New effects of

matter

in neutrino oscillations

$$\Downarrow$$

NEW NEUTRINO

RESONANCES IN

MOVING AND POLARIZED

MATTER

The MSW effect in moving and polarized matter

4/03/2002

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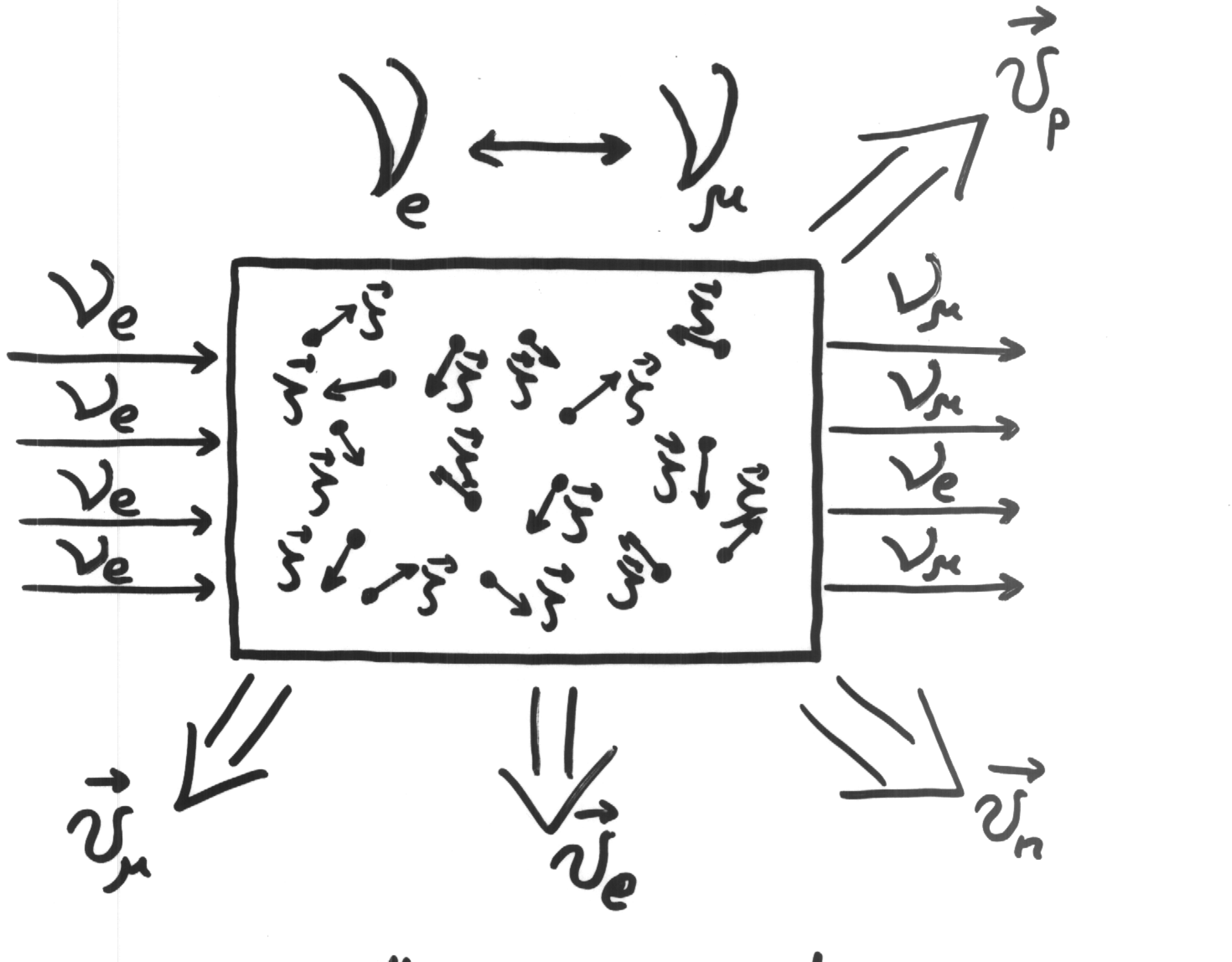


moving matter components

$f = e, n, p, \mu$, etc

with polarizations $\vec{\xi}, \vec{\xi}, \vec{\xi}, \vec{\xi}$, etc

RESULTS (1999 – 2002)

(I) Lorentz invariant approach to ν spin evolution in arbitrary e.m. field

A. Egorov, A. Lobanov, A. Studenikin, hep-ph/9902447, hep-ph/9910476;
in: Results and Perspectives in Particle Physics, ed. M. Greco, 2000, p. 117;
Phys. Lett. B 491 (2000) 137;
M. Dvornikov, A.S., Phys. Atom. Nucl. 64 (2001), hep-ph/0202113; 0102099; hep-ph/0107109.

(II) $\nu_L \leftrightarrow \nu_R$ in arbitrary e.m. field and moving polarized matter

(*) "increase" ("decreas") effect

A. Lobanov, A.S. hep-ph/0106101, Phys. Lett. B 515 (2001) 94.

(III) $\nu_\ell \leftrightarrow \nu_{\ell'}$ flavour oscillations in moving and polarized matter

A. Grigoriev, A. Lobanov, A.S. hep-ph/0112304, hep-ph/0202276.

AND

PERSPECTIVES (2002– ...)

... detectable consequences in astrophysical and cosmology setting

Neutrino flavour oscillations $\nu_\ell \leftrightarrow \nu_{\ell'}$ in moving and polarized matter

For each of matter components

$$f = e, n, p, \mu, \dots, \nu_e, \nu_\mu, \dots \quad :$$

- $\vec{v}_f \rightsquigarrow$ speed of reference frame in which mean momentum of fermions f is zero (rest frame of background fermions f)

  total speed of matter

- $n_f^{(0)} \rightsquigarrow$ number density of f in r.f. of f

  invariant number density

$$\circledast \quad n_f = \frac{n_f^{(0)}}{\sqrt{1 - v_f^2}}$$

in the laboratory frame

- $\vec{\zeta}_f \rightsquigarrow$ mean value of polarization vectors of f in r.f.

delicate procedure... 5'

Matter ($f = e, n, p, \dots$) motion and polarization in laboratory frame:

the frame of oscillation experiment

$$j_f^\mu = (n_f, n_f \vec{v}_f),$$

currents

fermions

polarizations

$$\lambda_f^\mu = \left(n_f \vec{\zeta}_f \vec{v}_f , \; n_f \vec{\zeta} \sqrt{1 - v_f^2} + \right.$$

$$\left. + \frac{n_f \vec{v}_f (\vec{\zeta}_f \vec{v}_f)}{1 + \sqrt{1 - v_f^2}} \right).$$

For slowly moving matter component f ($v_f \approx 0$ in lab. frame):

$$j_f^\mu = (n_f^{(0)}, 0, 0, 0),$$

$$\lambda_f^\mu = (0, n_f^{(0)} \vec{\zeta}_f).$$

Let us suppose that at least one of matter components is moving with

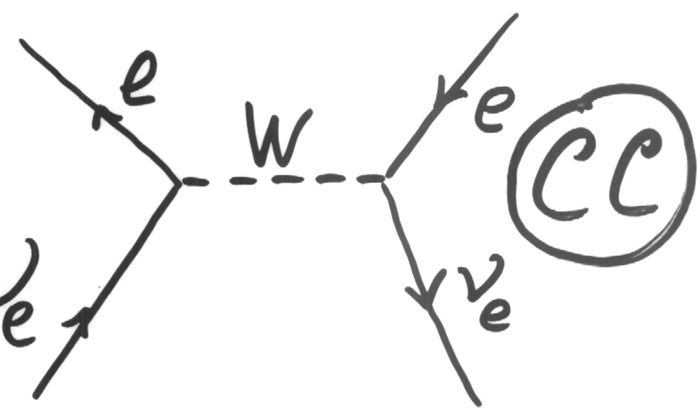
$$v_f \approx 1 \, .$$

Two-flavour oscillations $\nu_e \leftrightarrow \nu_\mu$ in matter $f = e$ (gas of electrons with $v_e \sim 1$).

Matter effect in $\nu_e \leftrightarrow \nu_\mu$ (elastic forward scattering of ν's on background electrons):

$$\Delta V = V_{\nu_e} - V_{\nu_\mu} \Longleftarrow$$

effective neutrino potentials



NC contributes in case of $\nu_{active} \longleftrightarrow \nu_{sterile}$

Two-step averaging procedure

In the reference frame $\vec{v}_f = 0 \Rightarrow$

$$\lambda_f^\mu = (0, n_f \vec{\xi}_f),$$

$$(0 \le |\vec{\xi}_f|^2 \le 1).$$

Mean value of polarization vectors of f:

A. Lobanov, A.S., 2001

$$\text{II} \quad \vec{\xi}_f = \sum_{\{n\}} \langle \vec{O} \rangle \rho_f(\{n\}) \Big/ \sum_{\{n\}} \rho_f(\{n\}),$$

fermion distribution (Fermi-Dirac)

$$\text{I} \quad \langle \vec{O} \rangle = \int \Psi_f^+(x) \vec{O} \Psi_f(x)\, dx,$$

fermion quantum state in e.m. field

$$\vec{O} = \gamma_0 \vec{\Sigma} - \gamma_5 \frac{\vec{p}}{E} - \gamma_0 \frac{\vec{p}(\vec{p}\vec{\Sigma})}{E(E+m)},$$

fermion momentum, energy, mass

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}.$$

relativistic spin operator of fermion f

Total Lagrangian density for ν_e

(CC interaction with $f=e$ is included)

$$\mathcal{L} = \bar{\Psi}_{\nu_e}(i\not{\partial} - m_{\nu_e})\Psi_{\nu_e} + \bar{\Psi}_e(i\not{\partial} - m_e)\Psi_e -$$

$$- \frac{G_F}{\sqrt{2}}\left(\bar{\Psi}_{\nu_e}\gamma_\mu(1+\gamma_5)\Psi_{\nu_e}\right)\left(\bar{\Psi}_e\gamma^\mu(1+\gamma_5)\Psi_e\right)$$ (Fierz transformed)

$\Longrightarrow$

additional matter term

$$\Delta\mathcal{L}_{eff} = - f_\mu\left(\bar{\nu}\gamma_\mu\frac{1+\gamma_5}{2}\nu\right),$$

$$f_\mu = \sqrt{2}\,G_F\left(j_e^\mu - \lambda_e^\mu\right)$$

electrons current and polarization

modifies the Dirac equation

$$\left(\gamma_0 E_{\nu_e} - \vec{\gamma}\vec{p}_{\nu_e} - m_{\nu_e}\right)\Psi_{\nu_e} = \left(\gamma_\mu f^\mu\right)\Psi_{\nu_e}.$$

$\Downarrow$

$$\boxed{E_{\nu_e} = \sqrt{(\vec{p}_{\nu_e} - \vec{f})^2 + m_{\nu_e}^2} + f^0}$$

neutrino dispersion relation in matter

... in the limit of weak potential

$$|\vec{f}| \ll p_0 = \sqrt{\vec{p}_\nu^{\,2} + m_\nu^2} \quad \Longrightarrow$$

Effective energy of ν_e in moving and polarized matter ($f = e$):

$$\boxed{E_{\nu_e} = p_0 + \sqrt{2}\, G_F \underline{\underline{n_e}} \left\{ (1 - \vec{\zeta}_e \vec{v}_e)(1 - \vec{\beta}_\nu \vec{v}_e) + \sqrt{1 - v_e^2} \left[\vec{\zeta}_e \vec{\beta}_\nu - \frac{(\vec{\beta}_\nu \vec{v}_e)(\vec{\zeta}_e \vec{v}_e)}{1 + \sqrt{1 - v_e^2}} \right] \right\} + O(\gamma_\nu^{-1}),}$$

$\vec{\beta}_\nu$ – neutrino speed , $\gamma_\nu = \dfrac{1}{\sqrt{1-\beta_\nu^2}}$

$\vec{v}_e$ – speed of matter (electrons)

$\vec{\zeta}_e$ – mean polarization vector of electrons in r.f. of matter

$$n_e = \frac{n_e^{(0)}}{\sqrt{1 - v_e^2}}$$

$n_e^{(0)}$ ← invariant number density (in r.f. of matter)

If $v_e \approx 0$ (and also $\zeta_e \approx 0$)
non-moving and unpolarized matter

$$E_{\nu_e} = p_0 + \underbrace{\sqrt{2}\, G_F\, n_e^{(0)}}$$ ← Wolfenstein matter term,

No matter term for ν_μ ($f = e, \cancel{\mu}, \cancel{\tau}$).

Important new phenomenon

dependence on matter total speed $\vec{v}_e$, polarization $\vec{\xi}_e$, neutrino speed $\vec{\beta}_\nu$, and correlations $(\vec{v}_e\vec{\beta}_\nu)$, $(\vec{v}_e\vec{\xi}_e)$, $(\vec{\beta}_\nu\vec{\xi}_e)$

Probability of oscillations (in adiabatic limit)

$$\boxed{\begin{aligned} P_{\nu_e\leftrightarrow\nu_\mu}(x) &= \sin^2 2\theta_{eff}\,\sin^2\frac{\pi x}{L_{eff}}, \\ \sin^2 2\theta_{eff} &= \frac{\Delta^2\sin^2 2\theta}{(\Delta\cos 2\theta - \underline{\underline{A}})^2+\Delta^2\sin^2 2\theta}, \\ L_{eff} &= 2\pi\left((\Delta\cos 2\theta-\underline{\underline{A}})^2+\Delta^2\sin^2 2\theta\right)^{-1/2}, \end{aligned}}$$

$$\Delta = \frac{\delta m_\nu^2}{2|\vec{p}|}, \quad \delta m_\nu^2 = m_2^2 - m_1^2$$

$$A = \sqrt{2}\,G_F\frac{n_e^{(0)}}{\sqrt{1-\underline{\underline{v_e^2}}}}\Big\{(1-\underline{\underline{\vec{\beta}_\nu}}\,\underline{\underline{\vec{v}_e}})(1-\underline{\underline{\vec{\xi}_e}}\,\underline{\underline{\vec{v}_e}}) + \sqrt{1-\underline{\underline{v_e^2}}}\Big[\underline{\underline{\vec{\xi}_e}}\,\underline{\underline{\vec{\beta}_\nu}} - \frac{(\underline{\underline{\vec{\beta}_\nu}}\,\underline{\underline{\vec{v}_e}})(\underline{\underline{\vec{\xi}_e}}\,\underline{\underline{\vec{v}_e}})}{1+\sqrt{1-\underline{\underline{v_e^2}}}}\Big]\Big\}$$

The resonance condition in moving and polarized matter

Mikheyev, Smirnov, Wolfenstein 1978, 1985

$$\frac{\delta m_\nu^2}{2|\vec{p}|}\cos 2\theta = A,$$

$$A = \sqrt{2}\, G_F \frac{n_e^{(0)}}{\sqrt{1-v_e^2}} \left\{ (1-\vec{\beta}_\nu \vec{v}_e)(1-\vec{\xi}_e \vec{v}_e) + \right.$$

$$\left. + \sqrt{1-v_e^2}\left[\vec{\xi}_e \vec{\beta}_\nu - \frac{(\vec{\beta}_\nu \vec{v}_e)(\vec{\xi}_e \vec{v}_e)}{1+\sqrt{1-v_e^2}} \right] \right. .$$

The resonance in $\nu_e \leftrightarrow \nu_\mu$ can occur for moving matter even if for given δm_ν^2, $|\vec{p}|$, θ, and $n_e^{(0)}$ the resonance in non-moving matter is impossible

$$\frac{\delta m_\nu^2}{2|\vec{p}|}\cos 2\theta \neq \sqrt{2}\, G_F\, n_e^{(0)} .$$

Unpolarized but moving matter

$$(\vec{\zeta}_e = 0,\ v_e \neq 0)$$

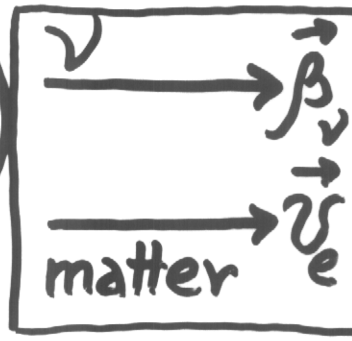
Resonance condition:

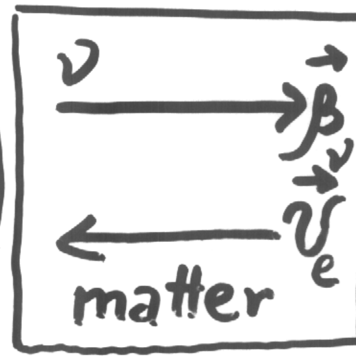
- $$\frac{\delta m_\nu^2}{2|\vec{p}|}\cos 2\theta = \sqrt{2}\, G_F\, n_0 \frac{1-\vec{\beta}_\nu \vec{v}_e}{\sqrt{1-v_e^2}} \, .$$

invariant matter density in r.f.

If $\longleftrightarrow \vec{\beta}_\nu$, $\longleftrightarrow \vec{v}_e$:

- $$\left.\frac{1-\vec{\beta}_\nu \vec{v}_e}{\sqrt{1-v_e^2}}\right|_{\beta_\nu \approx 1} = \begin{cases} \left.\sqrt{\dfrac{1-v_e}{1+v_e}}\right|_{v_e \approx 1} \approx \dfrac{\sqrt{1-v_e}}{\sqrt{2}} \\[3ex] \left.\sqrt{\dfrac{1+v_e}{1+v_e}}\right|_{v_e \approx 1} \approx \dfrac{\sqrt{2}}{\sqrt{1-v_e}} \end{cases}$$





Relativistic motion of matter along (against) neutrino propagation could provide resonance in $\nu \leftrightarrow \nu$ if matter density n_0 is too high (low) for resonance appearance in non-moving matter.

Matter polarization effect ($\xi_e \neq 0$, $v_e \neq 0$)

$$A = V^V + V^A ,$$

$$V^A = \sqrt{2}\, G_F\, n_0 \left\{ \vec{\xi}_e \vec{\beta}_\nu - \frac{(1-\vec{\beta}_\nu \vec{v}_e)(\vec{\xi}_e \vec{v}_e)}{\sqrt{1-v_e^2}} - \frac{(\vec{\beta}_\nu \vec{v}_e)(\vec{\xi}_e \vec{v}_e)}{1-\sqrt{1-v_e^2}} \right\}$$

- *Non-moving matter* ($v_e \approx 0$):

$$V^A = \sqrt{2}\, G_F\, n_0 (\vec{\xi}_e \vec{\beta}_\nu).$$

H. Nunokawa, V. Semikoz, A. Smirnov, J. W. F. Valle, 1997

- The same result for transversal matter polarization $(\vec{\xi}_e \vec{v}_e) = 0$ and arbitrary (direction and value) speed v_e $(0 \leq v_e < 1)$, also for $v_e \sim 1$:

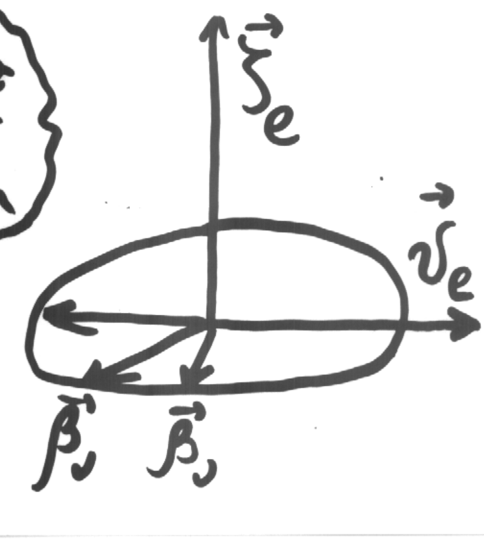
⊛ $\dfrac{dV^A}{dv_e} = 0$,

Lorentz invariant formalizm

⊛ $V^A = 0$.

only if $(\vec{\xi}_e \vec{\beta}_\nu) = 0$

not for any directions of $\vec{\beta}_\nu$



Critical dependence of V^A on correlations between unite vectors

$$\vec{k}_\nu = \frac{\vec{\beta}_\nu}{\beta_\nu} \longleftarrow \text{neutrino propagation}$$

$$\vec{k}_e = \frac{\vec{v}_e}{v_e} \longleftarrow \text{motion of matter.}$$

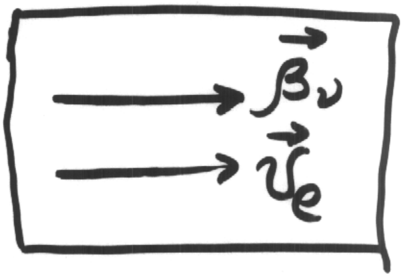
If $(\vec{k}_\nu, \vec{k}_e) = \pm 1$, $\longleftrightarrow \vec{\beta}_\nu$, $\longleftrightarrow \vec{v}_e$:

$$\bullet\ V^A = \sqrt{2}\, G_F n_0 (\vec{\xi}_e \vec{k}_\nu) \begin{cases} \sqrt{\dfrac{1-v_e}{1+v_e}}, & \xrightarrow{\vec{\beta}} \ \xrightarrow{\vec{v}_e} \\[2ex] \sqrt{\dfrac{1+v_e}{1-v_e}}, & \xrightarrow{\vec{\beta}} \ \xleftarrow{\vec{v}_e} \end{cases}$$

Sufficient increase (decrease) of matter polarization effect for relativistic ($v_e \sim 1$) motion of matter against (along) neutrino propagation

Total neutrino potential $A = \Delta V_{\nu_e \nu_\mu} = V^V + V^A$

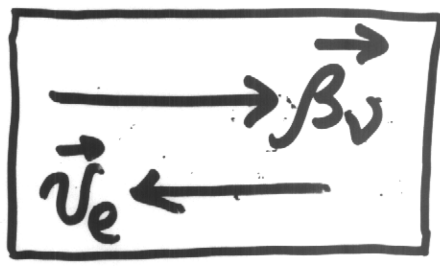
$$\vec{k}_\nu \vec{k}_e = 1$$



$$* \; A = \sqrt{2}\, G_F\, n_0 \sqrt{\frac{1-v_e}{1+v_e}} \left(1 + \vec{\xi}_e \overset{\vec{k}_e}{\vec{k}_\nu}\right),$$

- potential is suppresed in relativistic case $v_e \sim 1$ and
- is zero for complete polarization of matter $(\vec{\xi}_e \vec{k}_e) = -1$ (left-h.)

$$\vec{k}_\nu \vec{k}_e = -1$$



$$* \; A = \sqrt{2}\, G_F\, n_0 \sqrt{\frac{1+v_e}{1-v_e}} \left(1 + \vec{\xi}_e \overset{-\vec{k}_e}{\vec{k}_\nu}\right),$$

- potential is increased in relativistic case $v_e \sim 1$ and
- is zero for complete longitudinal polarization of matter $(\vec{\xi}_e \vec{k}_e) = \underset{(r.-h.)}{1}$.

Conclusions

* Lorentz invariant formalizm for neutrino flavour oscillations $\nu_e \leftrightarrow \nu_{e'}$

* Matter motion and polarization shift ν resonance condition of MSW effect
  - increase (decrease) of matter effect in $\nu_e \leftrightarrow \nu_{e'}$ for relativistic motion of matter against (along) ν propag.
  - critical dependence of matter effect on matter polarization:

    $A=0$ for complete l.h. $(\vec{v}_e \uparrow\downarrow \vec{\xi}_e)$ polarization if $\vec{v}_e \uparrow\uparrow \vec{\beta}_\nu$,

    $A=0$ for r.h. $(\vec{v}_e \uparrow\uparrow \vec{\xi}_e)$ polarization if $\vec{v}_e \uparrow\downarrow \vec{\beta}_\nu$

* Different types of $\nu_e \leftrightarrow \nu_{e'}$ in matter composed of $f = e, p, n, \mu, \gamma, \ldots$

* New phenomena in astrophysics possibilities

$\nu_e \leftrightarrow \nu_{\text{sterile}}$ in matter $f = e, p, n$, $\vec{v}_e$, $n_e^{(0)}$, $\vec{v}_p$, $n_p^{(0)}$, $\vec{v}_n$, $n_n^{(0)}$

$(\vec{\xi}_i = 0)$

$\vec{v}_i$ – speeds of matter components,

$n_i^{(0)}$ – invariant number densities.

Resonance condition: $\frac{\delta m_\nu^2}{2|\vec{p}|}\cos 2\theta = A_\nu$, effective potential

$$A = \frac{G_F}{\sqrt{2}}\left\{ \frac{n_e^{(0)}(1+4\sin^2\theta_W)}{\sqrt{1-v_e^2}}(1-\vec{\beta}_\nu\vec{v}_e) + \frac{n_p^{(0)}(1-4\sin^2\theta_W)}{\sqrt{1-v_p^2}}(1-\vec{\beta}_\nu\vec{v}_p) + \frac{n_n^{(0)}}{\sqrt{1-v_n^2}}(1-\vec{\beta}_\nu\vec{v}_n)\right\}$$

$A = \sqrt{2}G_F \times (n_e - \frac{1}{2}n_n)$ for $v_i = 0$, $n_e^0 = n_p^0$ neutral matter

if $\vec{v}_e \neq \vec{v}_p$ then NC ν_e scattering on e, p does not cancel even for $n_e^{(0)} = n_p^{(0)}$.