The Muon $g-2$ Revisited

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$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c}\vec{s}\,, \qquad \text{and} \qquad \underbrace{g_\mu = 2}_{\text{Dirac}}(1+a_\mu)\,.$$

$$a_\mu = \frac{1}{2}(g_\mu - 2) \quad : \text{anomalous magnetic moment}$$

World average experimental value:
(with the new BNL result included)

$$\boxed{a_\mu(\text{exp.}) = 11\ 659\ 202.3(15.1)\times 10^{-10}\ [1.3\text{ppm}]}$$

Standard Model "prediction" (before November '01)

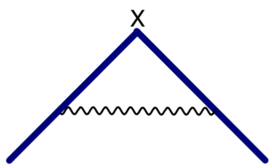
$$a_\mu^{\text{SM}} = (11\ 659\ 159.7 \pm 6.7)\times 10^{-10}\,.$$

- This 2.6σ "discrepancy" has triggered an *avalanche* of theroretical papers.

- Today I shall review the present status on the Standard Model prediction.

16th Rencontres de Physique de la Vallée d'Aoste, 5th March 2002

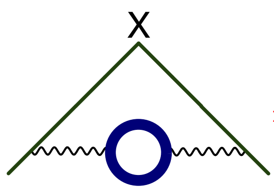
Some Theoretical Comments

- 

  $= a_\mu = a_e = \frac{1}{2}\frac{\alpha}{\pi}$ Schwinger '48

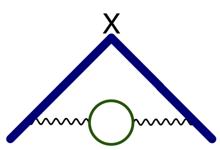
- Loops with *different masses* $\Rightarrow a_\mu \neq a_e$

  – Internal *LARGE* masses decouple:



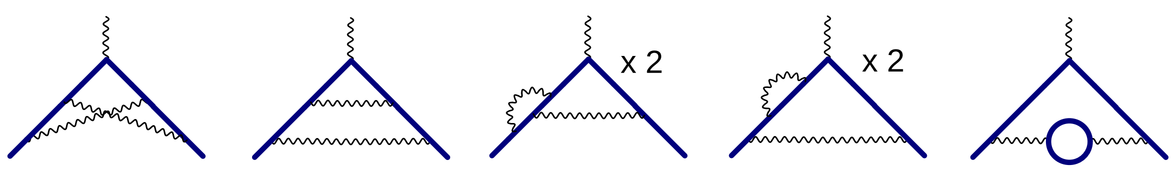
$$= \left[\frac{1}{45}\left(\frac{m_e}{m_\mu}\right)^2 + \mathcal{O}\left(\frac{m_e^4}{m_\mu^4}\log\frac{m_\mu}{m_e}\right)\right]\left(\frac{\alpha}{\pi}\right)^2 .$$

  – Internal *SMALL* masses give rise to log's of mass ratios:



$$= [\underbrace{\left(\frac{2}{3}\right)}_{\beta_1}\left(\frac{1}{2}\right)\log\frac{m_\mu}{m_e} - \frac{25}{36} + \mathcal{O}\left(\frac{m_e}{m_\mu}\right)]\left(\frac{\alpha}{\pi}\right)^2 .$$

- Two loops: *7 Feynman diagrams (with common fermion lines)*



$$a_l^{(4)} = \left\{\frac{197}{144} + \frac{\pi^2}{12} - \frac{\pi^2}{2}\ln 2 + \frac{3}{4}\zeta(3)\right\}\left(\frac{\alpha}{\pi}\right)^2$$

Peterman '57
Sommerfield '57

- Three loops: *72 Feynman diagrams (I can show you)* Laporta– Remiddi '96

- Four loops: *891 Feynman diagrams (I wan't show you)* Kinoshita, (in progress)

The Muon Anomaly

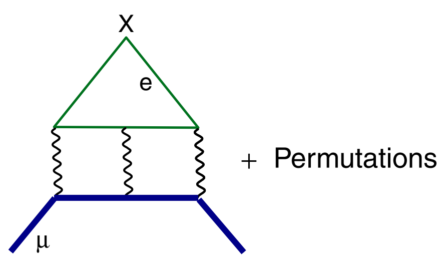
$$a_\mu = [a_e]_{\rm QED} + \underbrace{a_\mu(e,\tau) + a_\mu(\text{hadrons})}_{SU(3)\times SU(2)\times U(1)}$$

- Vacuum Polarization from electron loops
  - Enhanced by QED short–distance logarithms
  - Obey Renormalization Group Equation $[\alpha \Rightarrow \alpha(m_\mu)]$

$$\left(m_e\frac{\partial}{\partial m_e}+\beta(\alpha)\alpha\frac{\partial}{\partial\alpha}\right)a_\mu^{(\infty)}\left(\frac{m_\mu}{m_e},\alpha\right)=0$$

Lautrup–de Rafael '74

- Light–by–Light Scattering from electron loops



  - Enhanced by QED infrared logarithms

Kinoshita *et al* '69
Laporta–Remiddi '93

$$a_\mu^{(3)}|_{\rm l.byl.} = \left[\frac{2}{3}\pi^2 \ln\frac{m_\mu}{m_e} + \cdots\right]\left(\frac{\alpha}{\pi}\right)^3 = 20.947\ldots\left(\frac{\alpha}{\pi}\right)^3$$

- Vacuum Polarization and Light–by–Light Scattering from tau loops, suppressed by Mass Decoupling (but explicitly known)

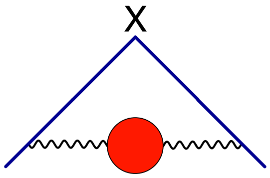
$$a_\mu(\text{ QED}) = (11\ 658\ 470.57 \pm 0.29) \times 10^{-10}$$

Recall $a_\mu(\text{ Exp.}) = (11\ 659\ 202.3 \pm 15.1) \times 10^{-10}$

Is this discrepancy due to Hadronic–Electroweak SM Interactions only ?

Hadronic Vacuum Polarization

- All calculations are based on the spectral representation



$$a_\mu^{(\text{h. v.p.})} = \frac{\alpha}{\pi}\int_0^\infty \frac{dt}{t}\frac{1}{\pi}\text{Im}\Pi(t)\int_0^1 \frac{x^2(1-x)}{x^2+\frac{t}{m_\mu{}^2}(1-x)}dx$$

$$\sigma(t)_{e^+e^-\to\ \text{hadrons}} = \frac{4\pi^2\alpha}{t}\frac{1}{\pi}\text{Im}\Pi(t)$$

- Compilation from Recent Estimates:

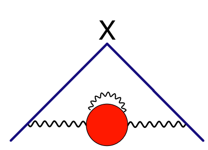
Authors	**Contribution to** $a_\mu \times 10^{10}$
Davier–Höcker	692.4 ± 6.2
Jegerlehner	697.4 ± 10.5
Narison	703.1 ± 7.7
de Trocóniz–Ynduráin (v5)	695.2 ± 6.4

- Higher Order Hadronic Vacuum Polarization

  Calmet, Narison, Perrottet, de Rafael '77; Krause '97

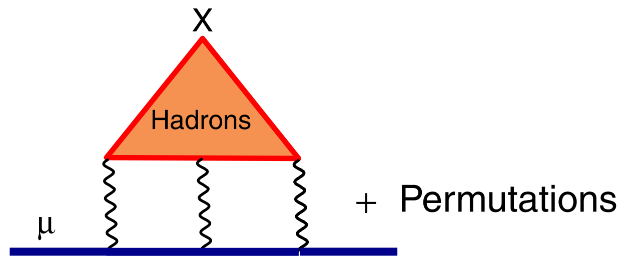
$$a_\mu^{(\text{h.o.-h. v.p.})} = -10.0(0.6) \times 10^{-10}$$

- Disagreement, at present, on Hadronic EM self–energy estimates:



Knecht, Nyffeler, de Rafael (work in progress)

Hadronic Light–by–Light Scattering



- All Estimates (so far) are model dependent
- The most recent estimates use models *compatible* with low–energy χPT behaviour and Large–N_c counting rules (de R '94)
  - ENJL–model Bijnens, Pallante, Prades '96

$$a_\mu^{(\text{h. l. by l.})} = (-9.2 \pm 3.2) \times 10^{-10}$$

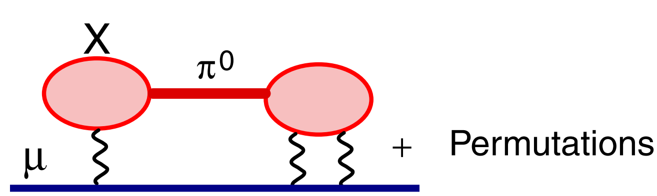
  - Vector Gauge Model Hayakawa – Kinoshita '98

$$a_\mu^{(\text{h. l. by l.})} = (-7.9 \pm 1.5) \times 10^{-10}$$

  - Minimal Hadronic Approximation to Large–N_c QCD Knecht–Nyffeler '01

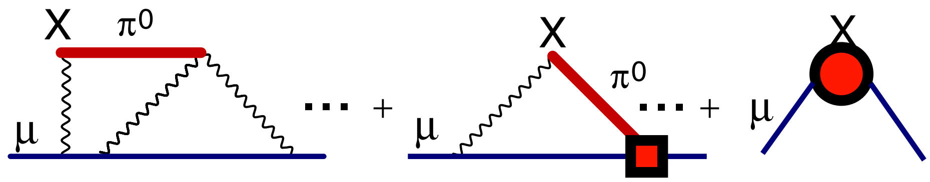
$$\boxed{a_\mu^{(\text{h. l. by l.})} = (+8.1 \pm 3.0) \times 10^{-10}}$$

- In these calculations, the dominant contribution comes from twice the anomalous VVP vertex



Effective Field Theory Approach

Knecht–Nyffeler–Perrottet–de Rafael '01



$$\underline{\underline{h_2 \log^2\left(\frac{m}{\mu}\right) + h_1 \log\frac{m}{\mu} + h_0}} + \underline{\chi(\mu)\left(j_1 \log\frac{m}{\mu} + j_0\right)} + \kappa(\mu)$$

$$\mu\frac{d}{d\mu}(\cdots) = 0 \Rightarrow \begin{cases} 2h_2 - j_1\gamma_\chi = 0 \\ \mu\frac{d}{d\mu}\kappa(\mu) + j_0\gamma_\chi - \chi(\mu)j_1 - h_1 = 0 \end{cases}$$

- Here, $\gamma_\chi = \mu\frac{d}{d\mu}\chi(\mu) = N_c$, known from $\pi^0 \to e^+e^-$ calculation
  Knecht, Peris, Perrottet, de Rafael '99
- To extract the $\log^2$ term, only j_1 needed; (calculation analogous to the $\gamma - Z$ contribution to $g_\mu - 2$; Peris, Perrottet, de Rafael '95)

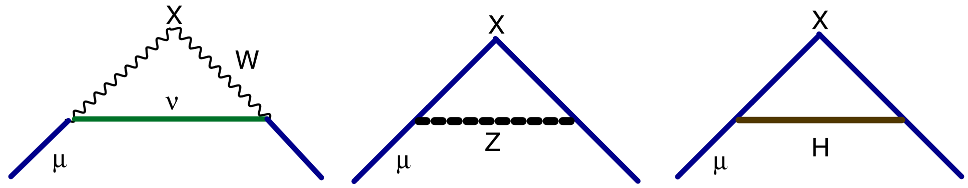
$$h_2 = \frac{1}{2}\gamma_\chi j_1 \quad \text{and} \quad j_1 = \left(\frac{\alpha}{\pi}\right)^3 \frac{N_c}{24\pi^2}\frac{m^2}{F^2}$$

- Also, $\kappa(\mu) = \frac{1}{2}\gamma_\chi j_1 \log^2\left(\frac{\mu}{\mu_0}\right)$
- Therefore,

$$a_\mu^{(\pi^0)} = \left(\frac{\alpha}{\pi}\right)^3 \left\{\frac{N_c^2}{48\pi^2}\frac{m^2}{F^2}\log^2\left(\frac{\mu_0}{m}\right) + \mathcal{O}\left[\log\left(\frac{\mu_0}{m}\right)\right]\right\} .$$

Weak Interactions

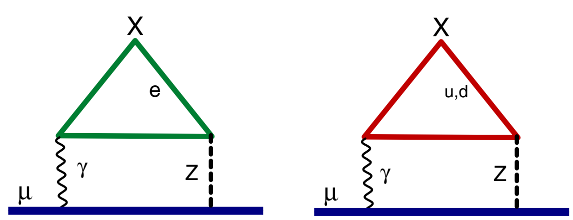
- One Loop



$$a_\mu^{(\mathrm{W})} = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left[\frac{5}{3} + \frac{1}{3}(1 - 4\sin^2\theta_W) + \mathcal{O}\left(\frac{m_\mu^2}{M_Z^2} \ln \frac{M_Z^2}{m_\mu^2}, \frac{m_\mu^2}{M_H^2} \ln \frac{M_H^2}{m_\mu^2} \right) \right]$$

$$= 19.48 \times 10^{-10}$$ Bardeen–Gastmans–Lautrup 72, (also others...)

- Electroweak Corrections to Two Loops
  - Rather large because $\sim G_F m_\mu^2 \frac{\alpha}{\pi} \times \ln \frac{M}{m_l}$ terms
  - Earlier estimate (Kukhto *et al*) incomplete because of separation between LEPTONS and QUARKS no longer possible
    Peris–Perrottet–de Rafael '95



  - Full Two–Loop Calculation Czarnecki–Krause–Marciano '96

$$a_\mu^{[\mathrm{EW}(2)]} = (-4.3 \pm \underbrace{0.4}) \times 10^{-10}\,.$$

  - Improvement on Hadronic Electroweak Calculation
    Knecht–Peris–Perrottet–de Rafael '02

$$a_\mu^{[\mathrm{EW}(2)]} = (-4.5 \pm 0.2) \times 10^{-10}\,.$$

Summary of Contributions

After Knecht–Nyffeler's hadronic light-by-light calculation

- Leptonic QED contributions

$$a_{\text{QED}}(\mu) = 11\ 658\ 470.57 \pm 0.29 \times 10^{-10} \quad * \ * \ *$$

- Hadronic Contributions
  - Vacuum Polarization

$$a^{(\text{VP})}_{\text{hadronic}} = [692.4 \pm \underbrace{6.2}_{?} - (10.0 \pm 0.6)]) \times 10^{-10} \quad *$$

  - Light–by–Light

$$a^{(\text{light by light})}_{\text{hadronic}} = \underbrace{(8.1 \pm 3.0)}_{\text{work in progress}} \times 10^{-10} \quad * \ *$$

- Electroweak Contributions

$$a_{\text{EW}} = (15.0 \pm 0.2) \times 10^{-10} \quad * \ *$$

- Total Standard Model Contribution

$$a_{\mu}^{\text{SM}} = (11\ 659\ 176.1 \pm 6.9) \times 10^{-10}$$

$$a_{\mu}^{\text{exp}} = (11\ 659\ 202.3 \pm 15.1) \times 10^{-10}$$

$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (26.2 \pm 16.6) \times 10^{-10} \quad 1.6\sigma$$