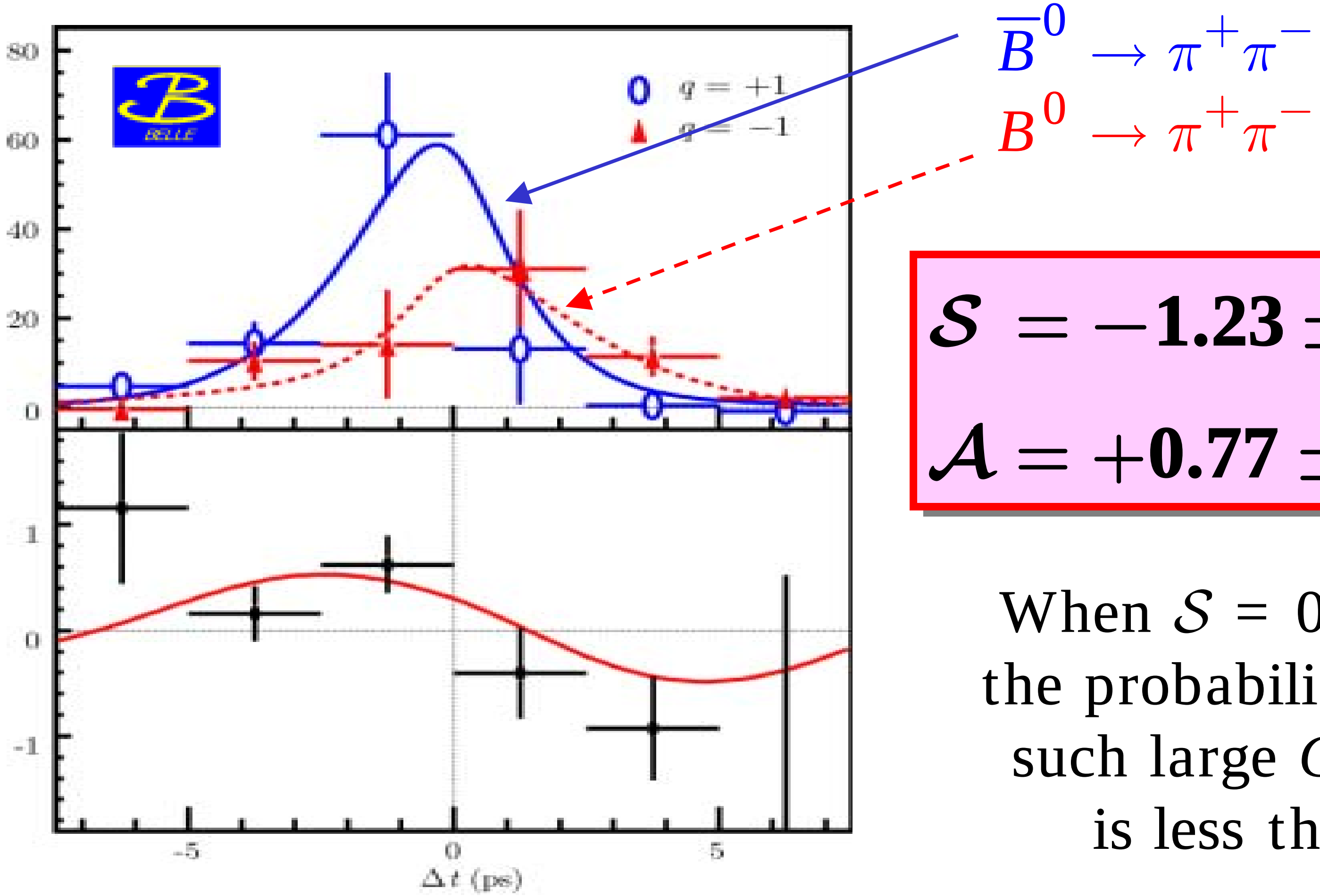
- **Maximum likelihood fit method**

$$L(\mathcal{S},\mathcal{A}) = \prod_{i=1}^{760} P(\Delta t_i; \mathcal{S},\mathcal{A}) \xrightarrow{\ maximize\ } \frac{\partial^2 L}{\partial\mathcal{S}\,\partial\mathcal{A}} = 0$$

$$P(\Delta t_i; \mathcal{S},\mathcal{A}) = \underbrace{f_{\rm sig}\cdot \mathcal{P}_{\rm sig}(\Delta t;\mathcal{S},\mathcal{A})\otimes R}_{\text{signal}} + \underbrace{(1-f_{\rm sig})\cdot P_{\rm bkg}(\Delta t)}_{\text{background}}$$

1. $f_{\rm sig}$: event by event signal probability
2. $P_{\rm sig}$: $\frac{e^{-|\Delta t|/\tau_B}}{4\tau_B}\left\{1+q(1-2w)\left[\mathcal{S}\sin(\Delta m_d \Delta t)+\mathcal{A}\cos(\Delta m_d \Delta t)\right]\right\}$
3. R : Δt resolution
4. $P_{\rm bkg}$: Δt distributiuon for background events

CP Violation in $B^0 \to \pi^+\pi^-$ Decays



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$$\mathcal{S} = -1.23 \pm 0.41\,^{+0.08}_{-0.07}$$

$$\mathcal{A} = +0.77 \pm 0.27 \pm 0.08$$

When $\mathcal{S} = 0$ and $\mathcal{A} = 0$,
the probability to observe
such large CP violation
is less than 0.1%.

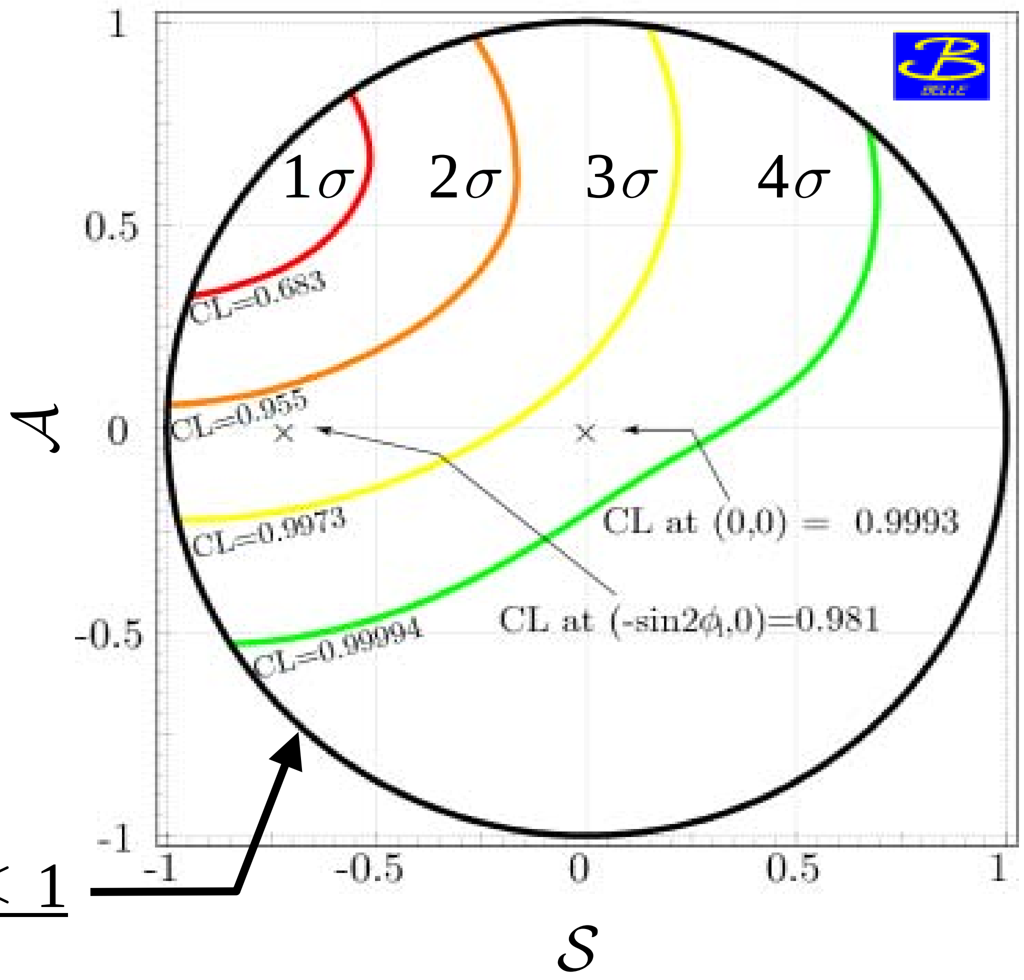
K. Abe *et al.* [Belle Collaboration], submitted to Phys. Rev. D, arXiv:hep-ex/0301032.

The Result Tells Us …

$$\mathcal{S} = -1.23 \pm 0.41 \,^{+0.08}_{-0.07}$$
$$\mathcal{A} = +0.77 \pm 0.27 \pm 0.08$$

1. *CP* conservation is ruled out at 3.4 σ confidence level.

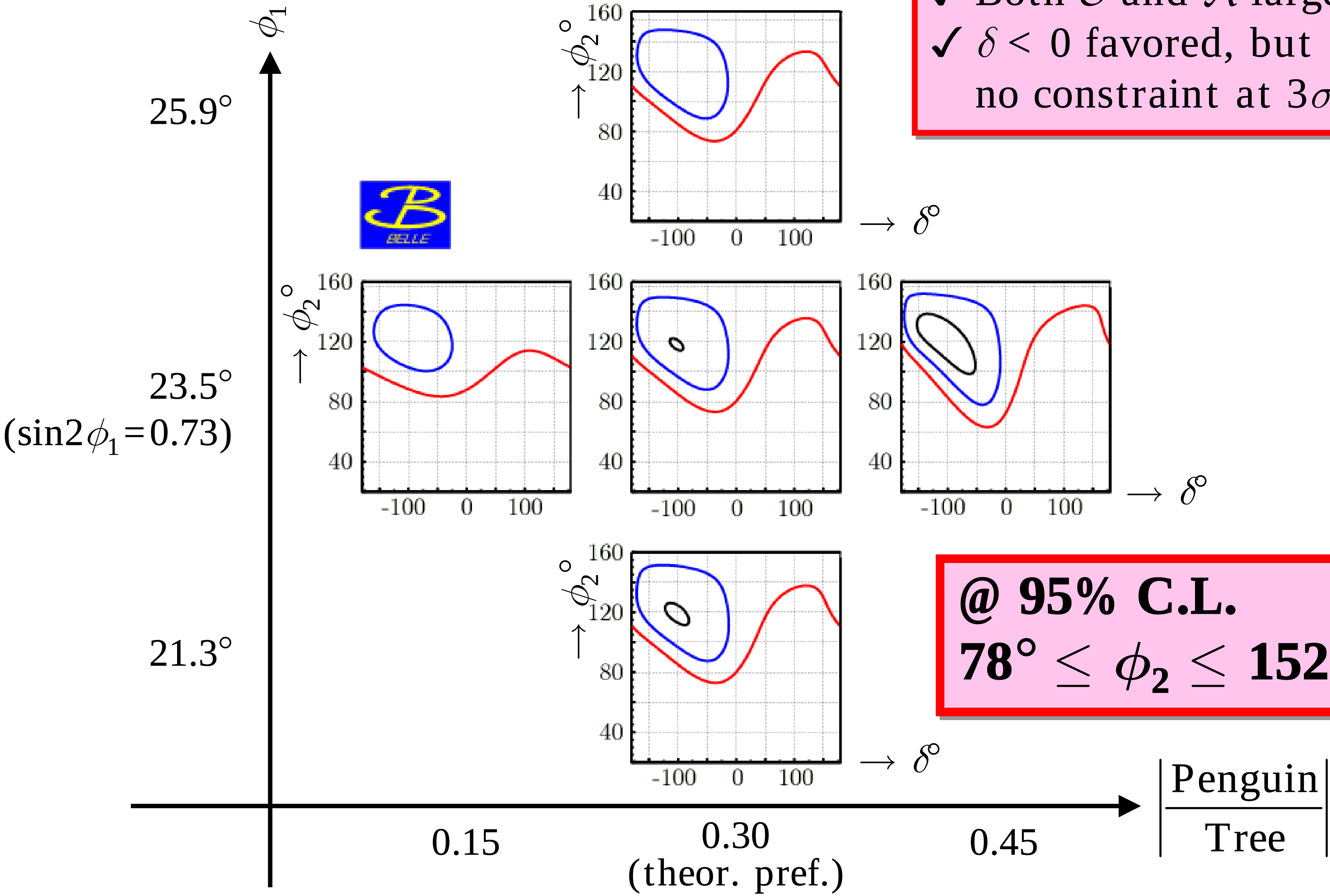
2. $\mathcal{A} \neq 0$ cannot be established yet.



<u>*Physical boundary:* $\mathcal{S}^2 + \mathcal{A}^2 < 1$</u>

The Result Tells Us ... – Constraint on ϕ_2

- ✓ Both $\mathcal{S}$ and $\mathcal{A}$ large.
- ✓ $\delta < 0$ favored, but no constraint at 3σ.



@ 95% C.L.

$$78^\circ \leq \phi_2 \leq 152^\circ$$

Consistency Checks

- **B^0-$\bar{B}^0$ mixing fit on Δt distribution: *OK***
  - ✓ $B^0 \to K^+\pi^-$: $\Delta m_d = 0.55^{+0.05}_{-0.07}\ \mathrm{ps}^{-1}$

    consistent with PDG2002 $0.489 \pm 0.008\ \mathrm{ps}^{-1}$

- **Lifetime fit on Δt distributions: *OK***
  - ✓ $B^0 \to \pi^+\pi^-$: $\tau_B = 1.42 \pm 0.14$ ps
  - ✓ $B^0 \to K^+\pi^-$: $\tau_B = 1.46 \pm 0.08$ ps

    consistent with PDG2002 1.542 ± 0.016 ps

- **Null asymmetry test: *OK***

  consistent with zero

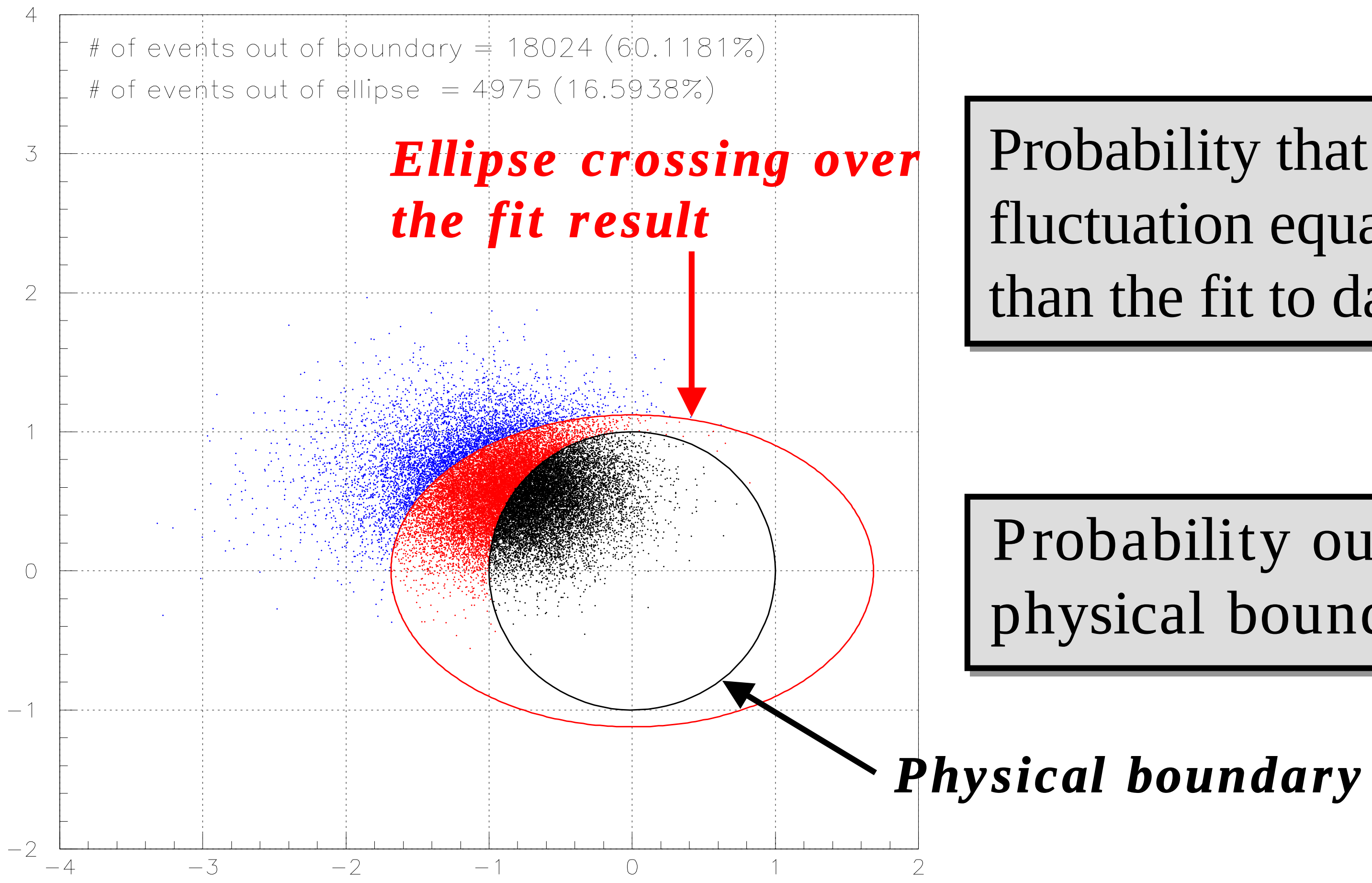
  - ✓ Non-CP sample: $\mathcal{S} = +0.045 \pm 0.033$, $\mathcal{A} = -0.015 \pm 0.022$
  - ✓ $B^0 \to K^+\pi^-$: $\mathcal{S} = +0.08 \pm 0.16$, $\mathcal{A} = -0.03 \pm 0.11$

  $\mathcal{A}$: consistent with counting method ($\to$ A.Drutskoy's talk)

Consistency checks ... OK

Probability to get result outside physical region

$\mathcal{S}$ and $\mathcal{A}$ distribution obtained from parameterized MC (30k events)



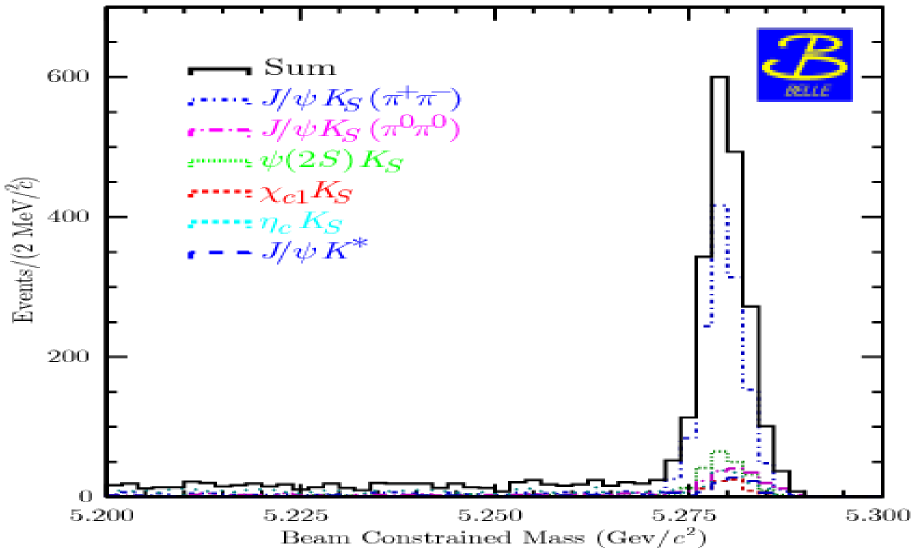
Input: $\mathcal{S} = -0.822$, $\mathcal{A} = +0.569$
(input values at the physical boundary)

Probability that we have fluctuation equal to or larger than the fit to data is 16.6%

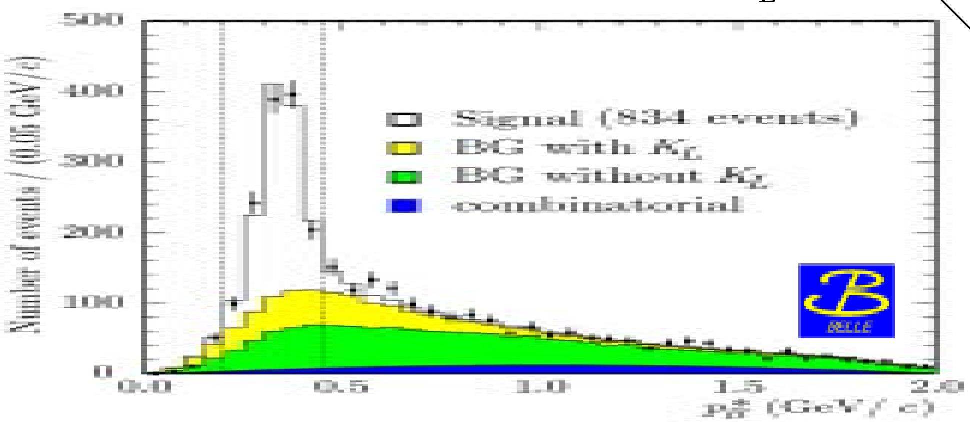
Probability outside the physical boundary = 60.1%

$b \to c\bar{c}s$ Reconstruction

Reconstructed B^0 mass



Reconstructed B^0 momentum for $B^0 \to J/\psi K_L$ decay



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Decay	CP	#	p
$J/\psi K_S(\pi^+\pi^-)$		1285	0.98
$J/\psi K_S(\pi^0\pi^0)$		188	0.82
$\psi(2S)(\ell^+\ell^-)K_S$		91	0.96
$\psi(2S)(J/\psi\pi^+\pi^-)K_S$		112	0.91
$\chi_{c1}(J/\psi\gamma)K_S$	−1	77	0.96
$\eta_c(K_SK^+\pi^-)K_S$		72	0.65
$\eta_c(K^+K^-\pi^0)K_S$		49	0.73
$\eta_c(pp)K_S$		21	0.94
$J/\psi K_L$	+1	1330	0.63
$J/\psi K^{*0}(K_S\pi^0)$	mix	101	0.92
Total		**3326**	

CP Violation in $b \to c\bar{c}s$ Transition

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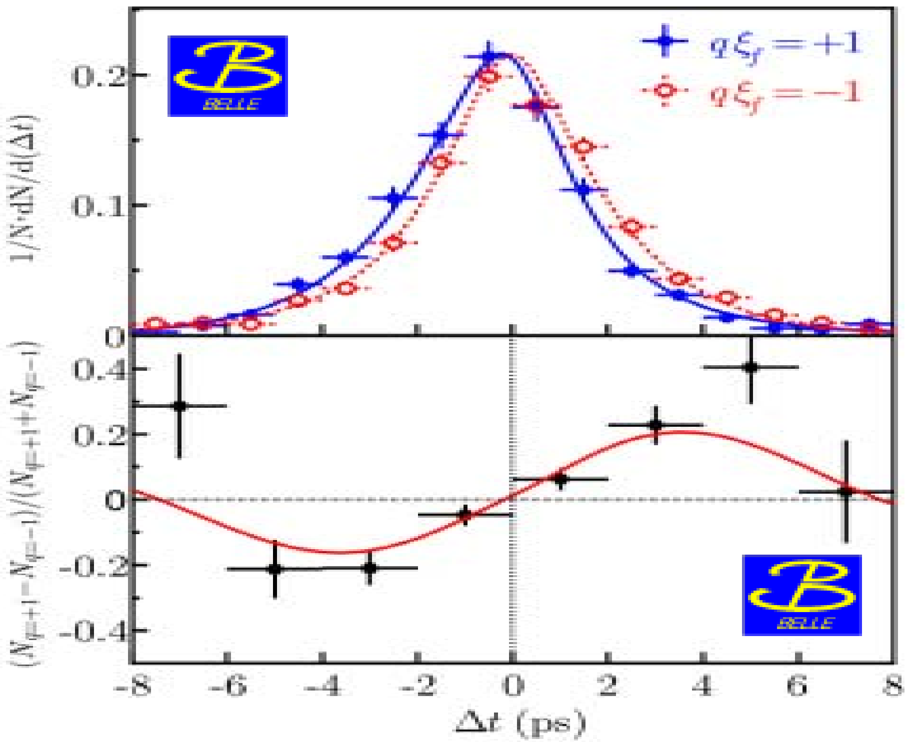
$$\sin 2\phi_1 = 0.719 \pm 0.074(\text{stat}) \pm 0.035(\text{syst})$$

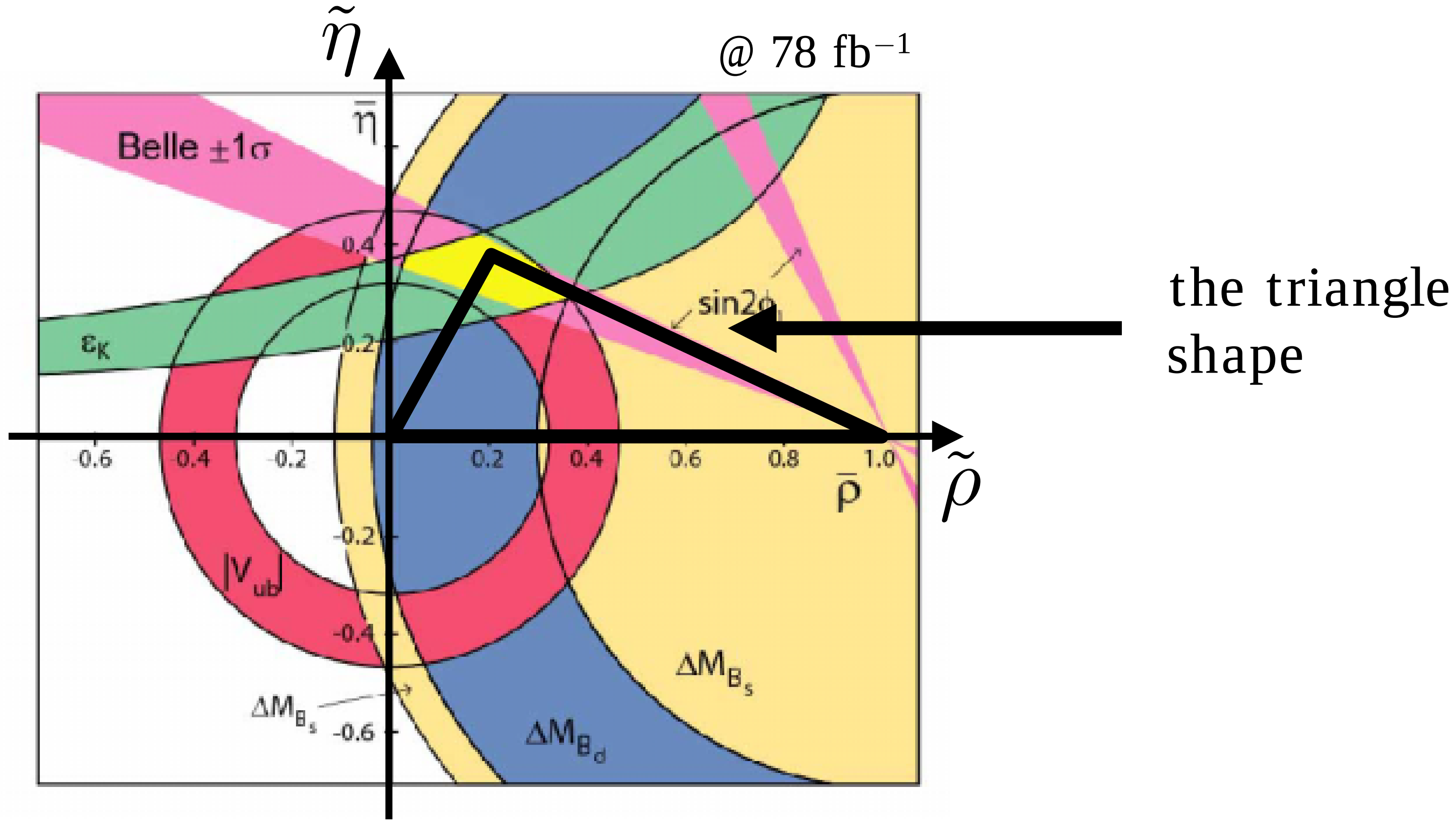


$\overline{B}^0 \to f_{CP}$

$B^0 \to f_{CP}$

← $\dfrac{N_{\overline{B}} - N_B}{N_{\overline{B}} + N_B}$ vs. Δt

Constraint on the Unitarity Triangle Shape

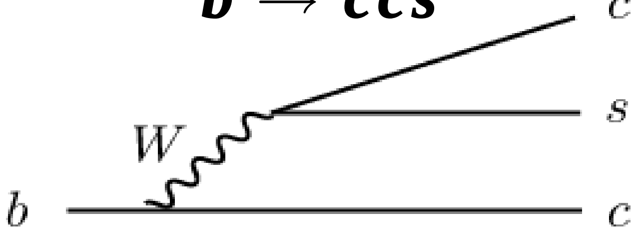


K. Abe *et al*. [Belle Collaboration], Phys. Rev. D **66**, 071102 (2002)

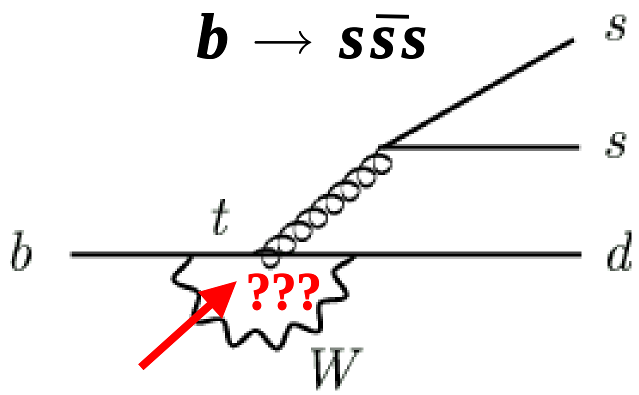
CP Violation in $b \to s\bar{s}s$ Transition

- **Standard model**
  - Same magnitude of CP violation in $b \to c\bar{c}s$ and $b \to s\bar{s}s$.

$b \to c\bar{c}s$



$b \to s\bar{s}s$



Unknown phase contributing to CP violation?

- **New physics**
  - New physics may be present in the penguin-loop, if we see different CP violation in tree and penguin.

$b \to s\bar{s}s$ Reconstruction

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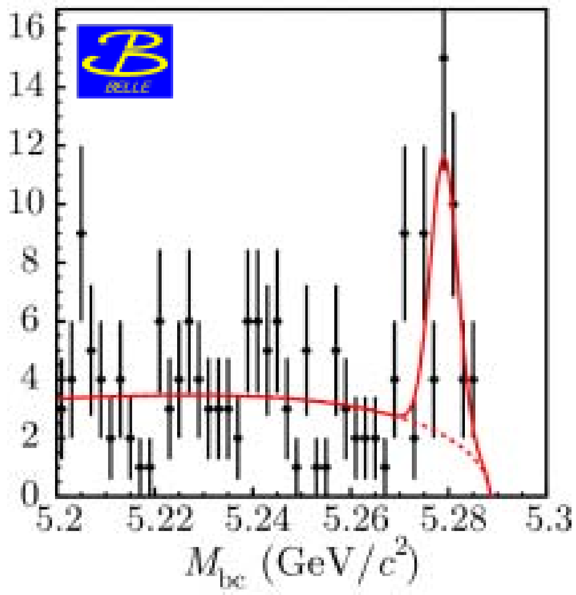
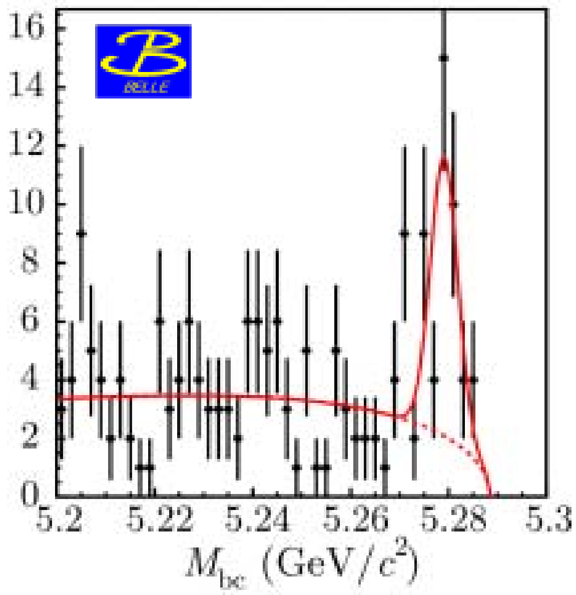
$B^0 \to \phi K_S$
$\phi \to K^+K^-$



$N = 53$

$p = 0.67^{+0.07}_{-0.05}$

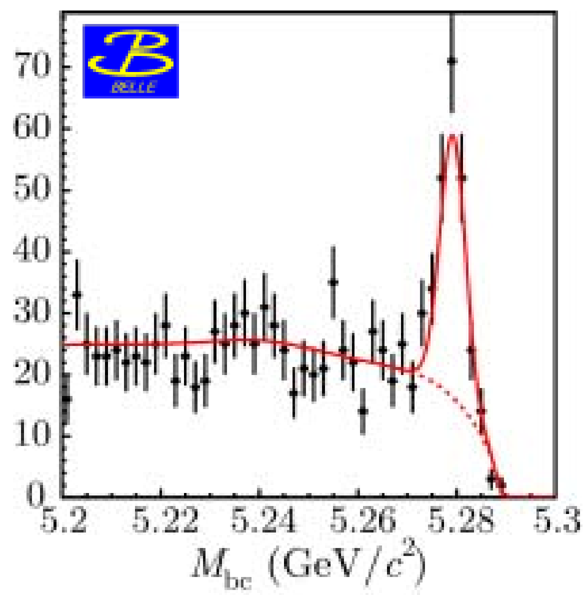
$B^0 \to K^+K^-K_S$
$(K^+K^- \neq \phi)$



$N = 191$

$p = 0.50^{+0.04}_{-0.03}$

$B^0 \to \eta' K_S$
$\eta' \to \pi^+\pi^-\eta,\ \rho\gamma$
$\eta' \to \gamma\gamma$



$N = 299$

$p = 0.49 \pm 0.05$

CP Violation in $b \to s\bar{s}s$ Transition

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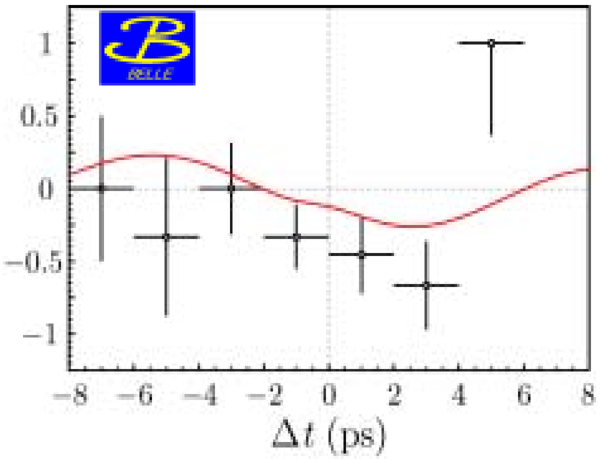
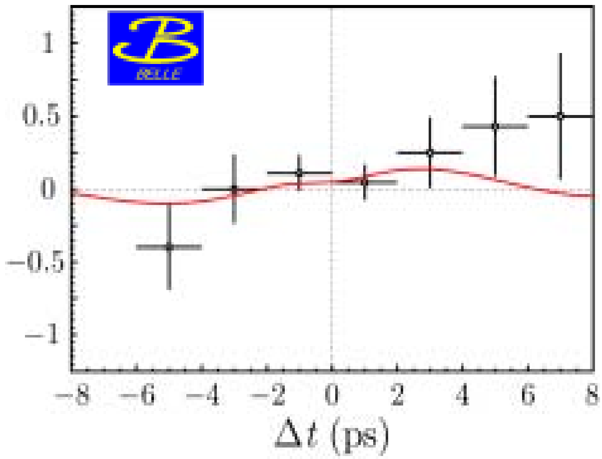
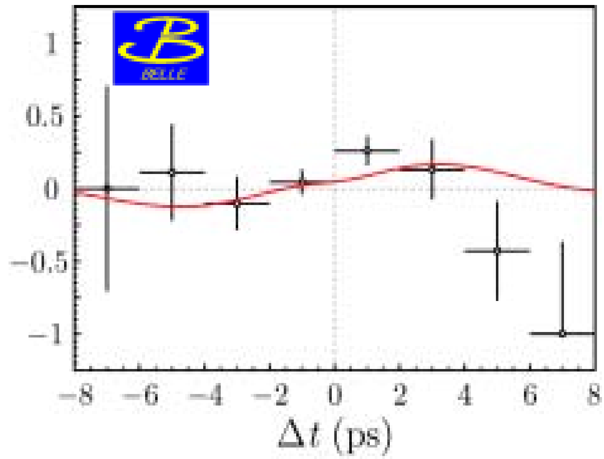
$B^0 \to \phi K_S$



$B^0 \to K^+K^-K_S$



$B^0 \to \eta' K_S$



mode	$B^0 \to \phi K_S$	$B^0 \to K^+K^-K_S$	$B^0 \to \eta' K_S$
$-\xi_f \mathcal{S}$	$-0.73 \pm 0.64 \pm 0.22$	$+0.49 \pm 0.43 \pm 0.11^{+0.33}_{-0.00}$	$+0.71 \pm 0.37\ ^{+0.05}_{-0.06}$
$\mathcal{A}$	$-0.56 \pm 0.41 \pm 0.16$	$-0.40 \pm 0.33 \pm 0.10^{+0.00}_{-0.26}$	$+0.26 \pm 0.22 \pm 0.03$

K. Abe *et al.* [Belle Collaboration], to be published in Phys. Rev. D.

The Results Tell Us ...

- **$\sin 2\phi_1$ world average from $b \to c\bar{c}s$ transition**
  - $\sin 2\phi_1 = +0.734 \pm 0.054$

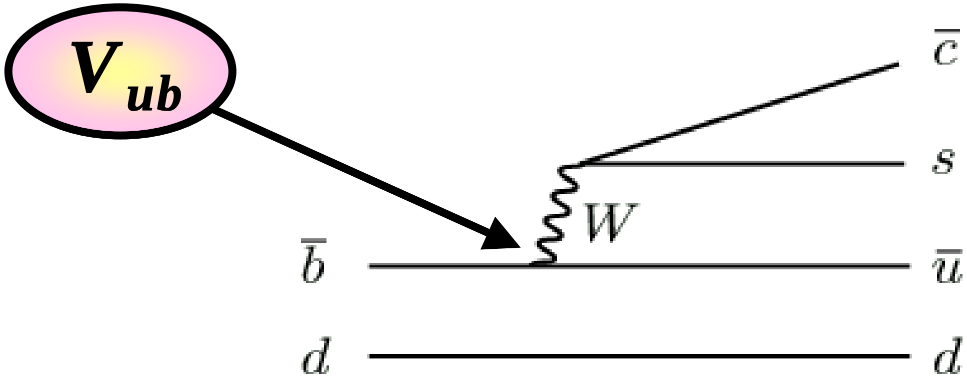
- **$B^0 \to K^+K^-K_S$ and $B^0 \to \eta' K_S$**
  - Results consistent with the world average

- **$B^0 \to \phi K_S$**
  - 2.1σ deviation from the world average.
  - A clue of new physics or just a statistical fluctuation?
    $\Rightarrow$ Need more data.

$|V_{ub}|$ Measurement

Hadronic B decays for $|V_{ub}|$

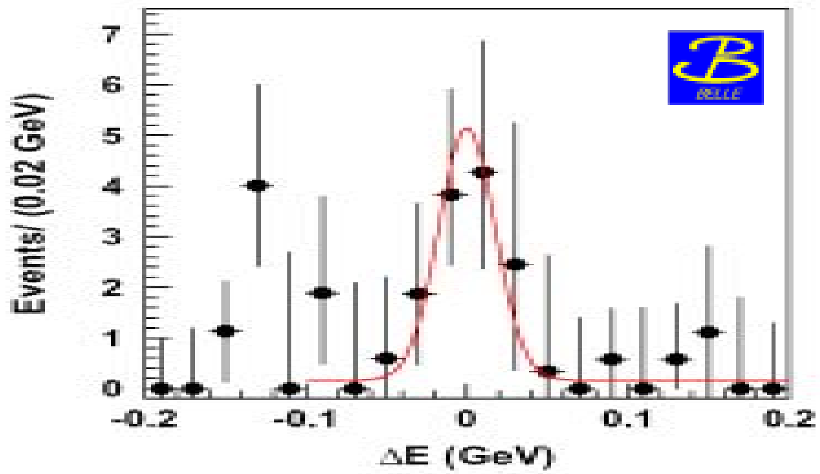
- **$B^0 \to D_s^+\pi^-$ decay**
  - The decay is dominated by $b \to u$ transition without penguin contribution.



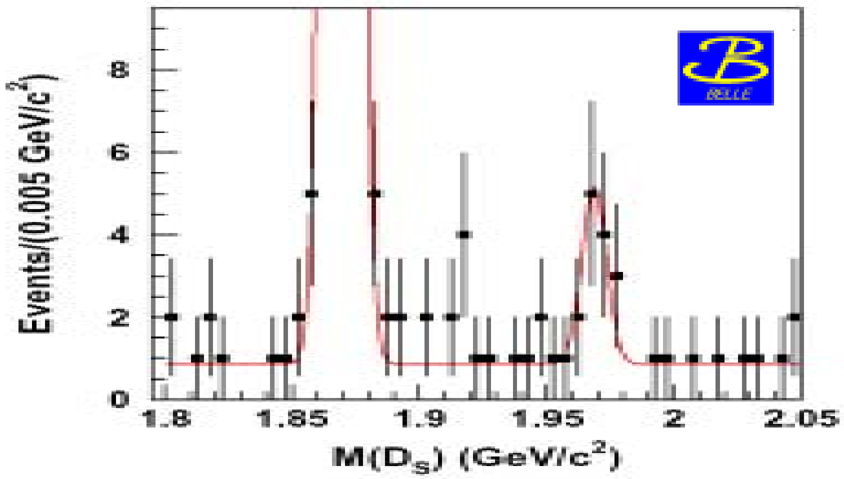
- **How do we determine $|V_{ub}|$?**
  - Reconstruct $B^0 \to D_s^+\pi^-$ decay.
  - Determine branching fraction of the decay.
  - Calculate $|V_{ub}|$ using the obtained fraction and other experimental results.

$B^0 \to D_s^+ \pi^-$ Reconstruction

B^0 energy difference



D_s mass spectrum



D_s^+ decay modes:

$$\phi\pi^+, K_S^0 K^+, \overline{K}^{*0} K^+$$

$B^0 \to D_s^+ \pi^-$

$$\mathcal{B} = (2.4^{+1.0}_{-0.8} \pm 0.7) \times 10^{-5}$$

3.6σ significance

Prediction
$\mathcal{B} = (2 - 6) \times 10^{-5}$

P. Krokovny *et al.* [Belle Collaboration], Phys. Rev. Lett. **89**, 231804 (2002)

$|V_{ub}/V_{cb}|$ from $B^0 \to D_s^+\pi^-$

Another result by Belle

$$\mathcal{B}(B^0 \to D_s^+\pi^-) \times \mathcal{B}_{\phi\pi} = (8.6^{+3.7}_{-3.0} \pm 1.1) \times 10^{-7}$$

CLEO collab. PRD 53, 4734 (1996)

$$\mathcal{B}(B^0 \to D_s^+ D^-) \times \mathcal{B}_{\phi\pi} = (3.0 \pm 1.1) \times 10^{-4}$$

Kim *et al.* PRD 63, 094506 (2001)

$$\frac{\mathcal{B}(B^0 \to D_s^+\pi^-)}{\mathcal{B}(B^0 \to D_s^+ D^-)} = (0.424 \pm 0.041) \times \left|\frac{V_{ub}}{V_{cb}}\right|^2$$

$$|V_{ub}/V_{cb}|$$
$$(8.2^{+3.5}_{-2.9} \pm 3.4) \times 10^{-2}$$

Using PDG2002 for V_{cb}, $|V_{cb}| = (41.2 \pm 2.0) \times 10^{-3}$

$$|V_{ub}| = (3.5^{+1.0}_{-0.9}) \times 10^{-3}$$

Conclusions

- **CP violation measurement**
  - ϕ_2: $\mathcal{S} = -1.23 \pm 0.41\,^{+0.08}_{-0.07}$, $\mathcal{A} = +0.77 \pm 0.27 \pm 0.08$

    $78^\circ \leq \phi_2 \leq 152^\circ$ @ 95% C.L.
  - ϕ_1 ($b \to c\bar{c}s$): $\sin 2\phi_1 = 0.719 \pm 0.074 \pm 0.035$
  - ϕ_1 ($b \to s\bar{s}s$): consistent to $\sin 2\phi_1$ for $K^+K^-K_S$ and $\eta' K_S$

    while 2.1σ deviation is observed in ϕK_S
- **$|V_{ub}|$ measurement**
  - $\mathcal{B}r(B^0 \to D_s^+ \pi^-) = (2.4^{+1.0}_{-0.8} \pm 0.7) \times 10^{-5}$
  - $|V_{ub}/V_{cb}| = (8.2^{+3.5}_{-2.9} \pm 3.4) \times 10^{-2}$
  - $|V_{ub}| = (3.5^{+1.0}_{-0.9}) \times 10^{-3}$

Backup Slides

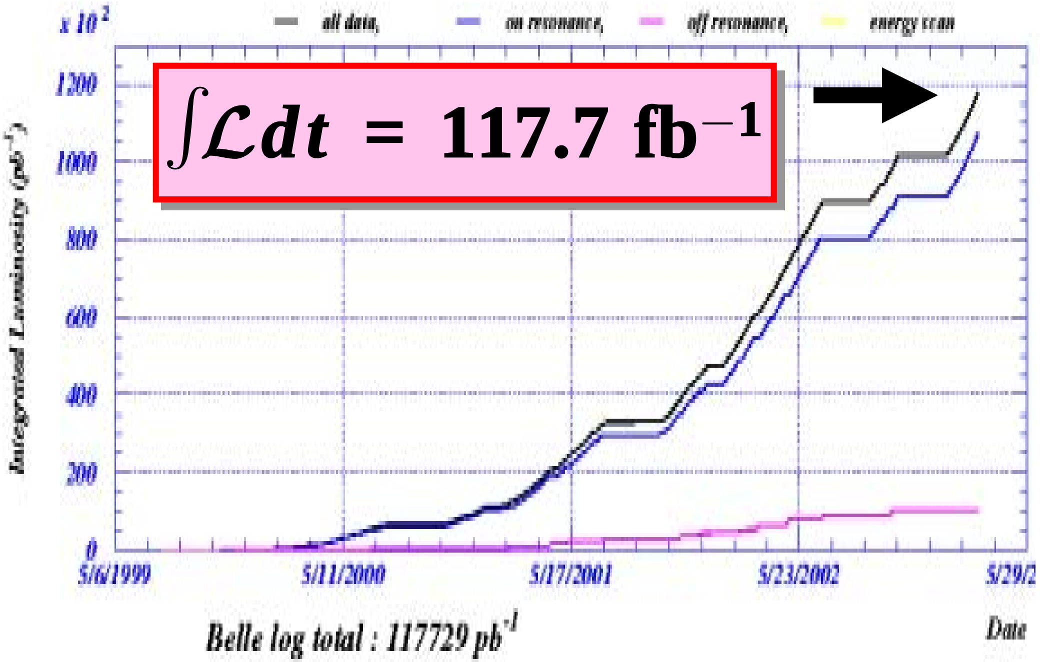
The KEKB Accelerator

- e^+e^- **collider**

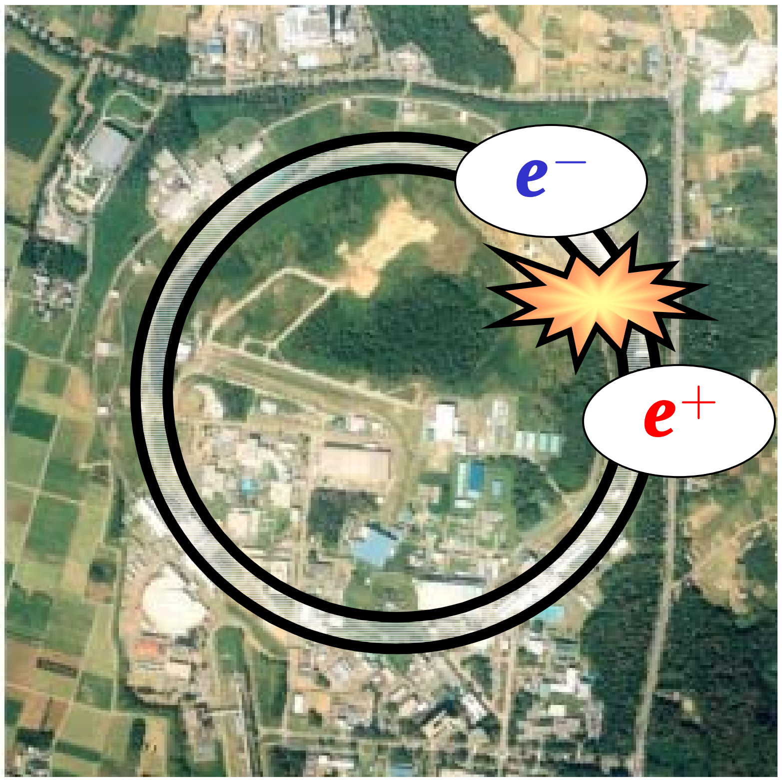
World Record

$$\mathcal{L} = 8.26 \times 10^{33}\ \mathrm{fb}^{-1}$$

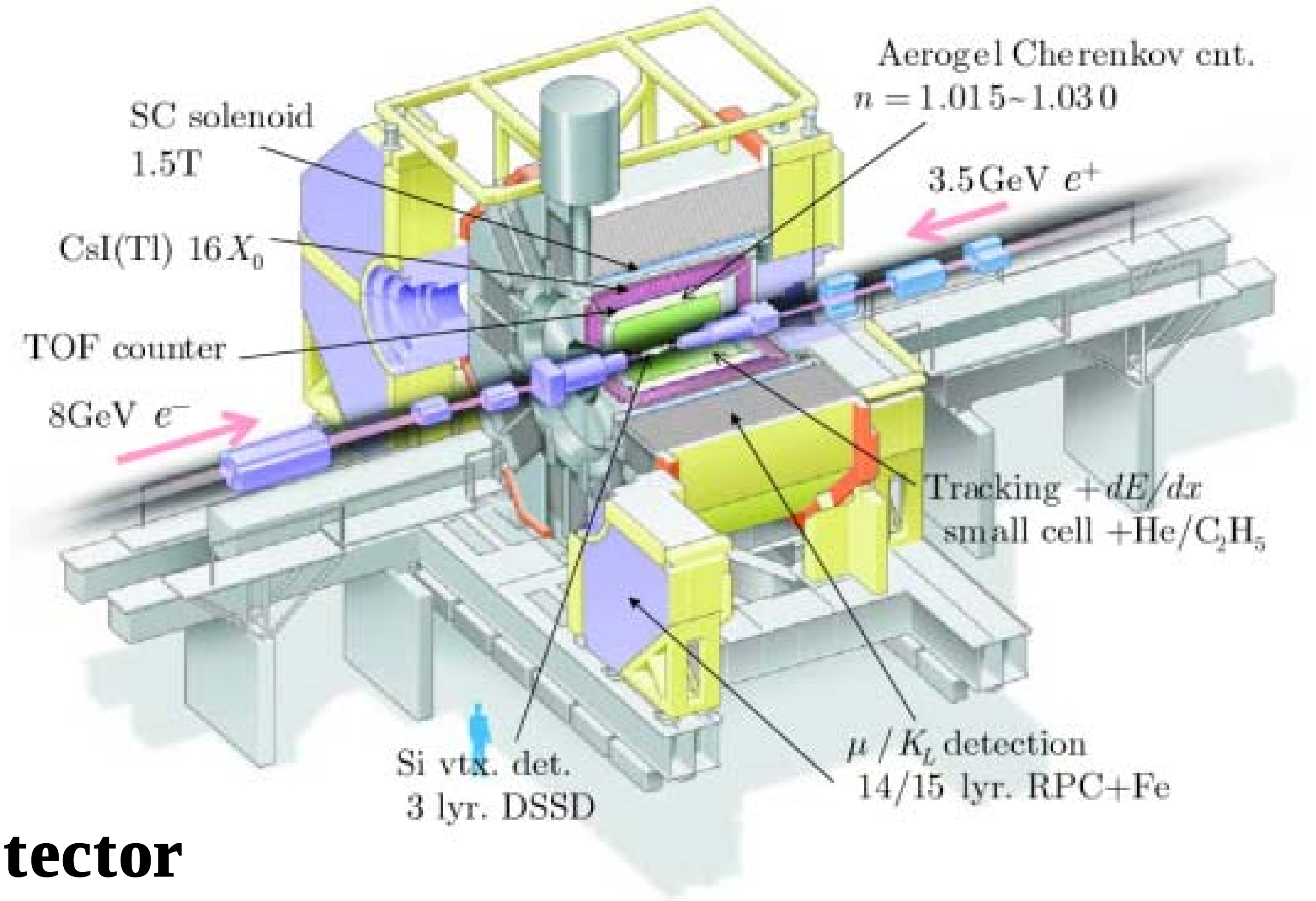
KEKB history (2003/3/10)



KEKB
uest for CPV



The Belle Detector



- **Vertex detector**
- **Momentum and energy detector**
- **Particle identification**

Previous Results at Belle

$$@\ 42\ \text{fb}^{-1}: \mathcal{S} = -1.21\,^{+0.38}_{-0.27}\,^{+0.16}_{-0.13}, \quad \mathcal{A} = +0.94\,^{+0.25}_{-0.31} \pm 0.09$$

K. Abe *et al*. [Belle Collaboration], Phys. Rev. Lett, **89**, 071801 (2002)

Results indicated large *CP* asymmetries → Need more data.

Changes in new analysis

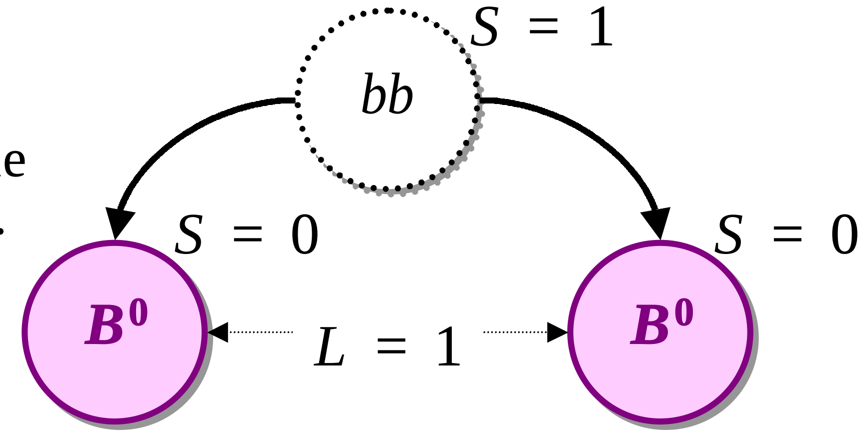
- More data = 78 fb^{-1}.
- Improvements to the analysis.
  - Better track reconstruction algorithm.
  - More sophisticated Δt resolution function.
  - Inclusion of additional signal candidates by optimizong event selection.
- Frequentist statistical analyses.
  - use of MC pseudo-experiments based on control samples.

Flavor Tagging

- **Determine flavor of B_{CP}**
  - We can never know the B_{CP}'s flavor from its decay products, because the final state is *CP* eigenstate.
- **Knack of flavor tagging**

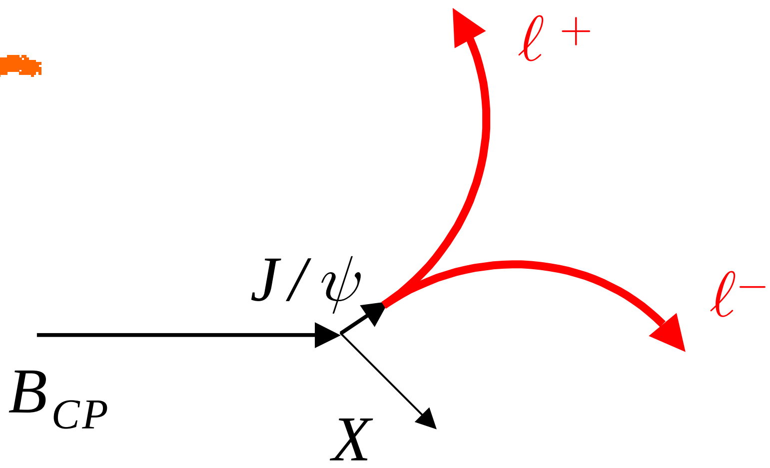
We can know B_{CP}'s flavor
from examination of its partner *B*'s flavor

- **Bose statistics**
  - A wave function of a same particle pair, B^0 or B^0, originates from *bb* resonance (S=1) is symmetric due to Bose statistics.
  - However, same particle pair has L=1 and it is forbidden because the wave function gets anti-symmetric.
  - Flavor of B_{CP} is always opposite to its partner *B*'s flavor.

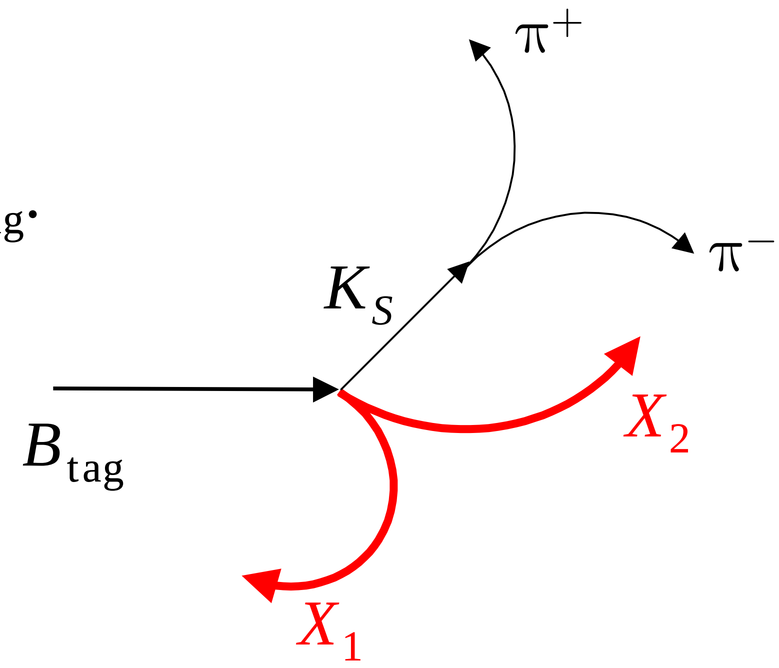


Vertex Reconstruction

- $\boldsymbol{B_{CP}}$ **vertex:** $\boldsymbol{z_{CP}}$
  - Use 2 tracks from J/ψ of $B_{CP} \to J/\psi X$ decay.



- $\boldsymbol{B_{\mathrm{tag}}}$ **vertex:** $\boldsymbol{z_{\mathrm{tag}}}$
  - Use all decay products from B_{tag}.
  - Except long secondary particles originates from lived particles: charmed meson, K_S, etc.



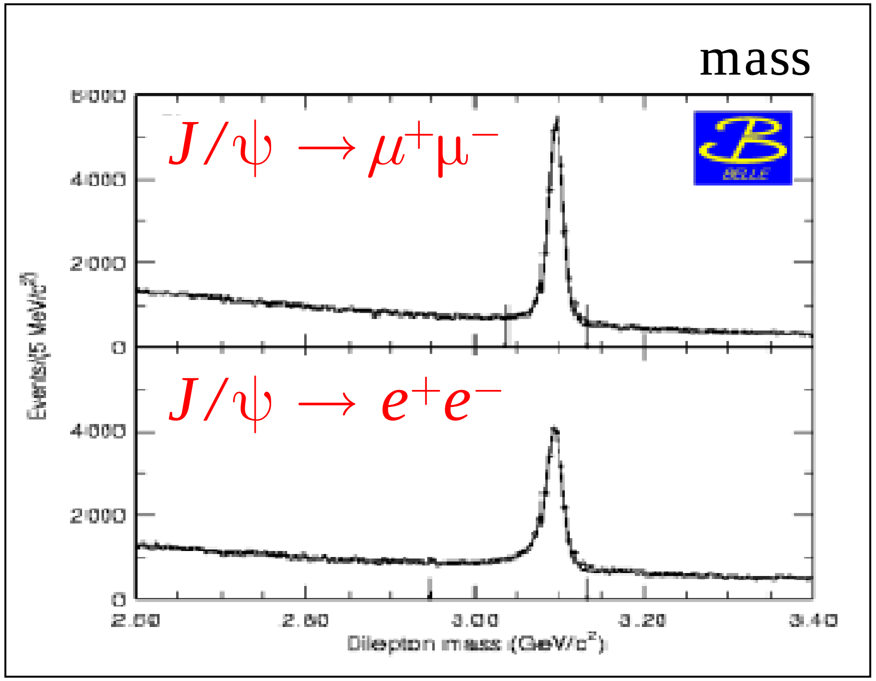
Vertex resolution: ~100 μm

Systematic uncertainties

Source	$\mathcal{S}$		$\mathcal{A}$	
	+error	−error	+error	−error
Background fractions	+0.044	−0.055	+0.058	−0.048
Vertexing	+0.037	−0.012	+0.044	−0.054
Fit bias	+0.052	−0.020	+0.016	−0.021
Wrong tag fraction	+0.015	−0.016	+0.026	−0.021
τ_B, Δm_d, $\mathcal{A}_{K\pi}$	+0.022	−0.022	+0.021	−0.014
Resolution function	+0.010	−0.013	+0.019	−0.020
Background shape	+0.007	−0.002	+0.003	−0.015
Total	+0.08	−0.07	+0.08	−0.08

$b \rightarrow c\bar{c}s$ Reconstruction

In case of $B^0 \rightarrow J/\psi K_S$ decay



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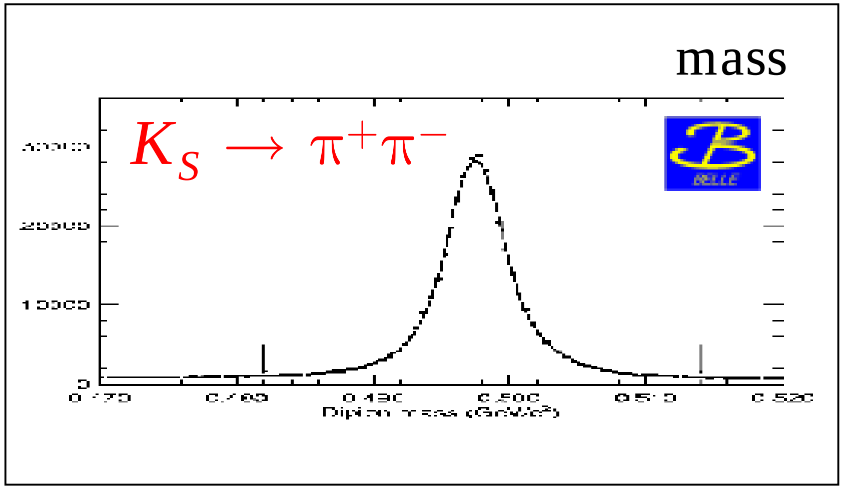

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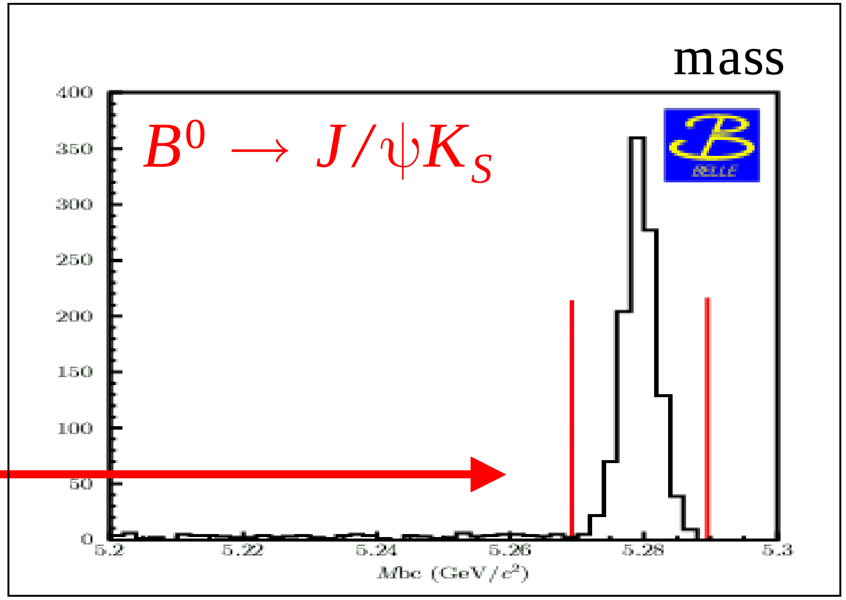


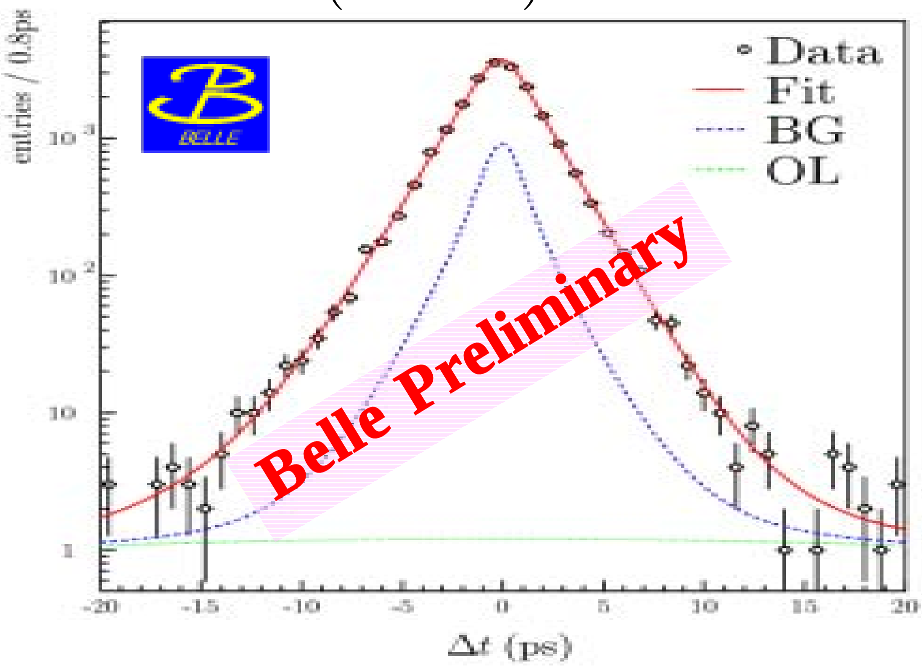
1285 $B^0 \rightarrow J/\psi K_S$ candidates

Δt Resolution Function $\equiv R(\Delta t)$

1. Detector resolution for z_{CP}, z_{tag}
2. Secondary track effect for z_{tag} reconstruction
3. Kinematic approximation

$$\Delta t \simeq \frac{\Delta z}{c(\beta\gamma)_\Upsilon}$$

$$\exp\left(|\Delta t| / \tau_B\right) \otimes R(\Delta t)$$



Belle

τ_B = **1.551±0.018 ps**

World average

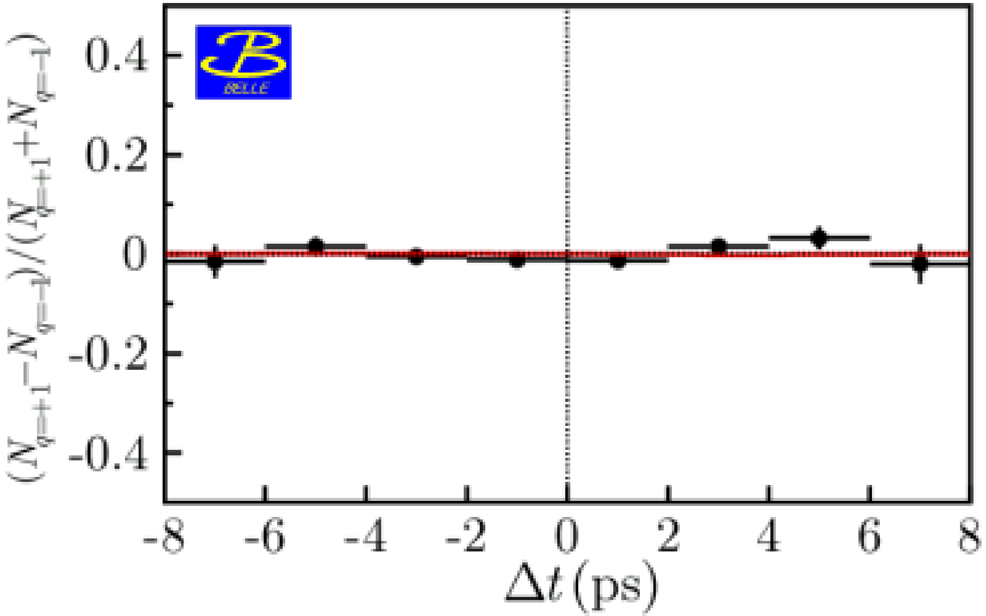
τ_B = **1.542±0.016 ps**

Consistent to world average

Fit Bias in $b \to c\bar{c}s$

- "$\sin 2\phi_1$" of non-CP final state should be 0

"f_{CP}" = $B^0 \to D^{*\pm}\pi^{\mp}$, $B^0 \to J/\psi K^{*0}(K^+\pi^-)$, $B^0 \to D^{*-}\ell^+\nu$

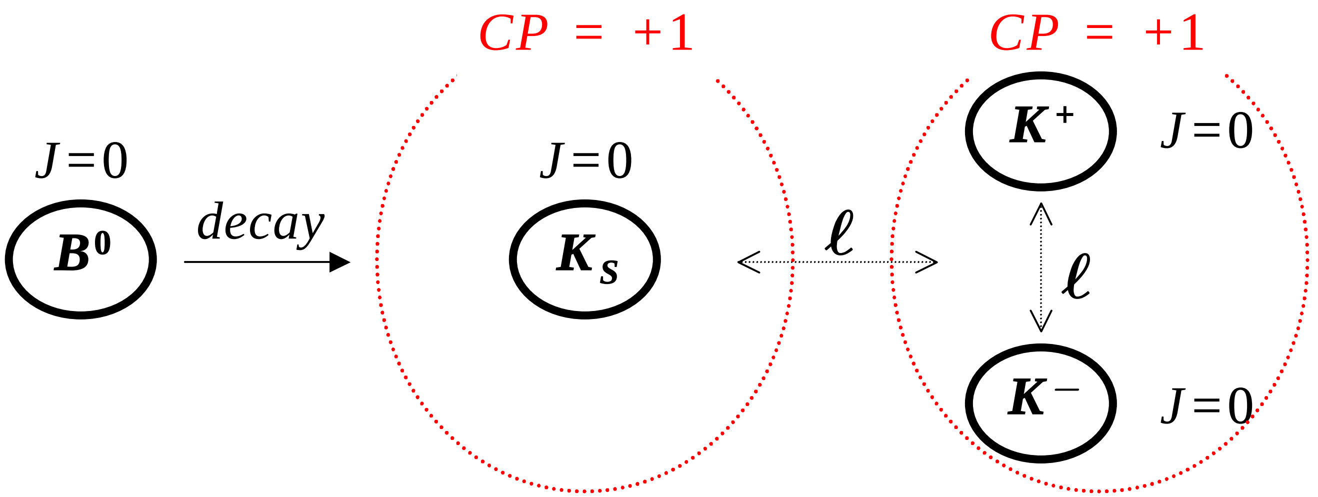


"$\sin 2\phi_1$" = $+0.005 \pm 0.015$(stat) ⇒ **No bias**

$B^0 \to K^+K^-K_S$: $CP = \pm 1$ Mixture

Since $B^0 \to K^+K^-K_S$ is 3-body decay,
the final state is a mixture of $CP = \pm 1$.
How can we determine the mixing fraction?

$CP = \pm 1$ fraction is equal to that of ℓ=even/odd



$CP = (-1)^{\ell}$

$B^0 \to K^+K^-K_S$: $CP = \pm 1$ Mixture — Cont'd

ℓ-even fraction in $|K^0K^0>$ can be determined by $|K_SK_S>$ system

$$\left|K^0\overline{K}^0\right\rangle = \frac{\alpha}{\sqrt{2}}\left(\left|K_SK_S\right\rangle + \left|K_LK_L\right\rangle\right) + \beta\left|K_SK_L\right\rangle$$

$CP = +1$ $\quad$ ℓ = even $\quad$ ℓ = odd

Add K^+ to above kets

$$\left|K^+K^0\overline{K}^0\right\rangle = \frac{\alpha}{\sqrt{2}}\left(\left|K^+K_SK_S\right\rangle + \left|K^+K_LK_L\right\rangle\right) + \beta\left|K^+K_SK_L\right\rangle$$

Using isospin symmetry

$$\mathcal{B}(B^+ \to K^+K^0\overline{K}^0) = \mathcal{B}(B^0 \to K^0K^+K^-) \times \frac{\tau_{B^+}}{\tau_{B^0}} = \frac{\mathcal{B}(B^0 \to K_SK^+K^-)}{2} \times \frac{\tau_{B^+}}{\tau_{B^0}}$$

$$\alpha^2 = 2\frac{\mathcal{B}(B^+ \to K^+K_SK_S)}{\mathcal{B}(B^0 \to K^0K^+K^-)} \times \frac{\tau_{B^0}}{\tau_{B^+}} = \frac{\mathcal{B}(B^+ \to K^+K_SK_S)}{\mathcal{B}(B^0 \to K_SK^+K^-)} \times \frac{\tau_{B^0}}{\tau_{B^+}}$$

$$= 1.04 \pm 0.19(\text{stat}) \pm 0.06(\text{syst})$$

100^{+0}_{-20} % CP even