Magnetic fields in cosmalogy

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Observations: galactic magnetic fields, coherent on galactic size, 10 kpc, $B_{gal} \sim$ (a few) $\cdot \mu G$

$$\rho_B = \frac{B^2}{8\pi} \simeq \rho_{CMBR} \simeq 10^{-3} \rho_{matter}$$

Intergalactic magnetic fields

$B_{ig} \sim 10^{-3} B_{gal}$ (?), $l_{coh} \sim$ Mpc

Origin – mysterious

For comparison: $B_{\otimes} = 0.5G$, $B_{\odot} \sim 10^3 G$

$B_{\text{white dwarfs}} \sim 10^9 G$; $B_{\text{neutron stars}} \sim 10^{13} G$

Possible explanations:

I. Conventional (astrophysical) – – stellar ejecta, magnetic line reconnection.

II. Early universe, inflation ⇒ large scales

⊕ dynamo amplification at later stage

III. Intermediate: late but before galaxy formation.
or during

(Almost) always either λ is too small or B is too weak

I. Astrophysical

Non-professional estimate

Energy/mass of galactic magnetic fields.

$$E^{magn}_{gal} = \frac{4}{3}\pi R^3_{gal} \cdot \rho_{CMBR} = 5.7 \cdot 10^{34}\, g \approx 30 M_\odot$$

$$R_{gal} = 10\, kpc = 3 \cdot 10^{22}\, cm$$

$$\rho_{CMBR} = 0.26\, eV/cm^3 = 0.5 \cdot 10^{-33} \frac{g}{cm^3}$$

Neutron stars: $B \sim 10^{13} G$, $R \sim 10^6 cm$

$$E^{magn}_{n\text{-}stars} = \frac{4}{3}\pi R^3_{ns} \cdot \frac{B^2}{8\pi} = 10^{-11} M_\odot \left(\frac{B}{10^{13} G}\right)^2$$

$$[\ \rho^{magn}(1G) = 2 \cdot 10^{-40}\, GeV^4 = 2.5 \cdot 10^{10} \frac{eV}{cm^3} = 5 \cdot 10^{-23} \frac{g}{cm^3}.$$

$M_{Galaxy} \sim 10^{11} M_\odot$

White dwarfs: $R \lesssim 10^9 cm$, $B \lesssim 10^{10} G$ ($10^8 - 10^{10} G$

$$E^{magn}_{WD} \leq 10^{-8} M_\odot. \quad \otimes\ \underbrace{3 \cdot 10^9}\ \Rightarrow 30 M_\odot$$

that many WD are needed if energy is not lost at reconnection of small scales to large scales

$N_{WD} \sim 10^{10}$ /per Galaxy

$N_{ns} \sim 10^9$ / — " —

Generation of seed magnetic fields in the early universe.

During inflation very long gravitational waves and large scale scalar field perturbations were generated.

Why not electromagnetic?

Because scalars (even with $m=0$) and gravitons are not conformally invariant, while photons are.

(CT = conf. inv.)

By rescaling of the fields one can exclude FRW-gravity.

Fortunately not for scalars, otherwise we would not be here.

(A) Possible breaking of conformal invariance in electrodynamics

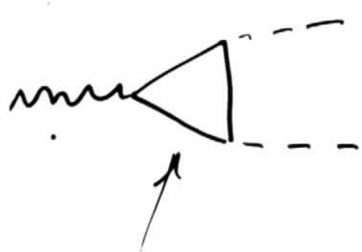
1. New interaction with gravity, non-minimal, non gauge invariant:

$$\mathcal{L} = c_1 R A_\mu A^\mu + c_2 R_{\mu\nu} A^\mu A^\nu + c_3 R_{''''} F^{''} F^{''} + \dots$$

2. Dilaton (spring inspired):

$$\mathcal{L} = -\frac{1}{4} e^{\theta} F_{\alpha\beta} F^{\alpha\beta}$$

3. Conformal anomaly:



$$T^\mu_\mu = \alpha\beta F^2 \neq 0$$

need many particles in the loop

B) Generation of vorticity perturbations,

$\text{rot}\,\vec{V} \neq 0 \Rightarrow \vec{J}_{em}$ due to different mobility of e^- and p or different interaction of particles and antiparticles

$\text{rot}\,\vec{J}_{em} \neq 0 \Rightarrow B \neq 0$

In the early universe:

1. First order phase transitions, bubbles of one phase inside another Characteristic scale is too small.

2. Breaking of electromagnetic gauge invariance in the early univers and generation of electric charge asymmetry Due to isocurvature fluctuations chaotic e.m. currents would be induced after symmetry restoration
~~8~~ Good result but ~~all~~ model is rather unusual

Large inhomogeneities in leptonic charge asymmetry at BBN ($t \sim 1$ sec)

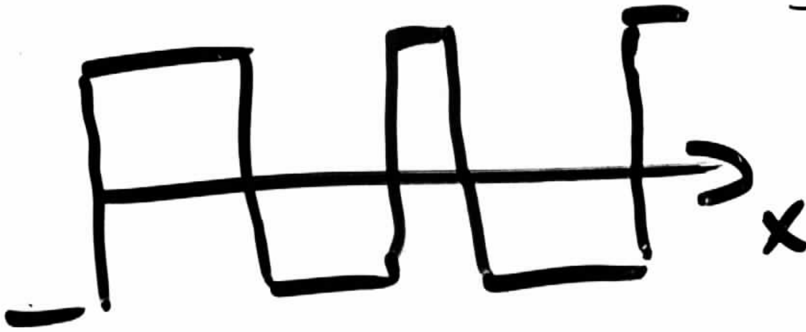
1. Resonance oscillations between ν_a ($a = e, \mu, \tau$) and ν_s (s = sterile) could create $\underline{L \sim 0.1 - 0.3}$

   (ν_s is needed)

   (compare $B \simeq 10^{-9}$)

2. L should be strongly inhomogeneous at horizon scale $t \sim 1$ sec $\Rightarrow \sim 100$ pc today

   $\underline{\delta L / L \sim 1}$



3. $\delta L \neq 0 \Rightarrow$ fluxes of ν and $\bar{\nu}$ interacting with $e^{\pm} \Rightarrow$ currents because of different $\sigma(\nu e)$ and $\sigma(\bar{\nu} e)$

4. Hydrodynamics with large Reynolds number $\Rightarrow$ turbulence $\Rightarrow$ vorticity (eddies)

Weak field and λ is not large enough but with dynamo and "Brownian" reconnection may be ok

Generation of B around hydrogen recombination epoch.

(standard physics)

$T \gtrsim 3000\,K \sim 0.25\,eV$

Usual density perturbations

$$\frac{\delta\rho}{\rho} = 10^{-4} - 10^{-5} \Longrightarrow \mathrm{rot}\,\vec{v} \neq 0$$

Hydrodynamics approximation:

$$\rho[\partial_t \vec{v} + (\vec{v}\vec{\partial})\vec{v}] = -\vec{\nabla} p + \partial_k[\eta(\partial_k \vec{v} + \vec{\nabla} v_k)] +$$
$$+ \vec{\partial}[(\zeta - \tfrac{2}{3}\eta)\partial_k v_k] \leftarrow (*)$$

Viscocity: $\eta = \rho\, l_{free} \equiv \eta\nu$, $\nu \simeq l_{free}$

turns into Navier–Stokes equ. for constant ρ, η, ζ

Reynolds number at scale λ:

$$R_\lambda = \frac{v\lambda}{l_{free}}$$

$$l^{\gamma}_{free} = (\sigma_T n_e)^{-1} = 25\,pc\left(\frac{eV}{T}\right)^3 \xrightarrow[\text{today}]{} 100\,kpc\left(\frac{eV}{T}\right)^2$$

Estimate V assuming homogeneous incompressible fluid:

ν = viscosity

$$\partial_t \vec{V} + (\vec{V}\vec{\nabla})\vec{V} - \nu \Delta \vec{V} = -\frac{\vec{\nabla} p}{\rho}$$

$$V_\lambda = \frac{\lambda}{3\nu}\,\delta_\lambda\left[1 - e^{-\frac{\nu t}{\lambda^2}}\right], \quad \delta_\lambda = \left(\frac{\delta\rho}{\rho}\right)_\lambda$$

$$p = \frac{\rho}{3}$$

$$\hookrightarrow R_\lambda^{max} = \frac{t}{3 l_{free}}\,\delta_\lambda \sim 10^3\,\delta_\lambda \left(\frac{T}{eV}\right)$$

Turbulence developes if $R > 20-30$,

needs $\delta_\lambda > 10^{-2}\left(\frac{eV}{T}\right)$ $\qquad$ $T > 100$ MeV for $\delta \sim 10^{-4}$

$$\lambda_{max} \sim t \sim 10^{12}\,sec \cdot \left(\frac{eV}{T}\right)^2 = 10\,kpc\left(\frac{eV}{T}\right)^2 \xrightarrow[today]{} 40\,Mpc\,\frac{eV}{T}$$

at RD-stage, if $T > eV$

Take $T \gtrsim 1000\,eV \Rightarrow \underline{\underline{R \sim 10^6\,\delta_\lambda}}$

$$\lambda_{max}^{today} \sim 40\,kpc$$

Pregalactic shrinking $\underline{\underline{1:100}}$ i.e. $\underline{1\,Mpc \to 10\,kpc}$

Laminar flow, $R \lesssim 1$

$$\vec{\omega} = \vec{\nabla} \times \vec{V}$$

$$\partial_t \vec{\omega} - \nu \Delta \vec{\omega} = -\vec{\nabla} \times \left(\frac{\vec{\nabla} p}{\rho} \right) \equiv \vec{S}$$

If $p = p(\rho)$ then $\vec{S} = -\vec{\nabla} \times \left(\frac{\vec{\nabla} p}{\rho} \right) = 0$

and $\vec{\omega} = 0$

Local thermal equilibrium is established and if $p = p(T)$, $\rho = \rho(T)$ and $T = T(x)$ still $\vec{S} = 0$

But chemical potentials are non-vanishing and inhomogeneous:

$$f_e = \exp\left[-\frac{E}{T(x)} + \xi(x) \right]$$

$$\xi(x) = \ln \beta(x) + \text{const}, \quad \beta(x) = \frac{n_B}{n_\gamma} \sim 6 \cdot 10^{-10}$$

$$S_\lambda \sim \frac{1}{3} \lambda^{-2} \underbrace{\left(\frac{\delta \rho}{\rho} \right)_\lambda \frac{\delta \beta}{\beta}}_{(10^{-8} - 10^{-9})} \cdot \underbrace{\frac{\rho_{matter}}{\rho_{tot}}}_{(10^{-1} - 10^{-2})} \sim \left(10^{-8} - 10^{-10} \right) \lambda^{-2}$$

even $\frac{\delta\beta}{\beta} \sim 10^{-3}$ is allowed by CMBR.

$$\omega = |\vec{\nabla} \times \vec{v}| \sim \frac{1}{l_{free}} \left(\frac{\delta\rho}{\rho}\right)^2 \left[1 - e^{-\frac{l_{free}\, t}{\lambda^2}}\right]$$

vorticity

Magnetic hydrodynamics with high conductivity:

$$\varkappa \simeq \frac{1}{\alpha} \frac{n_e}{n_\gamma} \frac{m_e^{3/2}}{T}$$

$$\partial_t \vec{B} = \vec{\nabla} \times (\vec{V} \times \vec{B}) + \frac{1}{\varkappa} \vec{\nabla} \times \vec{J}$$

$J \sim n_e V$; e are frozen in the γ-liquid, while p are (almost) free

$$\frac{B}{T^2} \sim \alpha e \frac{0{,}24\, T}{m_e} \left(\frac{t}{l_e}\right) \frac{\rho_e}{\rho_\gamma} \cdot \left(\frac{\delta\rho}{\rho}\right)^2 \sim 10^{-18} \left(\frac{T}{eV}\right)^2$$

$\otimes\, 10^4$ at contraction when galaxies are formed

$$\left(\frac{B}{T^2}\right)_{gal} \sim 10^{-14} \left(\frac{T}{eV}\right)^2 \to 10^{-10}$$

for $T \sim 100\,eV$

Thus large scale field ~~B~~ with $B \sim 10^{-10} B_{obs}$ can be generated

Galactic dynamo with relatively mild amplification 10^{10} is necessary (estimates give up to $10^{15} - 10^{18}$).

No problem with small scale amplification (?) if spectrum of B is cut-off at low λ.

Intergalactic fields are not explained.

Optimistic possibilities:

1. $(\delta\rho/\rho) \gtrsim 10^{-3}$ at $\underset{(Now)}{l} \sim 100$ kpc ~~x~~ (a few)

   ↳ large R and turbulent eddies
   Mechanisms ?

   CMBR - ?
   OK

2. ↳ Beyond hydrodynamics, suppressed but not as strong as $(\delta\rho/\rho)^2 \sim 10^{-9}$.