JET PHYSICS AT LEP & QCD

La Thuile
14.03.03

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1. the energy dependence of mean multiplicities,
2. oscillations of cumulant moments of multiplicity distributions as functions of their rank,
3. difference between quark and gluon jets,
4. the hump-backed plateau of inclusive rapidity distribution and energy dependence of its maxima,
5. difference between heavy- and light-quark jets,
6. color coherence in 3-jet events,
7. intermittency and fractality
8. the energy behavior of higher moments of multiplicity distributions,
9. subjet multiplicities,
10. jet universality.

... + EGT (12.03.03) ...
(EW effects at TESLA)

Recent review papers:

Dremin, Gary, Phys. Rep. 349 (2001) 301
Dremin, Physics-Uspekhi 172 (2002) 551
(www.ufn.ru → May 2002 issue)

Thanks to experimentalists at LEP.

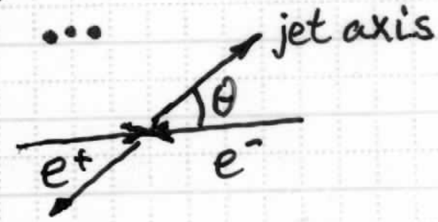
Early days

1. Jets (?)

$$S = \frac{3}{2} \left(\sum_i p_{i\perp}^2 \right)_{min} / \sum_i p_i^2 \quad \text{sphericity}$$

Spherocity, thrust ...

$\langle S \rangle \downarrow$, if $E \uparrow$ ...



2. Spin (?)

$$\frac{d\sigma}{d\theta} \sim 1 + \cos^2\theta \qquad \rightarrow \; 1/2 \; \rightarrow q(?)$$

3. Quarks (?)

$$R = \frac{\sigma_{tot}^{e^+e^- \to h}}{\sigma^{e^+e^- \to \mu^+\mu^-}} \simeq \sum_{i=1}^{n} Q_i^2$$

n - number of quarks $(u, d, s, c, b, \underset{\downarrow}{t})$

Jets evolution

Theory

The generating functional:

$$G(\{u\}, \varkappa_0) = \sum_n \int d^3k_1 \ldots d^3k_n\, u(k_1) \ldots u(k_n)\, P_n(k_1, \ldots, k_n; \varkappa_0)$$

$P_n(k_1, \ldots, k_n; \varkappa_0)$ - the probability density of exclusive production of particles with momenta $k_1, \ldots, k_n$ at the initial energy $\varkappa_0$

$u(k)$ - an auxiliary function

For $u(k) = u = \text{const}$, $P_n(\varkappa_0)$ - the multiplicity distribution

$G(\varkappa_0) = \sum_n u^n P_n(\varkappa_0)$ - the generating function

Symbolical equation (gluons) → system of two eqs for q and g jets

$$G' \sim \int \alpha_s K \left[G \otimes G - G \right] d\Omega$$

phase space

Cascade evolution (G') is determined by 3-gluon vertex ($\otimes$) (production) and escape of a single gluon (G), weighted by coupling strength α_s and a splitting function K defined by the interaction Lagrangian.

$$G'(y) = \int_0^1 dx\, K(x)\, \gamma_0^2 \left[G(y + \ln x)\, G(y + \ln(1-x)) - G(y) \right]$$

$\gamma_0^2 = \frac{2N_c \alpha_s}{\pi}$; $y = \ln \frac{p\theta}{Q_0}$; p - jet momentum, θ - opening angle

$K = \frac{1}{x} - (1-x)[2 - x(1-x)]$

LO ≡ DLA → $[\alpha_s \ln^2 s]^n$ terms summed

NLO ≡ MLLA → $[\alpha_s \ln s]^n$

2NLO, 3NLO ... → kinetic equation (?)

Gluodynamics

LO (DLA) $\sum_n [\alpha_s \ln^2 s]^n$

$$K(x) \approx \frac{1}{x} \quad ; \quad \alpha_s \approx \text{const (fixed)} \quad \text{or} \quad \alpha_s \sim \frac{1}{y} \text{ (running)}$$

$$G(y + \ln(1-x)) \approx G(y)$$

$$[\ln G(y)]' \approx \gamma_0^2 \int_0^1 \frac{dx}{x} [G(y + \ln x) - 1]$$

$$[\ln G(y)]'' \approx \gamma_0^2 [G(y) - 1] \quad \text{- LO equation}$$

b.c.: $G(0,u) = u$; $G(y,1) = 1$ $(= \sum_n P_n)$

NLO (MLLA) $\sum_n [\alpha_s \ln s]^n$

Renormgroup approach - A. Mueller (1984)

No systematic perturbative expansion till 1993 → Dremin (1993)

↓

„Exponential" perturbative series — modified perturbative expansion (MPE)

Taylor series expansion ≡ MPE
(of $G(y + \ln(1-x))$ in x)

Exact solution for fixed coupling - Dremin, Hwa (1994)

Scaling → power behavior

~~Parton and dipole approaches in QCD~~

~~I.M. Dremin[1], P. Edén[2]~~
~~[1] *Lebedev Physical Institute, Moscow 119991, Russia*~~
~~[2] *Nordita, Copenhagen, DK-2100, Denmark*~~

$$G(u,y) = \sum_n u^n P_n(y), \tag{1}$$

Factorial moments $F_q = \frac{d^q G(u)}{du^q}\Big|_{u=1}$; Cumulant moments $K_q = \frac{d^q \ln G(u)}{du^q}\Big|_{u=1}$; $H_q = \frac{K_q}{F_q}$

$$G' \sim \int \alpha_S K[G \otimes G - G] d\Omega. \tag{2}$$

init. cond. $P_n = \delta_{n1}$: $G_0 = u$. (3)

4.

System of 2 eqs for gluons (G) and quarks (F):

Energy conservation

$$G'_G = \int_0^1 dx K_G^G(x) \gamma_0^2 [G_G(y + \ln x) G_G(y + \ln(1-x)) - G_G(y)]$$

$$+ \quad n_f \int_0^1 dx K_G^F(x) \gamma_0^2 [G_F(y + \ln x) G_F(y + \ln(1-x)) - G_G(y)], \tag{4}$$

$$G'_F = \int_0^1 dx K_F^G(x) \gamma_0^2 [G_G(y + \ln x) G_F(y + \ln(1-x)) - G_F(y)], \tag{5}$$

$$y = \ln(p\Theta/Q_0) = \ln(2Q/Q_0)$$

~ Cut-off parameter (non-perturb.) [LPHD]

$$\gamma_0^2 = \frac{2N_c\alpha_S}{\pi}, \tag{6}$$

$$\alpha_S(y) = \frac{2\pi}{\beta_0 y}\left(1 - \frac{\beta_1}{\beta_0^2} \cdot \frac{\ln 2y}{y}\right) + O(y^{-3}), \tag{7}$$

$$K_G^G(x) = \frac{1}{x} - (1-x)[2 - x(1-x)], \tag{8}$$

$$K_G^F(x) = \frac{1}{4N_c}[x^2 + (1-x)^2], \tag{9}$$

$$K_F^G(x) = \frac{C_F}{N_c}\left[\frac{1}{x} - 1 + \frac{x}{2}\right], \tag{10}$$

DG (01) + LPHD (!)

Experiment & QCD

EQS for $\langle n \rangle$ and higher moments.

1. $\langle n(E) \rangle$ — mean multiplicity

$$\left.\frac{dG}{du}\right|_{u=1} = \sum n P_n = \langle n(E) \rangle \sim e^{\int^y \gamma(y')dy'}$$

Pert. expansion of exponent: MPE $\rightarrow$ $\gamma = \underline{\gamma_0}(1 - a_1\gamma_0 - a_2\gamma_0^2 - \ldots)$

Prediction: $\boxed{\langle n \rangle \sim \exp[c\sqrt{\ln s}]}$

DLA, running coupling $[\gamma_0 \sim \frac{1}{\sqrt{y}}]$

MLLA $\rightarrow$ factor $\ln^{-d} s$ ...

Hydrodynamics; fixed coupling QCD $\rightarrow$ power behavior

Multiperipheral; flat rapidity plateau (Feynman) $\rightarrow$ log behavior

$\oplus$ Fig. 1

13 $\leftarrow$

2. H_q-moments — shape of the distribution

$$\left.\frac{1}{\langle n \rangle^q}\frac{d^qG}{du^q}\right|_{u=1} = \frac{\langle n(n-1)\ldots(n-q+1)\rangle}{\langle n \rangle^q} = F_q \quad \text{factorial moments}$$

$$\left.\frac{1}{\langle n \rangle^q}\frac{d^q \ln G}{du^q}\right|_{u=1} = K_q \quad \text{cumulant moments} \quad (\text{„genuine" correlations})$$

$$H_q = \frac{K_q}{F_q}$$

Prediction: DLA $\rightarrow H_q = 1/q^2$

New effect predicted: MLLA $\rightarrow$ minimum at $q \approx \frac{24}{11\gamma_0} + 0.5 \approx 5$

2NLO... $\rightarrow$ oscillations

Dremin 1993; Dremin Nechitailo 1993

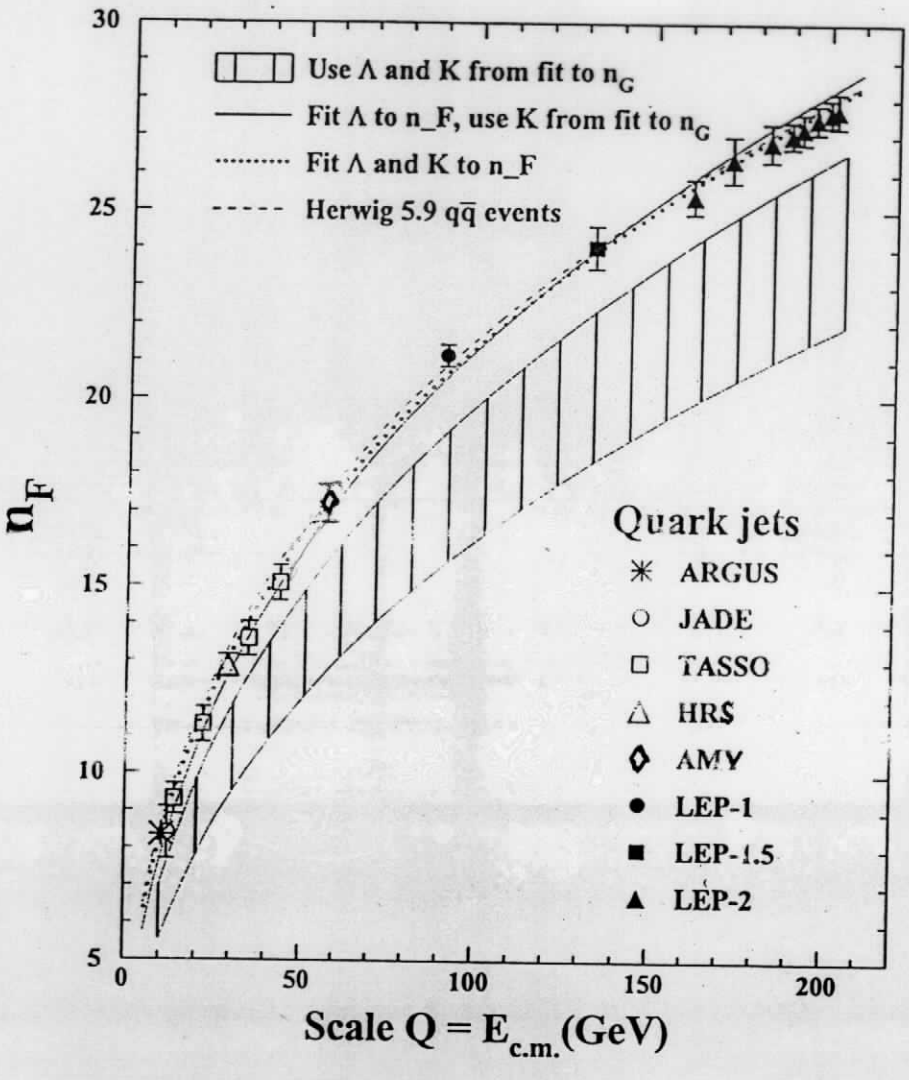
Poisson: $H_q \equiv 0$

Negative Binomial: $H_q = \frac{\Gamma(q)\Gamma(k+1)}{\Gamma(k+q)} = kB(q,k) > 0$

Fig. 2, 2a

(clusters) attraction $\leftrightarrow$ repulsion (separated jets)

(nothing in NBD!)



- Mueller
- Webber
- Dremin, Nechitailo
- Capella, Dremin, Gary, Nechitailo, Tran Thanh Van

Fig. 1

Perturbative solutions → Taylor series expansion (Dr. 93)

$$r = \frac{\langle n_G \rangle}{\langle n_F \rangle} \; ; \quad \gamma = \frac{\langle n_G \rangle'}{\langle n_G \rangle} = \left[\ln \langle n_G \rangle \right]' \; ; \quad \gamma_F = \frac{\langle n_F \rangle'}{\langle n_F \rangle} = \left[\ln \langle n_F \rangle \right]'$$

(ratio) (logarithmic slopes)

$$\left[\begin{array}{ll} r = r_0 (1 - r_1 \gamma_0 - r_2 \gamma_0^2 - r_3 \gamma_0^3) + O(\gamma_0^4) & (*) \\ \gamma = \gamma_0 (1 - a_1 \gamma_0 - a_2 \gamma_0^2 - a_3 \gamma_0^3) + O(\gamma_0^5) & (**) \end{array} \right.$$

$$\alpha_s = \frac{2\pi}{\beta_0 y} \left[1 - \frac{\beta_1}{\beta_0^2} \cdot \frac{\ln 2y}{y} \right] + O(y^{-3}) \rightarrow \text{2-loop} \; ; \quad \gamma_0 = \sqrt{\frac{2 N_c \alpha_s}{\pi}}$$

(~ 0.5 at present energies)

tends to 0 at $y \to \infty$ as $y^{-1/2}$ (as. freedom)

$$\beta_0 = \frac{11 N_c - 2 n_f}{3} \; ; \quad \beta_1 = \frac{51 N_c - 19 n_f}{3}$$

Analytic formulas for a_i, r_i: [r_i → $= 0$ in SUSY-QCD]

Numerical values:

$r_0 = \frac{C_A}{C_F} = \frac{9}{4}$ → LO ≡ DLA (NLO ≡ MLLA)

n_f	r_1	r_2	r_3	a_1	a_2	a_3
3	0.185	0.426	0.189	0.280	−0.379	0.209
4	0.191	0.468	0.080	0.297	−0.339	0.162
5	0.198	0.510	−0.041	0.314	−0.301	0.112
	NLOr 2NLO	NNLOr 3NLO	3NLOr 4NLO	NLO	NNLO	3NLO

Figs. Ratio: $2.25 \rightarrow 2.03 \rightarrow 1.77 \rightarrow 1.74$ at Z^0 for $\gamma_0 \approx 0.5$, $n_f = 4$

Anomalous dimension γ determines energy dependence:

Fig. $$\langle n_G \rangle = A \exp \left[\int_{y_0}^{y} \gamma(y) \, dy \right]$$

↓ non-perturbative normalization constant (fitted parameter)

Other NP effects:
α_s
K's
integration limits + Eqs. ? partons ?

Perturbative vs non-perturbative regions in Eqns:

$\int_0^1 (\ldots) dx$ includes both P and NP-regions but admits purely perturbative solutions (*) and (**)

$\int_{e^{-y}}^{1-e^{-y}} (\ldots) dx$ includes just P region ($Q_{jet} \geq \frac{1}{2} Q_0$) but gets NP-terms $\sim e^{-y} \sim \frac{1}{Q}$ (no analytic solution)

} ?

$r_{ch.} = \frac{\langle n_{ch.} \rangle_{gluon}}{\langle n_{ch.} \rangle_{quark}} = \boxed{1.51 \pm 0.02 \pm 0.05}$

$E_{jet} \simeq 41.6$ GeV

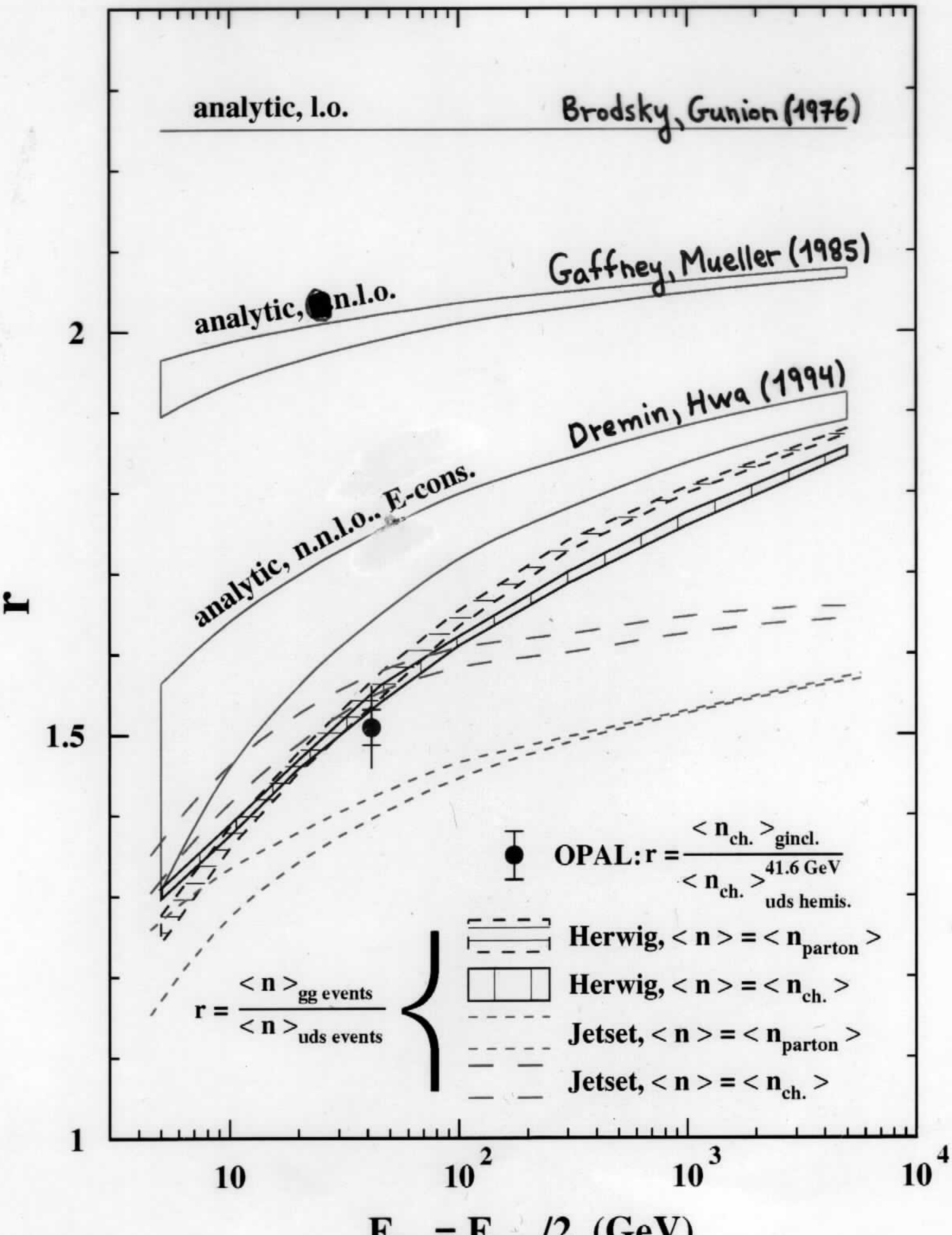


Fig. 3

<u>Energy dependence</u>:

$$\langle n_G\rangle = A\underbrace{y^{-a_1c^2}}_{NLO}\exp\Big\{\underbrace{2c\sqrt{y}}_{LO}+\underbrace{\frac{c}{\sqrt{y}}\Big[2a_2c^2+\frac{\beta_1}{\beta_0^2}(\ln 2y+2)\Big]}_{2NLO}+\underbrace{\frac{c^2}{y}\Big[a_3c^2-a_1\frac{\beta_1}{\beta_0^2}(\ln 2y+1)\Big]}_{3NLO}\Big\}$$

$c=\sqrt{\dfrac{4N_c}{\beta_0}}$; $y\to\ln\dfrac{E_{cm}}{\Lambda}$ Two parameters: A, Λ

($c\simeq 1.2$ for $n_f=4$)

$$\delta(y)=\frac{c}{\sqrt{y}}\Big[2a_2c^2+\frac{\beta_1}{\beta_0^2}(\ln 2y+2)\Big]+\frac{c^2}{y}\Big[a_3c^2-a_1\frac{\beta_1}{\beta_0^2}(\ln 2y+1)\Big]\approx \text{const}$$

(at present energies!)

↓

NNLO + 3NLO

<u>Conclusion</u>: NNLO+3NLO renormalize A, and the energy dependence is well approximated by NLO≡MLLA.

$$\langle n_F\rangle=\frac{\langle n_G\rangle}{r(y)}=\frac{\langle n_G\rangle}{r_0(1-r_1\gamma_0-r_2\gamma_0^2-r_3\gamma_0^3)}$$

↓ rewrite ~~it~~ in the exponent ↓

$$\langle n_F\rangle\simeq\frac{A}{r_0}y^{-a_1c^2}\exp\Big\{2c\sqrt{y}+\frac{c}{\sqrt{y}}\Big[\underline{r_1}+2a_2c^2+\frac{\beta_1}{\beta_0^2}(\ln 2y+2)\Big]+$$
$$+\frac{c^2}{y}\Big[c^2a_3+\underline{r_2}+\frac{r_1^2}{2}-a_1\frac{\beta_1}{\beta_0^2}(\ln 2y+1)\Big]\Big\}$$

1) No term with r_3! It would need a_4 to be self-consistent in pQCD. Conventional definition of LO, NLO, ...

2) Large value of $2r_2/r_1\sim 5$ (!) (compare with $a_3\sim 0.15\div 0.2$)

3) The same dependence of $\langle n_G\rangle$ and $\langle n_F\rangle$ in MLLA ≡ NLO !

$\gamma_F=\gamma-\dfrac{r'}{r}$; $r'\sim\gamma_0'\sim\gamma_0^3$ → SLOPES (!)

↓

NNLO correction in γ_F

$\gamma_F<\gamma$

$\gamma_{F_{NLO}}=\gamma_{NLO}$

OK! → $\boxed{r<r^{(1)}<r^{(2)}<\dfrac{C_A}{C_F}=2.25}$

smaller corrections ! → $r^{(1)}=\dfrac{\langle n_G\rangle'}{\langle n_F\rangle'}$, ratio of slopes; $r^{(2)}=\dfrac{\langle n_G\rangle''}{\langle n_F\rangle''}$ ... curvat.

→ ~~Figs.~~

<u>NP effects</u>

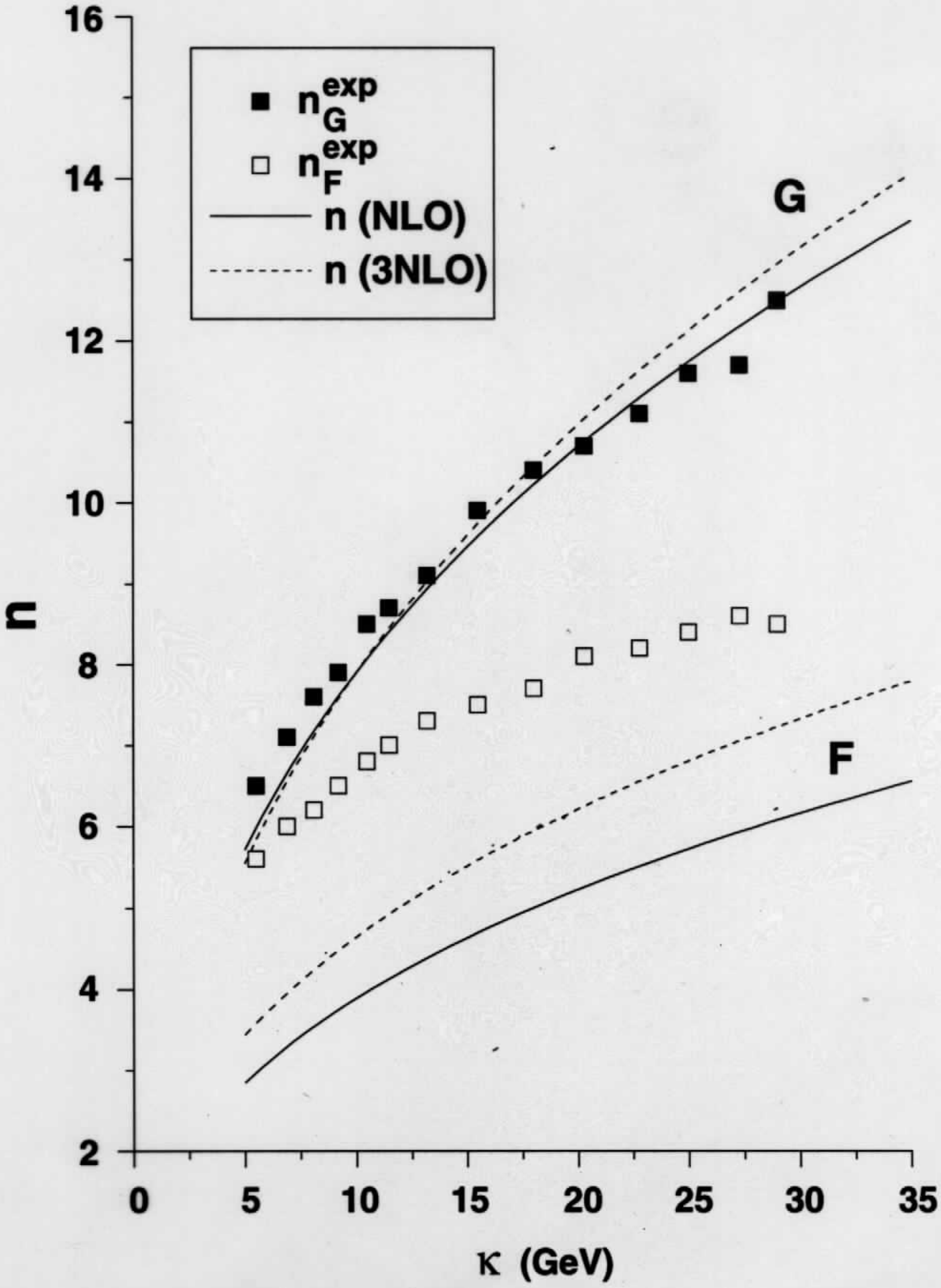


Figure 2: The average multiplicities of gluon (G) and quark (F) jets in the NLO and 3NLO approximations, compared to data [13]. For the theory, the gluon jet normalization is fit to the data; the normalization of the gluon jet curve fixes the normalization of the quark jet curve. The theoretical results are obtained using $n_f = 4$.

• Dremin, Gary (99) Fig. 3a

e+e− 91./91.5 GeV DELPHI Coll.

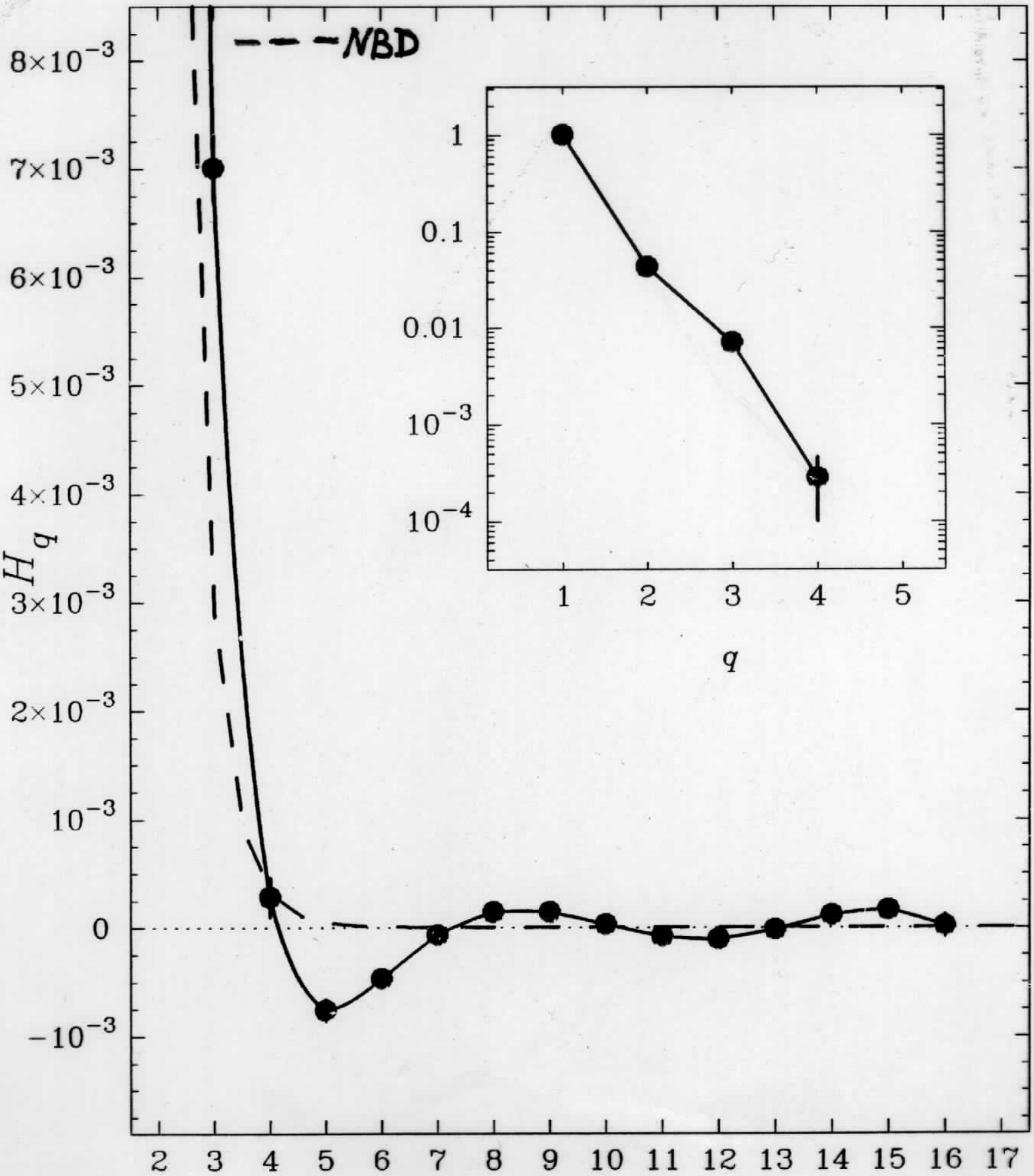
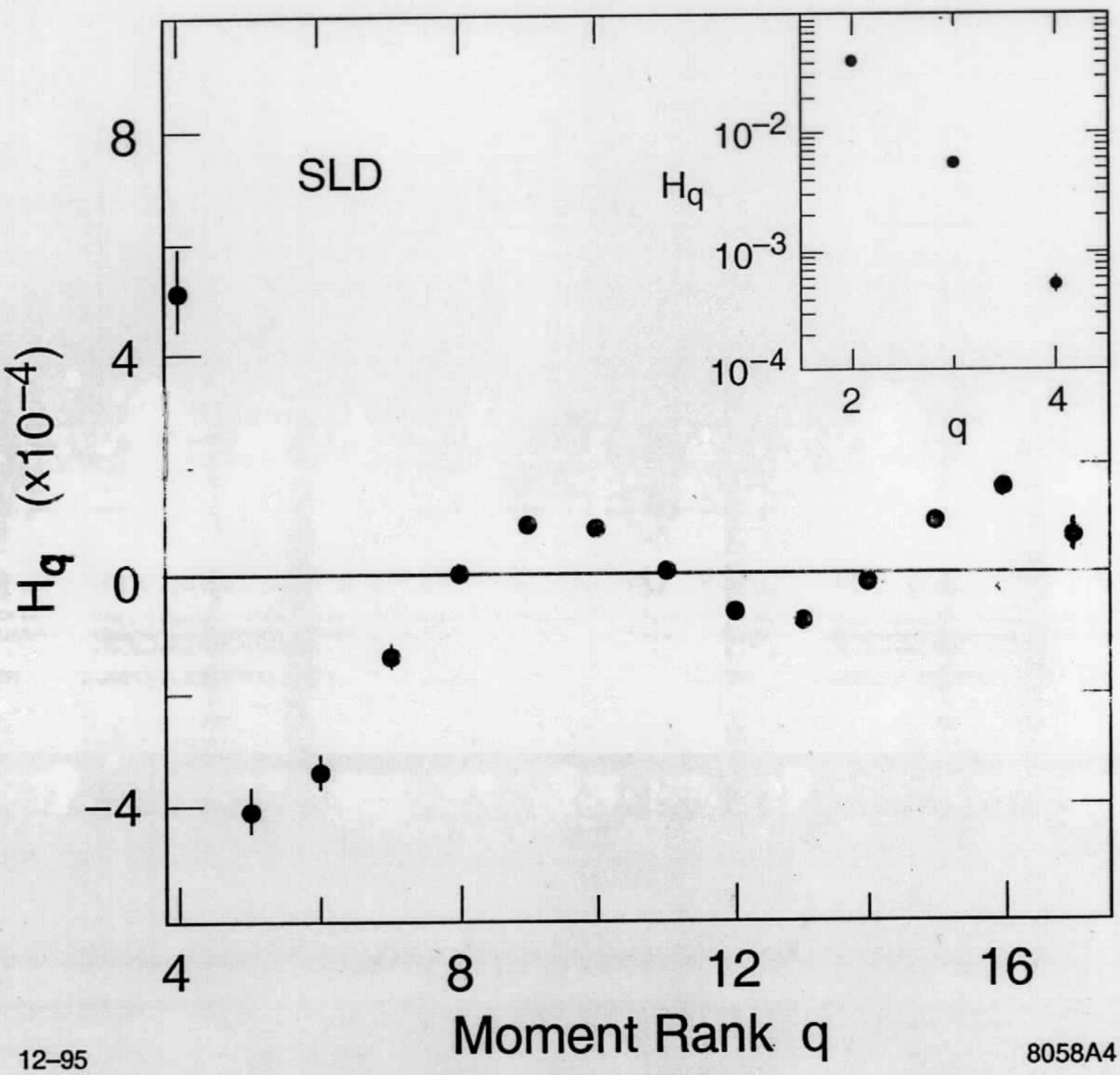


Fig. 2 $\quad q$ Dr. Arena et al Phys. Lett. B336 (1994) 119

$q_{min} \approx \frac{1}{h_1\gamma_0} + \frac{1}{2} \approx 5$; $h_1 = \frac{11}{24}$; $\gamma_0 \approx 0.48$ at Z^0 + [SLD Collaboration] ...



- Dremin (93)
- Dremin, Nechitailo (93)
- Lupia (98)

Fig. 2a

3. q & g jets

$r = \frac{\langle n_g \rangle}{\langle n_q \rangle}$ → LO+NLO terms cancel! Very sensitive to higher corrections!

and $\gamma_G > \gamma_F$ → different mean multiplicities and anomalous dimensions

Prediction:

DLA → $r_0 = \frac{C_A}{C_F} = \frac{9}{4}$; $r = \frac{9}{4}(1 - r_1\gamma_0 - r_2\gamma_0^2 - \dots)$

Fig. 3

$\gamma_G \simeq \gamma_F$ MLLA → $\gamma_G = \gamma_F$ $\gamma_F = \gamma_G - \frac{r'}{r} < \gamma_G$

Fig. 3a → **Fig. 1**

4. Hump-backed plateau **(vs Feynman flat plateau)**

Inclusive distribution → eq. for generating functional

Prediction:

1) Maxima $\left(\xi = \ln\frac{1}{x} = \ln\frac{E_j}{p}\right)$ → **Fig. 4**
2) Shape - ≈ Gaussian → **Fig. 5**

angular ordering and color coherence | MLLA

3) Maxima positions and energy dependence (widths...) $(e^+e^-, ep, p\bar{p}, \dots)$ → **Fig. 6**, **Fig. 6a**

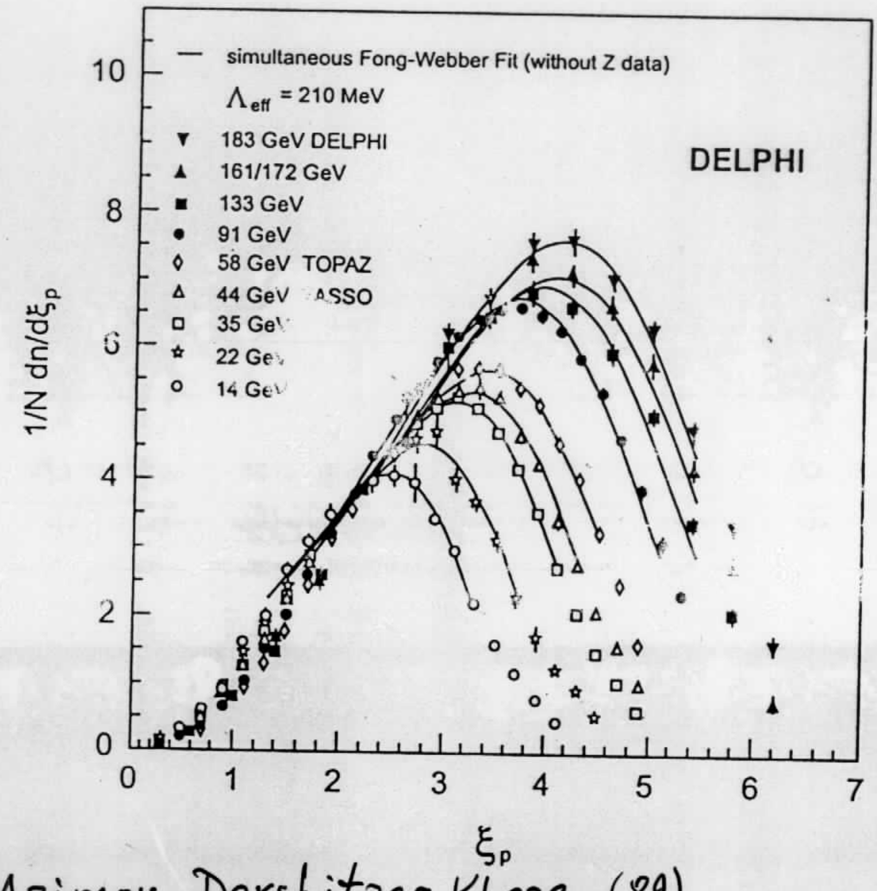
5. Heavy- and light-quark jets

Analogy: Bremsstrahlung of e and μ; role of mass in the propagator $\frac{1}{k^2+m^2}$

Prediction:

Energy-independent difference of companion mean multiplicities of heavy- and light-quark jets of equal energy

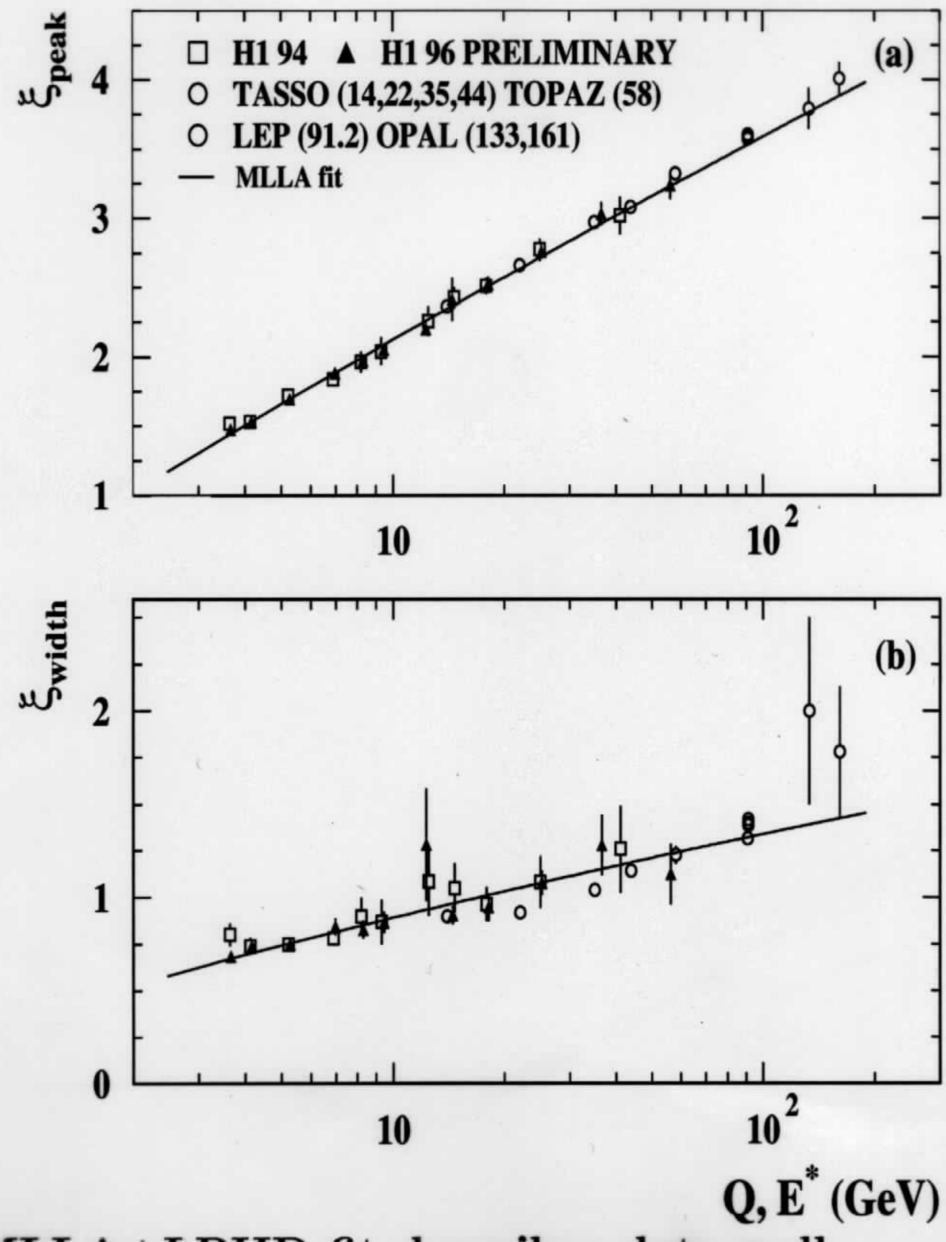
Fig. 7 → compare to „naive" model



- Azimov, Dokshitzer, Khoze (82)
- Dokshitzer, Fadin, Khoze (82, 83)
- Bassetto, Ciafaloni, Marchesini, Mueller (82)

Fig. 4; 5

Peak Position and Width of ξ of Charged Particle Spectra



- MLLA+LPHD fit describes data well
- No deviation from e^+e^- behaviour
  - Fong, Webber (89-91)
  - Dokshitzer, Khoze, Troyan (91)

Fig. 6

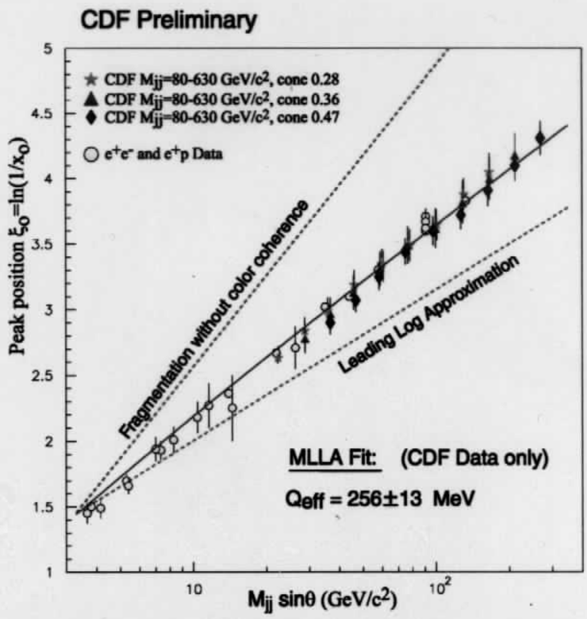


Figure 10: Peak position ξ^* of the inclusive ξ distribution plotted against di-jet mass $\times$ $\sin(\Theta)$ in comparison with the MLLA prediction (central curve); also shown are the double logarithmic approximation (lower curve with asymptotic slope $\xi^* \sim Y/2$) and expectation from cascade without coherence. Result by CDF Collaboration.[92]

data at full angle $\Theta = \pi/2$ from e^+e^- and ep collisions whereby the variable $Y = \ln(P \sin\Theta/Q_0)$ has been used. The data scatter around the expected curve (79) for $n_f = 3$. Taking instead the scaling variable $Y = \ln(2P\sin(\Theta/2)/Q_0)$ the full angle data would be shifted to the right by a factor $2\sin(\pi/4) \sim 1.4$. This would correspond essentially to a change of the next-to-next-to-leading order term in (79) but would not change the slope.

The slope is nicely confirmed and the leading DLA contribution ($\xi^* \sim Y/2$) is shown for comparison as the lower curve in Fig. 10, adjusted in height; the upper curve represents the spectrum for the incoherent cascade which peaks near the maximum ($\xi^* \sim Y$). Apparently the data support the prediction from the parton cascade with suppression of soft particles due to coherent gluon emission in a large energy range $2E_{jet}\sin\Theta \sim 4 - 300$ GeV.

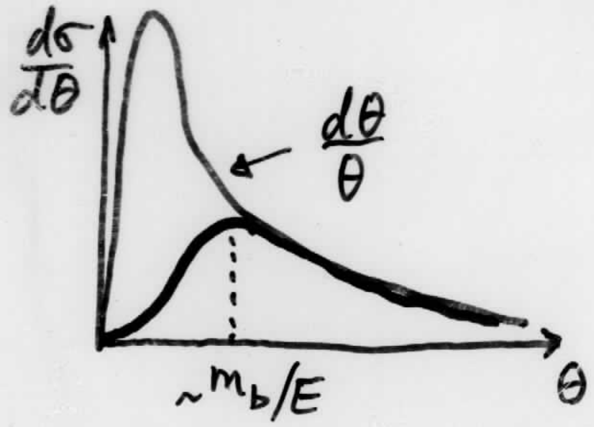
The analytical results for the particle spectrum near the soft limit (90) are nicely confirmed by the data. In these calculations the model (35) for mass effects has been used. The experimental data from the available range

Fig. 6a

Specifics of heavy-quark jets

1. Nothing very special in H_q → similar oscillations (Dr., Nechitailo)
2. Accompanying gluon radiation

$$\frac{d\sigma_b/d\theta}{d\sigma_{uds}/d\theta} \simeq \frac{1}{3}\frac{\theta^4}{\left(\theta^2+\frac{m_b^2}{E^2}\right)^2}$$



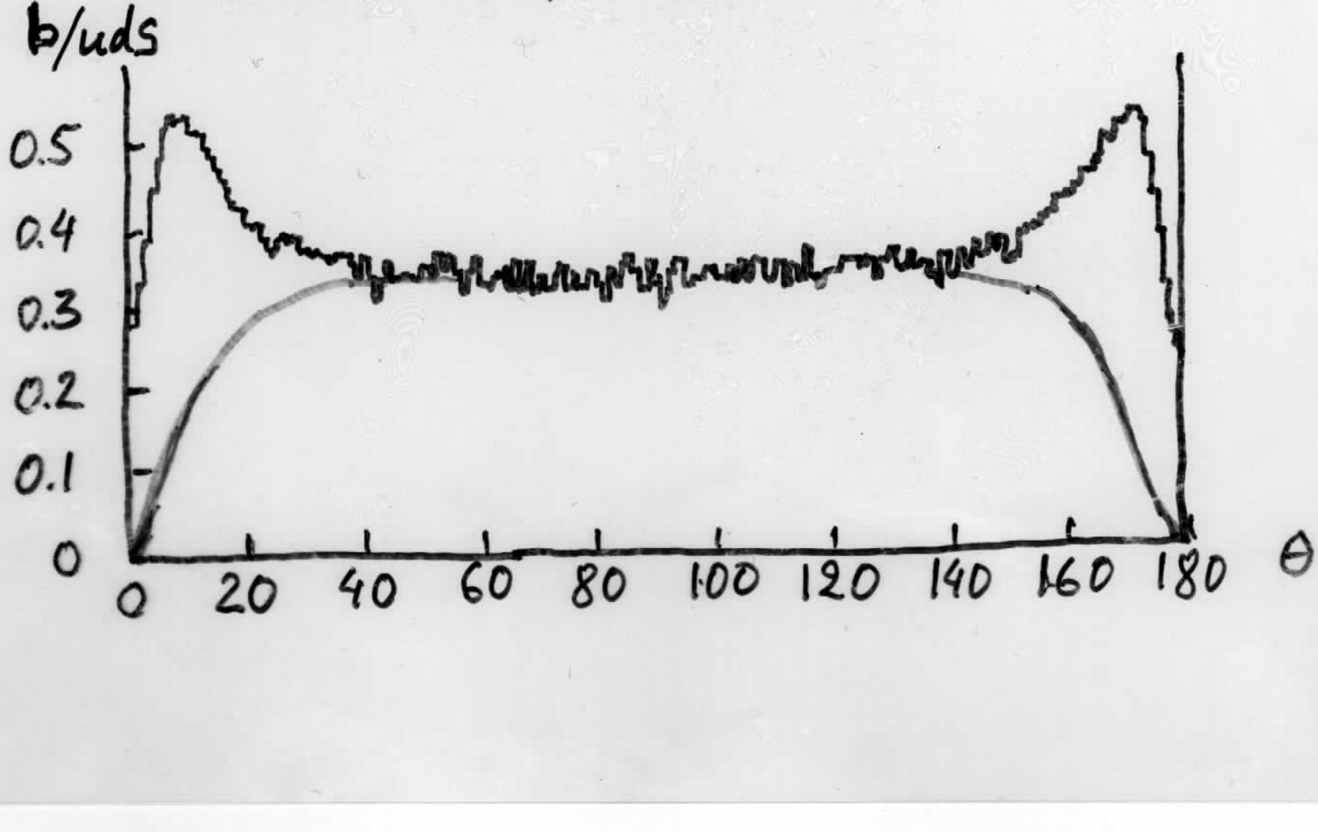
Dr. JETP Lett. 30, 140 (1979) → Cher. gluons;

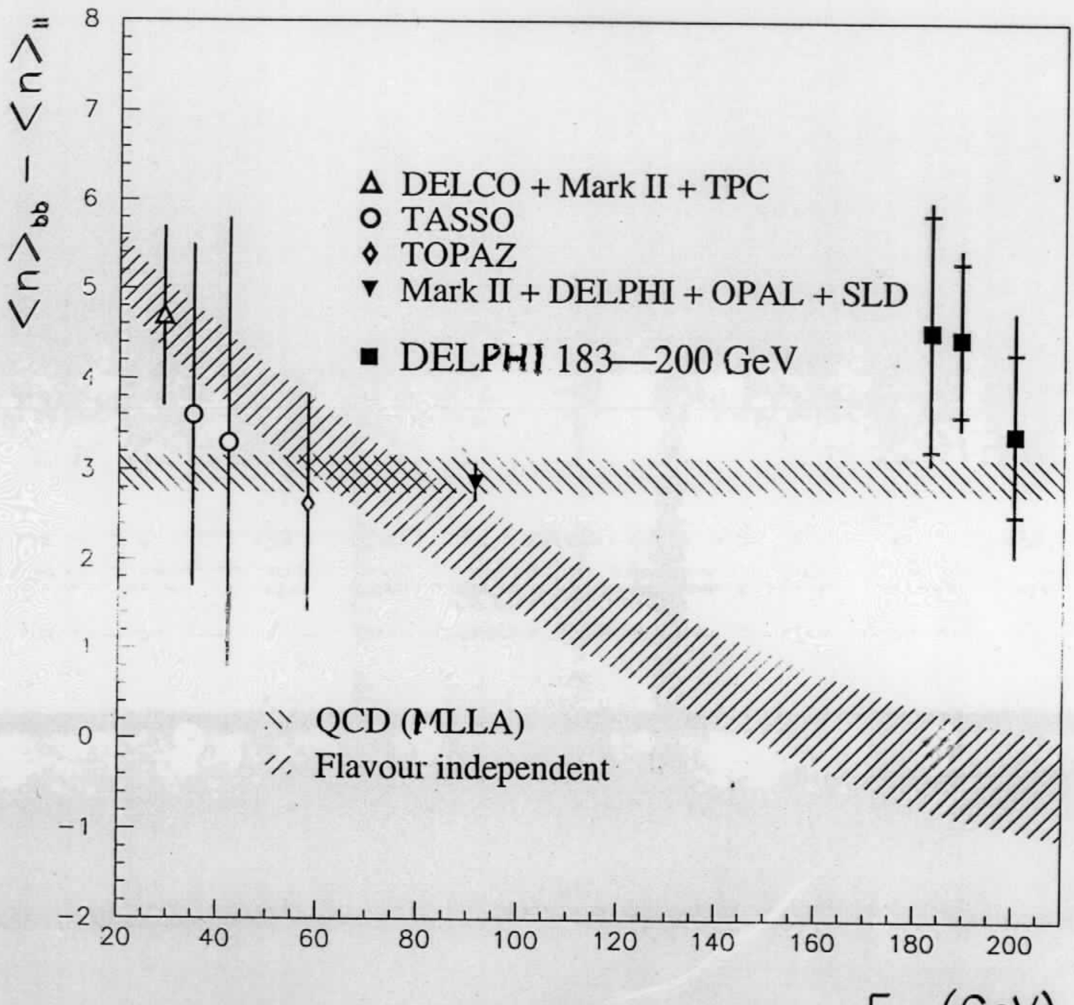
Ring-like (Dr. JETP Lett. 34 (1981) 594)... t-quark.

Dead-cone (Schumm, Dokshitzer, Khoze, Kotke Phys. Rev. Lett. 69 (92) 3029)

$\langle n_a \rangle_b < \langle n_a \rangle_{uds}$; does not depend on W.

DELPHI data (unpublished)

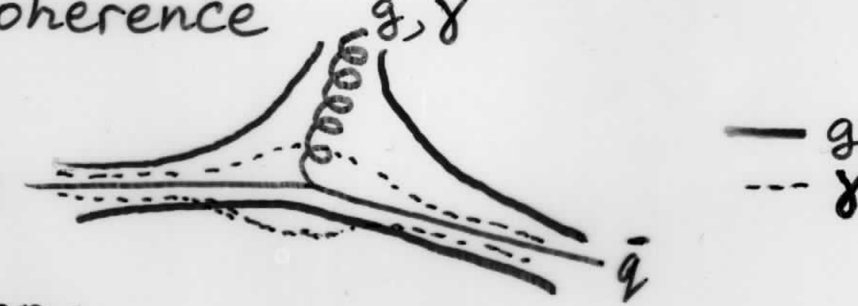




- Azimov, Dokshitzer, Khoze (82)
- Dokshitzer, Fadin, Khoze (82, 83)
- Petrov, Kisselev (88, 95)
- Dremin (79, 81)

Fig. 7

6. Color coherence



Prediction:

1. Particle flows → enlarged in the partons directions, depleted in-between
2. qg valley / $q\bar{q}$ valley = 2.4 (theor) 2.23±0.37 (exper.)

Fig. 8

3. $R_\gamma = \dfrac{N_{q\bar{q}}(q\bar{q}g)}{N_{q\bar{q}}(q\bar{q}\gamma)}$ = 0.61 (theor) 0.58 ± 0.06 (exper.)
4.  $\theta \to 0$ $n_{q\bar{q}g}\downarrow$, $n_\perp \downarrow$
5. azimuthal correlations

7. Intermittency and fractality

Self-similarity of the parton cascade
Analogy with turbulence → „jets inside jets inside..."

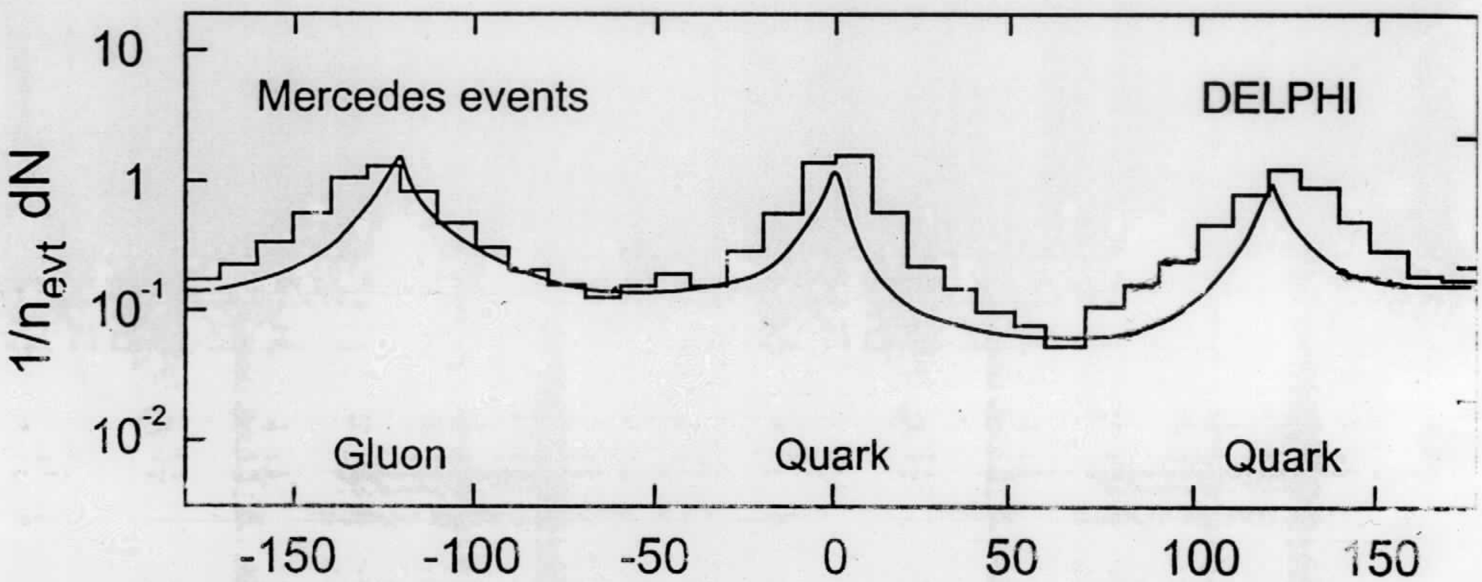
QCD **Prediction:**

Moments increase for bins decreasing:
a) linearly on log-log plot for large bins,
b) flattening at small bins due to α_s running.

a) → monofractal ; b) → multifractal
Slopes at different ranks q → Renyi dimensions

Fig. 9

Qualitatively OK, quantitatively - not
but DLA + some MLLA terms

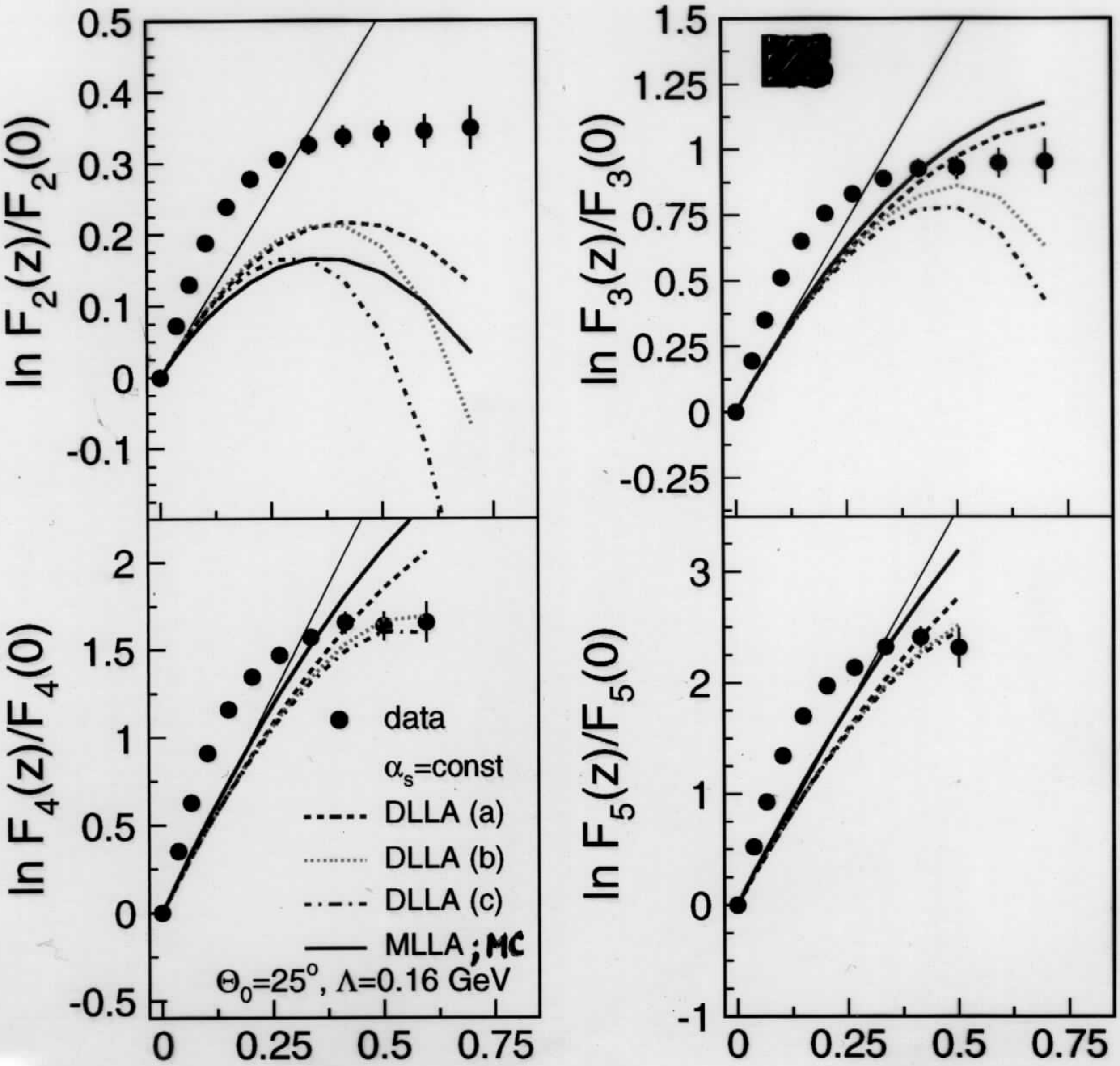


- Andersson, Gustafson, Sjöstrand (80)
- Azimov, Dokshitzer, Khoze, Troyan (85,86)

Fig. 8

L3 21

"Genuine" multiplicity correlations in ring regions



QCD
- (a) Dokshitzer, Dremin (93)
- (b) Brax, Peschanski (94)
- (c) Ochs, Wosiek (92,93)

- Bialas, Peschanski (86)
- Dremin (87)

$\Lambda = 0.16$ GeV

disagreement for $\Lambda = 0.16$ GeV ($n_f=3$)

Fig. 9

8. Higher moments

Prediction:

1. Increase with rank and energy ~OK
2. Asymptotic values

$DLA \Rightarrow F_2^G = \frac{4}{3}$; $F_2^F = \frac{7}{4}$ – very far from Z^0-values

3. At Z^0:

$MLLA \rightarrow F_2^G \simeq 1.039$ 1.023 (exper.)

$\rightarrow F_2^F \simeq 1.068$ 1.082 (exper.)

Failure of analytic approach at higher orders!
Soft partons play a crucial role in correlations.
(Dremin, Lam, Nechitailo (2000)) ($p_T \rightarrow$ Dremin, Eden (2001))

9. Subjet multiplicities

Increase the resolution and get more subjets.
For very high resolution → final hadrons.

Prediction:

1) Asymptotic ratio (DLA) of subjet multiplicities in 3- and 2-jet events:

$$\frac{n_3^{sj}}{n_2^{sj}} = \frac{2C_F + C_A}{2C_F} = \frac{17}{8} \longleftrightarrow < 1.5 \text{ (exper. at } Z^0\text{)}$$

Depletion due to color coherence.

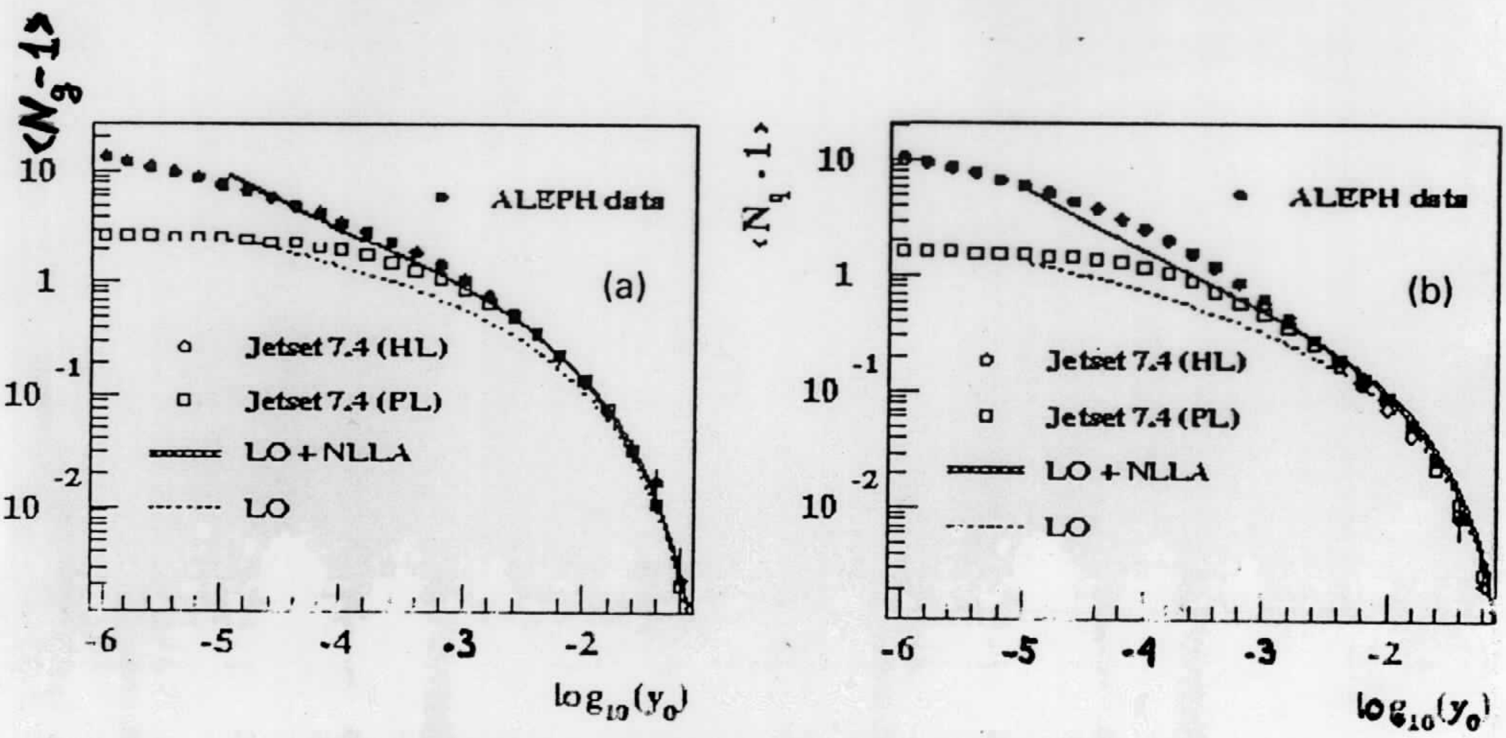
2) Subjet multiplicities in separated q and g jets
Fig. 10

10. Jet universality

Prediction:

Universal in diffent processes.

See Figs. above and next talks



- Dokshitzer, Khoze, Troyan (83)
- Ochs, Wosiek (93)
- Seymour (96)

Fig. 10

Conclusions

Success of Analytic pQCD

1. Qualitdtive predictions - OK!
2. Quantitatively → 20÷10% accuracy (in higher order corrections!)

Computer solution & Monte Carlo models lead to better fits

Main problems:
- Soft partons and non-perturbative terms. (near phase space boundary) (event shapes)
- Asymptotical nature of perturbative expansion, convergence and large value of the expansion parameter ($\gamma_0 \sim 0.5$)
- Probabilistic scheme limitations

OUTLOOK

RHIC, LHC,... - multiparticle production
$\langle n_{ch} \rangle \gtrsim 4000$ at 130 GeV Au-Au

QCD predictions - closer to asymptotics - - better estimation of background for new physics

Event-by-event analysis - pattern recognition - - wavelets as an analyzing tool

Color-suppressed effects, minijet properties, collective flows, ring-like events, event shapes, vacuum (?), ...