Theory of rare kaon decays

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Outline

- Introduction
- $K \to \pi\nu\bar{\nu}$
- Minimal Flavour Violation (MFV)
- $K \to \pi\ell\bar{\ell}$
- $K_L \to \mu\overline{\mu}$
- Conclusions

Introduction

- The SM with 3 families predicts weak interactions with an unitary matrix V_{ij}: 3 angles and 1 phase (CP violating)

$$\overbrace{V_{ud},\, V_{cb},\, V_{td}}^{\text{Wolfenstein}}$$
$$\Downarrow$$
$$\lambda,\, A\lambda^2,\, A\lambda^3(1-\rho-i\eta)$$

- FCNC only at 1-loop

- B-physics, V_{ij}-unitarity test: possible but not easy

- The area of all possible CKM-unitarity triangles is an invariant:

$$|J_{CP}| \stackrel{Wolfenstein}{\simeq} A^2\lambda^6\eta$$

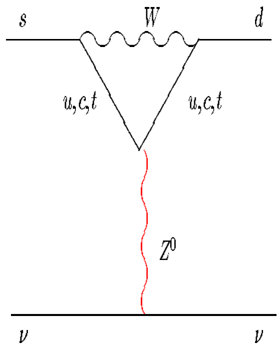
- As we shall see $B(K_L \to \pi^0 \nu\overline{\nu})$ will measure this area

- Already $B(K^+ \to \pi^+ \nu\overline{\nu})$ tests B-physics (ΔM_B, β)

see Isidori talk

$K \to \pi \nu \overline{\nu}$

$$A(s \to d\nu\overline{\nu}) \sim \overline{s}_L \gamma_\mu d_L \quad \overline{\nu}_L \gamma^\mu \nu_L \ \times \left[\sum_{q=c,t} V^*_{qs} V_{qd} \, m_q^2 \right]$$



$\sim$

$$[A^2 \lambda^5 \ (1 - \rho - i\eta) m_t^2 + \lambda m_c^2]$$

SM: $\underbrace{V - A \otimes V - A}_{\Downarrow}$ Littenberg

$$\Gamma(K_L \to \pi^0 \nu \overline{\nu}) \quad \begin{cases} \text{CP violating} \\ \Rightarrow \ J = A^2 \lambda^6 \eta \\ \text{Only } top \end{cases}$$

- K^+: $t >$ charm $\stackrel{\text{NLO}-\text{QCD}}{\Longrightarrow}$ K^+ $\mathcal{O}(5\%)$, K_L $\mathcal{O}(1\%)$

- $K_{l3} \stackrel{SU(2)\text{isospin}}{\Longrightarrow} \langle \pi | \overline{s} \gamma_\mu d | K \rangle$ ($\leq 1\%$ Marciano-Parsa)

- SM- BR($K \to \pi\nu\overline{\nu}$) Buchalla-Buras

$$K^+ \quad (0.72 \pm 0.21) \cdot 10^{-10} \qquad K_L : (2.8 \pm 1.0) \cdot 10^{-11}$$

CKM 10% accuracy Kopio,KEK

- 2 K^+ candidates for BNL-E787 E949 should improve

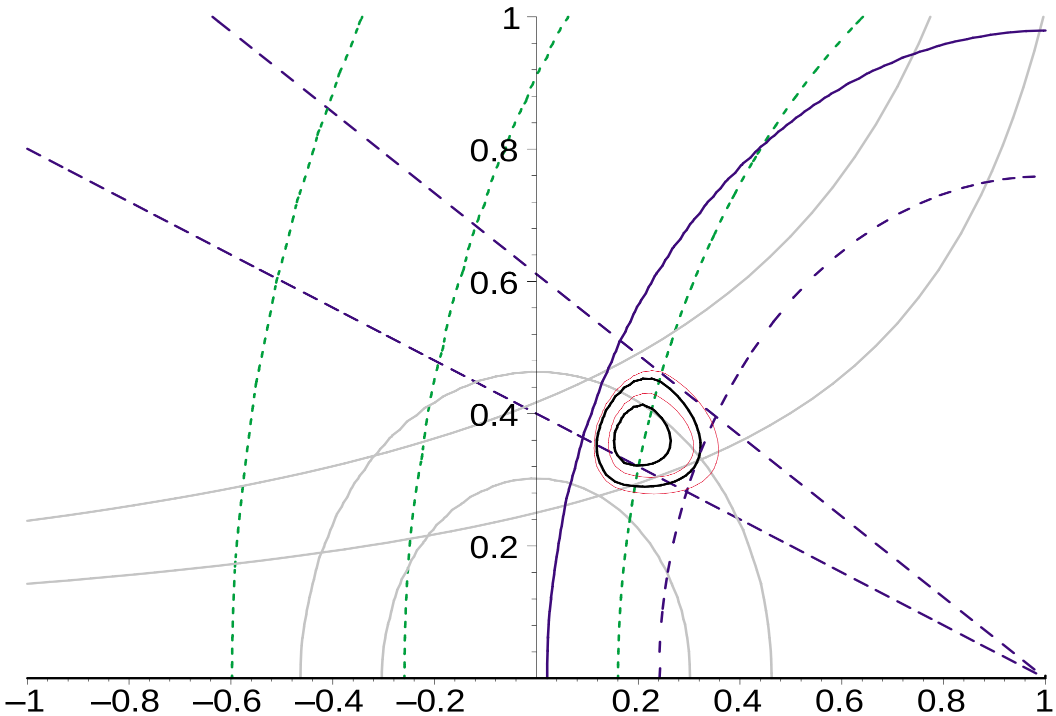
$$B(K^+) = (1.57^{+1.75}_{-0.82}) \cdot 10^{-10}$$

- K_L Model-independent bound, based on $SU(2)$ properties dim-6 operators for $\overline{s}d\overline{\nu}\nu$ Grossman-Nir

$$B(K_L) \;\leq \frac{\tau_L}{\tau_+} \times B(K^{\pm}) < 1.7 \times 10^{-9} \quad \text{at } 90\% C.L.$$

Impact of BNL-E787 in flavour physics

G.D. Isidori



- two scenarios
  - NP in $\overline{s}d\overline{\nu}\nu \stackrel{NP}{\Longrightarrow} \overline{b}s\overline{\nu}\nu$
  - NP in $B-\overline{B}$-mixing

Effective supersymmetry, 3 s-family lighter ≤ 1 TeV, why?

Minimal Flavour violation

- naturalness ? From $K - \overline{K}$-mixing $\Longrightarrow \Lambda_{NP} > 100$ TeV, stability of the Higgs potential $\Longrightarrow \Lambda_{NP} < 1$TeV

- NP is not generic, but has a symmetry (MFV), which generalizes the GIM mechanism. We want some global symmetry (G_F) allowing only th SM Yukawas

$$\mathcal{L} = \bar{Q}_L Y_D D_R H + \bar{Q}_L Y_U U_R H_c + \bar{L}_L Y_E E_R H$$

$G_F = U(3)^5$: 3 families of Q, U, D, L, E_R

Chivukula-Georgi

$\Lambda_{NP} > 100$ TeV: All allowed

$\Lambda_{NP} < 100$ TeV: NP dynamics preserves G_F

↓

$\Lambda < 1$ TeV: SM gauge group,G_F, $Y_{U,D,L}$

$$\Delta F = 2 \qquad \mathcal{L}_{NP} \stackrel{U(3)^5}{\Longrightarrow} \mathcal{L}_{MFV} =$$

$$= \frac{C}{\Lambda^2_{MFV}} [\frac{(V^*_{td} m_t^2 V_{tb})^2}{v^4} (\bar{d}_L \gamma^\mu b_L)^2 + \frac{(V^*_{td} m_t^2 V_{ts})^2}{v^4} (\bar{d}_L \gamma^\mu s_L)^2]$$

- Fixing $C \sim 1 \Longrightarrow$ bound on Λ^2_{MFV}

Buras et al., Nir et al., G.D.,Giudice,Isidori,Strumia

- We write the most general dim-6 $\mathcal{L}$ (a la Fermi) consistent con MFV and determine bounds/correlations

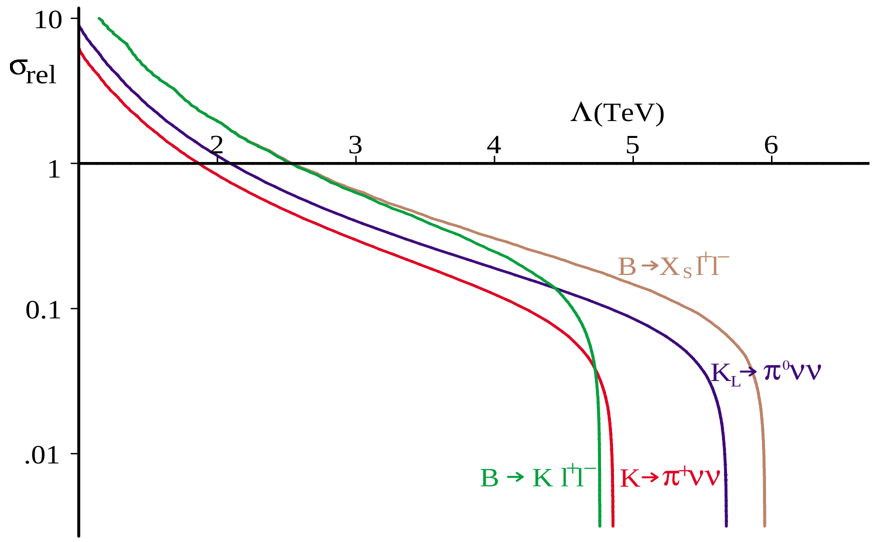
G.D.,Giudice,Isidori,Strumia

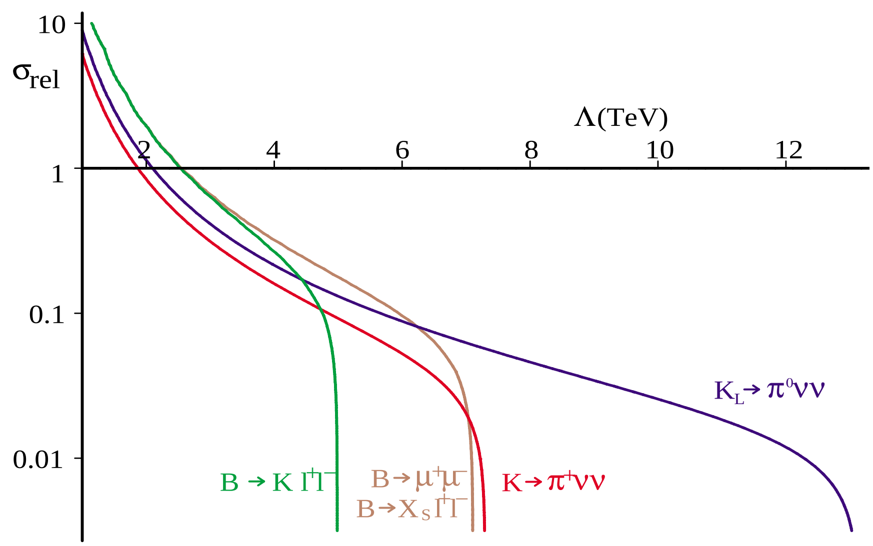
MFV dim-6 operator	main observables	Λ [TeV] $-$	$+$
$\mathcal{O}_0$	$\epsilon_K, \quad \Delta m_{B_d}$	6.4	5.0
$\mathcal{O}_{F1}$	$B \to X_s \gamma$	8.3	13.4
$\mathcal{O}_{\ell 1}$	$B \to (X)\ell\bar{\ell}, \quad K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	3.1	2.7
$\mathcal{O}_{\ell 2}$	$B \to (X)\ell\bar{\ell}, K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	3.4	3.0
$\mathcal{O}_{H1}$	$B \to (X)\ell\bar{\ell}, \quad K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	1.6	1.6

Bounds on MFV operators

Minimal Flavour violation and $K \to \pi\nu\bar{\nu}$

- Future experiments, in particular $K_L \to \pi^0\nu\bar{\nu}$ will probe deeply MFV





- Up: 10% accuracy on the CKM , Down: 1%

$K_L(p) \to \pi^0(p_3)\gamma(q_1)\gamma(q_2)$

- Lorentz + gauge invariance $\Rightarrow$

$$M \sim \quad A(y,z) \qquad\qquad B(y,z) \qquad\qquad \begin{cases} y = \frac{p\cdot(q_1-q_2)}{m_K^2} \\ z = \frac{(q_1+q_2)^2}{m_K^2} \end{cases}$$

$$\begin{array}{cc} \gamma\gamma & \gamma\gamma \\ J=0 & \mathrm{D-wave\ too} \\ F^{\mu\nu}F_{\mu\nu} & F^{\mu\nu}F_{\mu\lambda}\partial_\nu K_L \partial^\lambda \pi^0 \end{array}$$

- $$\frac{d^2\Gamma}{dydz} \sim z^2|A+B|^2 + \left(y^2 - \left(\frac{(1+r_\pi^2-z)^2}{4} - r_\pi^2\right)\right)^2 |B|^2$$

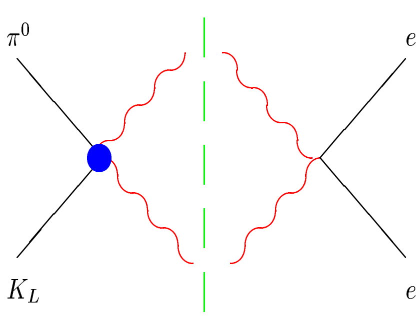
$$r_\pi = m_\pi/m_K$$

- Different gauge structure $\Rightarrow$ B$\neq$0 at $z \to 0$ (collinear photons).

Crucial role in $K_L \to \pi^0 e^+ e^-$

Morozumi et al, Flynn Randall

Sehgal Heiliger, Ecker et al., Donoghue et al.



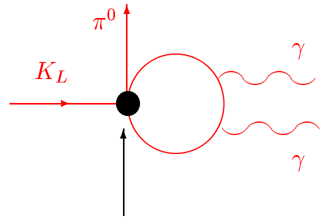
A suppressed by m_e/m_K

B is not

$K_L \to \pi^0\gamma\gamma$

Ecker, Pich, de Rafael
Cappiello,G.D

- $O(p^4)$ CT: 0; Loop:



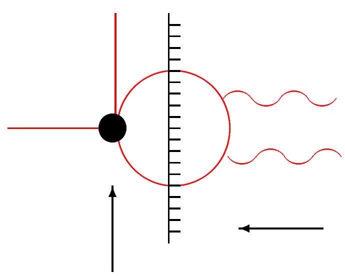
only A-amplitude

G_8 from $K_S \to 2\pi$

But $\Gamma(K_L \to \pi^0\gamma\gamma)_{p_4} \sim \frac{1}{2.5}\Gamma(K_L \to \pi^0\gamma\gamma)_{\rm exp}$

- $O(p^6)$ A, B amplitudes
  from:

3 CT′s
$F_{\mu\nu}F^{\mu\alpha}\partial_\alpha K_L \partial^\nu \pi^0$
$F^2 \; \partial K_L \partial \pi^0$
$F^2 \; m_K^2 K_L \pi^0$



Cappiello, G.D., Miragliulo
Cohen, Ecker, Pich

Full description of unitarity cut

$A(K \to 3\pi) = a + b\; Y + c\; Y^2 + d\; X^2$

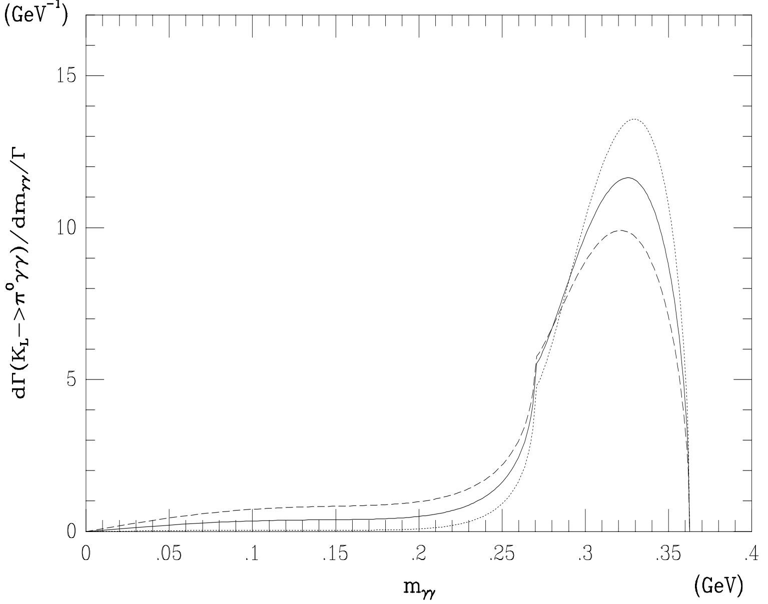
$$A_{\rm CT} = \alpha_1(z - r_\pi^2) + \alpha_2$$
$$B_{\rm CT} = \beta$$

VMD $\Rightarrow$ 1 coupling a_V ($\sim$−0.6 G.D., Portoles)

(Ecker, Pich, de Rafael)

(Sehgal et al.)

$$\alpha_1 = \frac{\beta}{2} = -\frac{\alpha_2}{3} = -4a_V \sim 2$$ n.d.a.$\sim$0.2

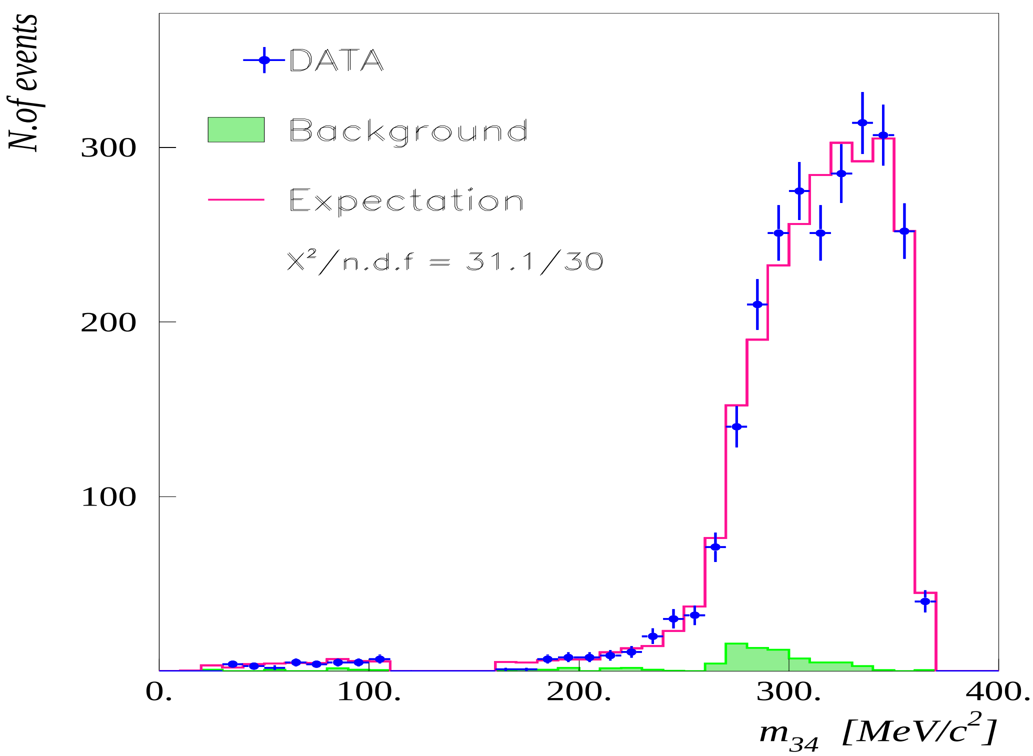


1 parameter fit:

	a_V	
...	-0.4	(NA48)
———	-0.7	(KTeV)
– – –	-1	

study of low $m_{\gamma\gamma}$-spectrum $\Rightarrow$ B$(K_L \to \pi^0 e^+ e^-)_{disc} \sim$

$$0.5 \cdot 10^{-12}$$



NA48, 02

$$a_V = -0.46 \pm 0.03 \pm 0.04 \quad \text{Br} = 1.36 \pm 0.03 \pm 0.03$$

Left-over questions

- Gabbiani, Valencia

	α_1	α_2	β
3 parameters fit	0	1.7	-5
(VMD	$-4a_V$	$12a_V$	$-8a_V$)

Taking B from rate and spectrum fit

large B   B$(K_L \to \pi^0 e^+ e^-)_{disc} \sim 1.5 \cdot 10^{-12}$

B$(K_L \to \pi^0 e^+ e^-)_{disc}$ expt. question

Dispersive: $\gamma\gamma$ off-shell?

Donoghue-Gabbiani: particular form factor, a factor 4∼ 5 larger

Important to measure $K_L \to \pi^0 \gamma \gamma^*$ (KTeV + NA48)

$K_S \to \gamma\gamma$

- No short distance contributions

Gaillard-Lee

- Neutral particles $(K_S) \Rightarrow$

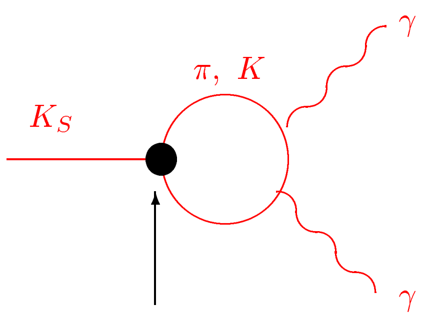
No $O(p^2)$

No $O(p^4)$ CT : $F_{\mu\nu}F^{\mu\nu}\langle\lambda_6 QU^+QU\rangle$

- Loop contribution finite

$\text{Br}_{\chi\text{PT}}(K_S \to \gamma\gamma) = 2.1\cdot 10^{-6}$

(G.D. and Espriu 86, Goity 87)



G_8 from $K_S \to \pi\pi$

$(2.78 \pm 0.072)\cdot 10^{-6}$

(NA48 '02)

scale independent and unambigous χPT prediction

- $O(p^6)_{\text{CT}}$

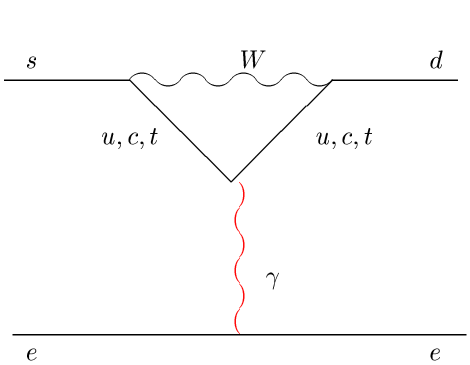
No VMD $\Rightarrow \dfrac{A^{(6)}}{A^{(4)}} \sim \dfrac{m_K^2}{(4\pi F_\pi)^2} \sim 0.2$

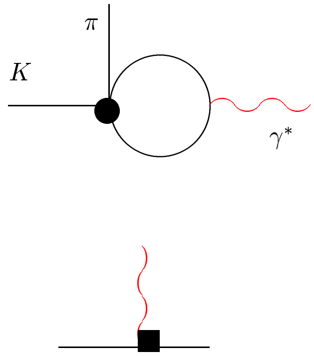
$F^{\mu\nu}F_{\mu\nu}\langle\lambda_6 Q^2 \mu M U^+\rangle$

(Not $\dfrac{m_K^2}{m_\rho^2} \sim 0.4$)

$K^{\pm}(K_S) \to \pi^{\pm}(\pi^0)\ell^+\ell^-$

- short distance << long distance





- 1 "γ" amplitude determined by form factor W

$$W^i = G_F m_K^2 (a_i + b_i z) + W^i_{\pi\pi}(z) \qquad i = \pm, S$$

$$a_i, b_i \sim O(1), \qquad z = \frac{q^2}{m_K^2}$$

- Observables $\Gamma(K^+ \to \pi^+ e^+ e^-)$, $\Gamma(K^+ \to \pi^+ \mu\overline{\mu})$, slopes

- a_i $\quad O(p^4)$ Ecker, Pich, de Rafael

- b_i $\quad O(p^6)$ G.D., Ecker, Isidori, Portoles

- $a_+,\ b_+$ in general not related to $a_S,\ b_S$

Expt. E865

- $K^+ \to \pi^+ e^+ e^-$: $\begin{array}{c} a_+ = -0.586 \pm 0.010 \\ b_+ = -0.655 \pm 0.044 \\ \frac{\chi^2}{n_{dof}} \sim \frac{13.3}{18} \end{array}$ $\begin{array}{c} \text{just linear slope} \\ (W_{\pi\pi} = 0) \\ \frac{\chi^2}{n_{dof}} \sim \frac{22.9}{18} \end{array}$

- $K^+ \to \pi^+ \mu \overline{\mu}$:

$$\begin{array}{ll} \mathrm{Br} = (9.22 \pm 0.77) \cdot 10^{-8} & \text{consistent with theory} \\ \text{not}(5. \pm 1.) \cdot 10^{-8} & (\text{E787, '97}) \end{array}$$

Problems: $\begin{array}{ccc} a_i & b_i & \text{same phenomenological size} \\ p^4 & p^6 & \text{different theoretical order} \end{array}$

Probably explained by large VMD

Then we can just parameterize

$$\mathrm{Br}(K_S \to \pi^0 e^+ e^-) = 4.6 \times 10^{-9} a_S^2$$

not predicted but dynamically interesting

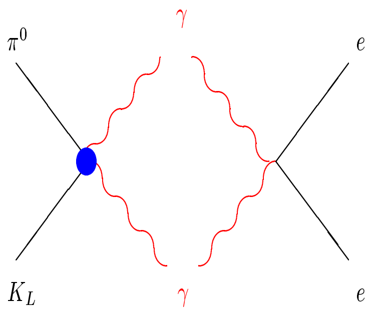
HyperCP has confirmed E865 (02) regarding $K^+ \to \pi^+ \mu \overline{\mu}$ and put a bound on the CP asymmetry (≤ 0.1)

$K_L \to \pi^0 e^+ e^-$: summary

$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) \leq 5 \cdot 10^{-10}$ KTeV

- V-A$\otimes$ V-A $\Rightarrow$ $\langle \pi^0 e^+ e^- | (\bar{s}d)_{V-A} (\bar{e}e)_{V-A} | K_L \rangle$ violates CP

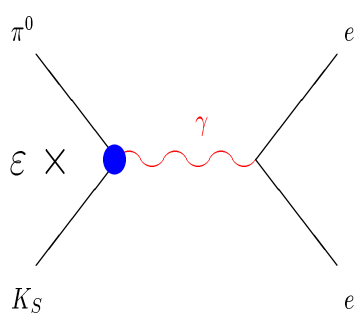
1)



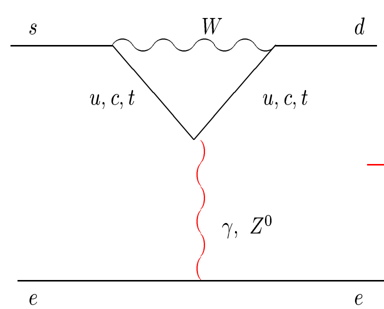
CP conserving

$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) \sim 10^{-12}$

2) $\varepsilon\ \times$



+ 3)



$\longrightarrow$ Im λ_t

$\lambda_t = V_{td} V_{ts}^*$

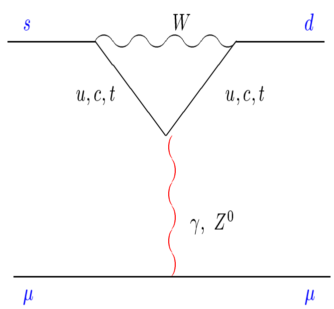
$\uparrow$ $\mathrm{B}(K_s \to \pi^0 e^+ e^-) = 4.6 a_S^2 \times 10^{-9}$

Possible large interference: $a_S < -0.5$ or $a_S > 1$

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-)_{\mathrm{CPV}} = \left[15.3 a_S^2 - 6.8 \frac{Im\lambda_t}{10^{-4}} a_S + 2.8 \left(\frac{Im\lambda_t}{10^{-4}} \right)^2 \right] \cdot 10^{-12}$$

short distance probe even for a_S large

$K_L \to \mu\overline{\mu}$



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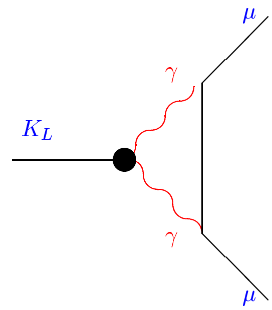


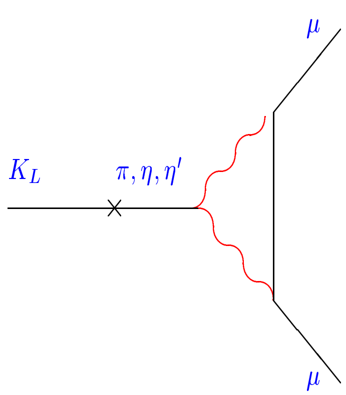
$$\mathrm{Br}(K_L \to \mu\overline{\mu}) = |ReA|^2 + |ImA|^2 = (7.18 \pm 0.17) \cdot 10^{-9}$$

$\swarrow$ $\quad K_L \to \gamma\gamma \downarrow \quad$ $AGS\ 871$

$Re(A_{\rm SD} + A_{\rm LD}) \qquad |ImA|^2 = (7.1 \pm 0.2) 10^{-9}$

Good determination of $A_{\rm LD} \Rightarrow A_{\rm SD} \Rightarrow$ SM test

Large N_c+$\mathrm{U}(3)_L \otimes \mathrm{U}(3)_R$ Gomez Dumm-Pich



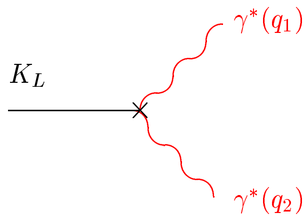
Local+non-local contribution
$|A_{\rm SD}|^2 < 2.9 \cdot 10^{-9}$
SM $(0.9 \pm 0.4) \cdot 10^{-9}$ Buchalla-Buras

$\mathrm{B}(K_L \to e\bar{e}) = (9.0 \pm 0.4) \cdot 10^{-12}$
BNL871 $(8.7^{+5.7}_{-4.1}) \cdot 10^{-12}$

1/N corrections & symmetry breaking? Large in $K_L \to \gamma\gamma$

Low energy form factor + matching with VMD to QCD

(G.D, Isidori, Portoles)

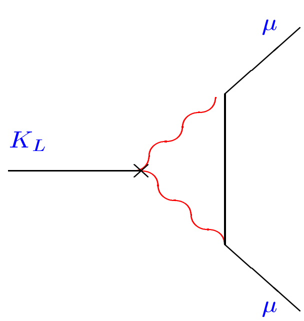


$$A(K_L \to \gamma^*\gamma^*) = A_{\gamma\gamma}^{\exp}\left[1 + \alpha\left(\frac{q_1^2}{q_1^2 - M_V^2} + \frac{q_2^2}{q_2^2 - M_V^2}\right) + \beta\frac{q_1^2 q_2^2}{(q_1^2 - M_V^2)(q_2^2 - M_V^2)}\right]$$

α fixed from $K_L \to \gamma\gamma^*$

β from $K_L \to e^+e^-\mu\bar{\mu}$ (KTeV, NA48, not yet)

Matching with QCD $q_1^2, q_2^2 \to \infty$ imposes



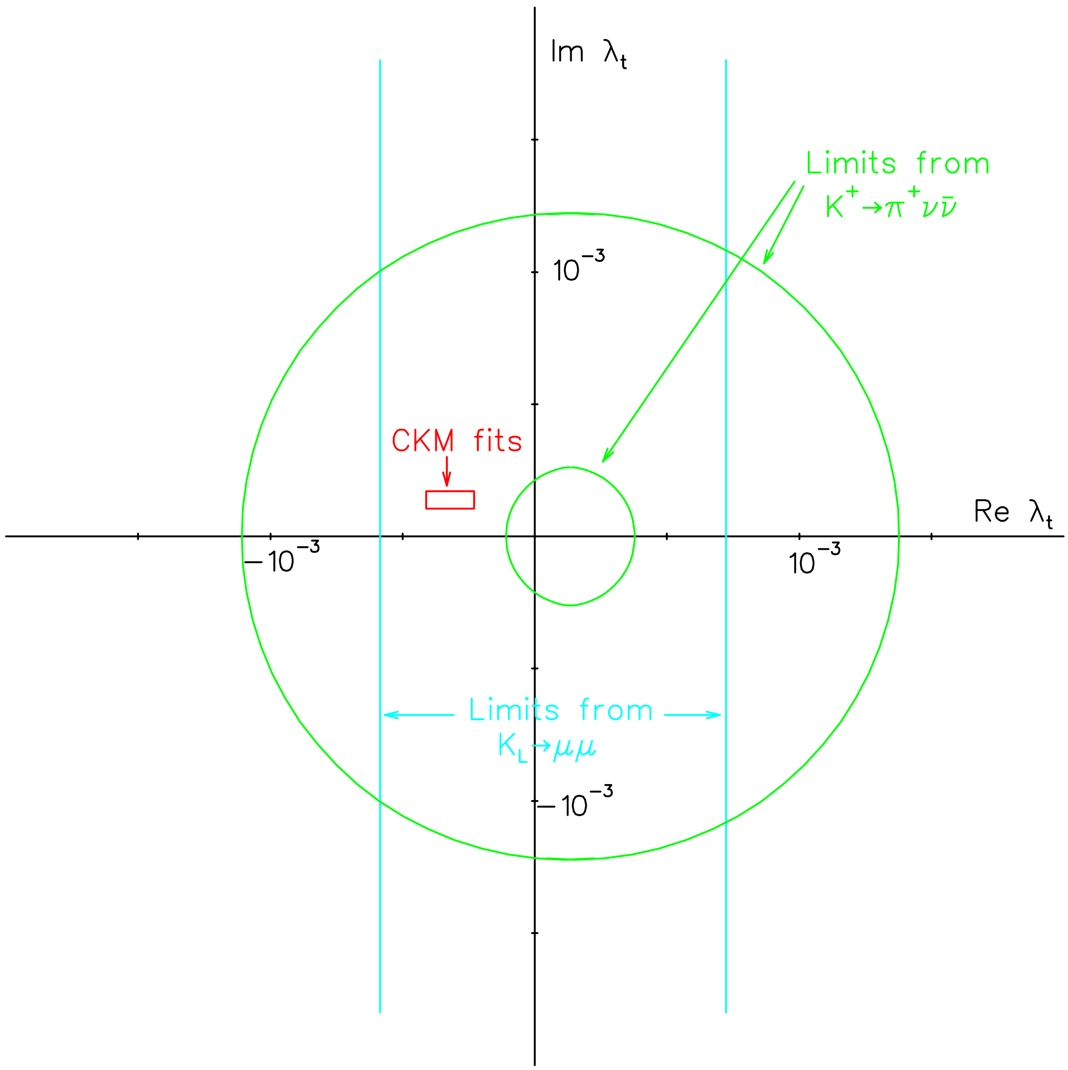
$$|1 + 2\alpha + \beta| \simeq 0.3$$

$\Rightarrow |A_{SD}|^2 < 2.8 \cdot 10^{-9}$

B($K_L \to e\bar{e}$) insensitive to α, β

Valencia alternative picture

From Littenberg hep-ex/0010048



Conclusions

- $K_L \to \pi^0 \nu\bar{\nu}$ a measurement with 10% accuracy $\Rightarrow$ would push to 10 TeV the validity of SM

- $K_L \to \pi^0 e\bar{e}$: if NA48 will tell us that $K_S \to \pi^0 e\bar{e}$ (a_S) is in an interesting interference range is worth pursuing this channel

- NA48 and KLOE:
  - $K_S \to \pi^0 e\bar{e}$ very interesting (a_S)
  - CP violation in $K^+ \to 3\pi / K^+ \to \pi^+\pi^0\gamma$ challenge
  - $K^+ \to \pi^+\gamma\gamma / K^+ \to \pi^+\pi^0\gamma$: chiral tests

- $K_S \to \gamma\gamma$ best chiral test, h.o. established: $\leq 15\%$ amplitude

- $K_L \to \mu\bar{\mu}$: full form factor close to be measured