THREE NEUTRINO MIXING AND OSCILLATIONS

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Convincing evidences of neutrino oscillations

in atmospheric and solar neutrino experiments

LSND indications requres confirmation; MiniBooNE in Fermilab started in 2002

Basics of neutrino mixing and oscillations

- Interaction of neutrinos with matter
  the CC and NC Standard Model interaction

$$j_\alpha^{\mathrm{CC}} = \sum_l \bar{\nu}_{lL} \gamma_\alpha l_L$$

$$j_\alpha^{\mathrm{NC}} = \sum_l \bar{\nu}_{lL} \gamma_\alpha \nu_{lL}$$

- CC interaction determine the *notion of flavor neutrinos and antineutrinos*

  in $\pi^+ \to \mu^+ + \nu_\mu$ the left-handed muon neutrino ν_μ is produced etc

- *Three flavor neutrinos* ν_e, ν_μ, ν_τ *exist in nature.*

  From the LEP measurement of the width of the decay $Z \to \nu + \bar{\nu}$

$$n_{\nu_f} = 3.00 \pm 0.06$$

Neutrino mixing

$$\nu_{lL} = \sum_i U_{li} \nu_{iL}$$

$U^\dagger \; U = 1$, ν_i is the field of neutrino with mass m_i.

- Neutrino mixing is different from quark mixing

  In the quark case hierarchy of couplings

$$\sin\theta_{12} = \lambda \simeq 0.22; \;\; \sin\theta_{23} \simeq \lambda^2; \;\; \sin\theta_{13} \simeq \lambda^3$$

  hierarchy of masses

$$m_d \ll m_s \ll m_b$$

In the neutrino case

$$\sin\,\theta_{23} \simeq 1;\;\; \sin\,\theta_{12} \simeq 1;\;\; |U_{e3}| = \sin\,\theta_{13} \ll 1$$

hierarchy of mass squared differences

$$\Delta m^2_{21} \ll \Delta m^2_{31}$$

the minimal mass m_1 is unknown

From^3H experiments upper bound $m_1 \leq (2.2 - 2.5)$ eV

hierarchy of masses (?)

Neutrino mixing can be fundamentally different from quark mixing

Quarks are charged Dirac particles

- For neutrinos two possibilities

  ν_i can be **Dirac particles**

  L(ν_i)=1,   L($\bar{\nu}_i$)= -1

  ν_i can be **Majorana particles**

  $\nu_i \equiv \bar{\nu}_i$

- The number of the massive neutrinos can be larger than three

  The mixing in this case

$$\nu_{lL} = \sum_{i=1}^{3+n_s} U_{li}\nu_{iL}$$

$$\nu_{sL} = \sum_{i=1}^{3+n_s} U_{si}\nu_{iL}$$

  n_s is the number of *sterile fields* (do not enter into CC and NC)

  $(\nu_{sL} = (\nu_{sR})^c \cdots)$

  signature: transition of flavor neutrinos into sterile states

  Different ways to search for such transitions

  $\sum_{l'=e,\mu,\tau} P(\nu_l \to \nu_{l'}) < 1$ requires NC measurements;

  more than two neutrino mass-suqred differences are necessary to describe data (the case if LSND is confirmed)

  matter effects.

- *Explanation of small neutrino masses requires new physics*

  See-saw. Lepton number is violated at a large scale $M \gg M_{EW}$

$$m_i \simeq \frac{(m_f^i)^2}{M} \ll m_f^i$$

  ν_i are *Majorana particles.*

  Hierarchy

$$m_1 \ll m_2 \ll m_3$$

  Heavy Majorana particles with mass $\simeq M$ must exist

  (a possible explanation of the barion asymmetry of the Universe).

  Smallness of neutrino masses can be due to **large extra dimentions**

  ν_i are *Dirac particles*

Neutrino oscillations

In case of neutrino mixing, the state of flavor neutrino with momentum $\vec{p}$ is a *coherent* state

$$|\nu_l>=\sum U_{li}^*|\nu_i>$$

$|\nu_i>$ is the state with momentum $\vec{p}$

energy $E_i=\sqrt{m_i^2+p^2}\simeq p+\frac{m_i^2}{p}$.

Transition probability

$$\mathrm{P}(\nu_\alpha\to\nu_{\alpha'})=|\delta_{\alpha'\alpha}+\sum_i U_{\alpha' i}U_{\alpha i}^*\;(e^{-i\Delta m_{i1}^2\frac{L}{2E}}-1)|^2$$

$$\Delta m_{i1}^2=m_i^2-m_1^2.$$

- depends on $\frac{L}{E}$
- oscillations can be observed if $\Delta m_{i1}^2\frac{L}{E}\gtrsim 1$ at least for one i

Varieties of $\frac{L}{E}$

$1-10^3$ (accelerators), 10^2-10^5 (reactors), $10-10^4$ (atmospheric neutrinos), $10^{10}-10^{11}$ (solar neutrinos)

Neutrino oscillations were first considered by *B.Pontecorvo* in 1958. Only one type neutrino was known at that time. Considered $\nu_e \to \bar{\nu}_{eL}$ (sterile state). Considered neutrino oscillations as lepton analogy of $K^0 \to \bar{K}^0$. In 1967 generalized idea oscillations for $\nu_\mu \to \nu_e$, predicted 1/2 suppression of the flux of solar ν_e etc.

In 1962 *Maki, Nakagawa, Sakata* proposed mixing of two neutrinos. Mentioned possibility of $\nu_\mu \to \nu_e$ transitions.

Oscillations between two types of neutrinos

$$\nu_\mu \to \nu_\tau \text{ or } \nu_\mu \to \nu_e \text{ etc}$$

$$\mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) = |\delta_{\alpha'\alpha} + U_{\alpha' 2} U^*_{\alpha 2} \ \ (e^{-i\Delta m^2 \frac{L}{2E}} - 1)|^2$$

$$\Delta m^2 = m_2^2 - m_1^2.$$

Appearence probability $(\alpha \neq \alpha')$

$$\mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) = \mathrm{P}(\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'}) = \frac{1}{2}\mathrm{A}_{\alpha';\alpha} \ \ (1 - \cos \Delta m^2 \frac{L}{2E})$$

The disappearence probability

$$\mathrm{P}(\nu_\alpha \to \nu_\alpha) = \mathrm{P}(\bar{\nu}_\alpha \to \bar{\nu}_\alpha) = 1 - \frac{1}{2}\mathrm{B}_{\alpha;\alpha} \ \ (1 - \cos \Delta m^2 \frac{L}{2E})$$

From the unitarity of the mixing matrix

$$\mathrm{B}_{\alpha;\alpha} = \mathrm{B}_{\alpha';\alpha'} = \mathrm{A}_{\alpha';\alpha} = \sin^2 2\theta$$

Evidence in favor of oscillations of atmospheric neutrinos

Produced in decays

$$\pi \to \mu + \nu_\mu \quad \mu \to e + \nu_\mu + \nu_e$$

Super-Kamiokande experiment

Water Cherenkov detector (50 ktons of water)

e and μ are detected

Cherenkov light was detected by 11200 photomultipliers

Large $\cos\theta_z$ *asymmetry of the high energy muon events was observed*

θ_z is zenith angle

If no neutrino oscillations

$$N_l(\cos\theta_z) = N_l(-\cos\theta_z) \quad (l = e, \mu)$$

For electron events a good agreement with the prediction

For the muon events in the Multi-Gev region
($E \geq 1.3$ GeV)

a significant $\cos\theta_z$ asymmetry

Fig

up/down asymmetry

$$\left(\frac{U}{D}\right)_{\mu} = 0.54 \pm 0.04 \pm 0.01\,.$$

For the high-energy neutrinos $\cos\theta_z$ is determined by the distance L

U is the the total number of the up-going muons (13000 -500 km)

D is the the total number of the down-going muons (20 - 500 km)

The S-K data are perfectly described by the *two-neutrino* $\nu_\mu \to \nu_\tau$ *oscillitations.*

Best-fit values from the combined fit

$$\Delta m^2_{\rm atm} = 2.5 \cdot 10^{-3}{\rm eV}^2 \quad \sin^2 2\theta_{\rm atm} = 1$$

$$(\chi^2_{min} = 142.1 \text{ at } 152 \text{ d.o.f.})$$

Evidence of oscillations of solar neutrinos

In all solar neutrino experiments the observed event rates are significantly smaller than the predicted rates

HOMESTAKE: $R = 0.34 \pm 0.03.$

GALLEX-GNO: $R = 0.58 \pm 0.05.$

SAGE : $R = 0.60 \pm 0.05.$

S-K : $R = 0.459 \pm 0.017.$

SNO: $R = 0.35 \pm 0.03.$

Comparison of the results of the SNO and the S-K experiments

In the SNO experiment

heavy water Cherenkov detector (1 kton of D_2O)

Solar neutrino are detected via

CC reaction

$$\nu_e + d \to e^- + p + p$$

elastic scattering ES

$$\nu + e \to \nu + e$$

now NC reaction

$$\nu + d \to \nu + n + p$$

During about one year 975.4 ± 39.7 CC events and 106.1 ± 15.2 ES events

Threshold T_{th}=6.75 MeV

Neutrinos from $^8B \to ^8 Be^* + e^+ + \nu_e$ are detected

Spectrum of ν_e known

ES event rate in agreement with more precise S-K event rate

No distortion of the spectrum of electrons

Event rate

$$R^{CC} = < \sigma_{\nu_e d} > \Phi^{CC}_{\nu_e}$$

$< \sigma_{\nu_e d} >$ is CC cross section averaged over initial spectrum of ^{8}B neutrinos (known)

$\Phi^{CC}_{\nu_e}$ is the flux of ν_e on the earth (unknown)

The SNO measured flux

$$(\Phi^{CC}_{\nu_e})_{SNO} = (1.75 \pm 0.07 \pm 0.12 \pm 0.05(\text{theor})) \cdot 10^6 \ \text{cm}^{-2}\text{s}^{-1}$$

In *S-K experiment* neutrinos are detected via

$$\nu + e \to \nu + e$$

during about 3.5 years

$18464 \begin{array}{c} +677 \\ -590 \end{array}$ events were observed

Energy threshould 5 MeV (^{8}B neutrinos)

No distortion of the recoil electron spectrum, no D/N asymmetry

All flavor neutrinos ν_e, ν_μ and ν_τ are detected

Sensitivity to ν_μ and ν_τ is lower than the sensitivity to ν_e

$\sigma_{\nu_{\mu,\tau} e} \simeq \frac{1}{6} \sigma_{\nu_e e}$

The event rate

$$R^{ES} = < \sigma_{\nu_e e} > \Phi_{\nu}^{ES}$$

The flux

$$\Phi_{\nu}^{ES} = \Phi_{\nu_e}^{ES} + \frac{< \sigma_{\nu_\mu e} >}{< \sigma_{\nu_e e} >} \sum_{l=\mu,\tau} \Phi_{\nu_l}^{ES}$$

$\Phi_{\nu_e}^{ES}$ is the flux of ν_e on the earth

$\sum_{l=\mu,\tau} \Phi_{\nu_l}^{ES}$ is the flux of $\nu_{\mu,\tau}$ on the earth

S-K measured flux

$$(\Phi_{\nu}^{ES})_{SK} = (2.32 \pm 0.03 \pm 0.08) \cdot 10^6 cm^{-2} s^{-1}$$

From the SNO and S-K fluxes

$$\sum_{l=\mu,\tau} \Phi_{\nu_l}^{ES} = (3.69 \pm 1.13) \cdot 10^6 \ \ cm^{-2} \, s^{-1}$$

Model independent evidence (at 3σ level) of the presence of $\nu_{\mu,\tau}$ in the flux of solar neutrinos on the earth

Remarks

- The total flux of the flavor neutrinos

$$\sum_{l=e,\mu,\tau} \Phi_{\nu_l}^{ES} = (5.44 \pm 0.99) \cdot 10^6 \;\; cm^{-2}\, s^{-1}$$

  Compatible with SSM BP00 value

$$(\Phi_{\nu_e}^0)_{SSM} = (5.93 \pm 0.89) \cdot 10^6 \;\; cm^{-2}.$$

- The probability of ν_e to survive

$$\mathrm{P}(\nu_{\mathrm{e}} \to \nu_{\mathrm{e}}) = 0.32 \pm 0.08$$

From the global analysis of the event rates measured in all experiments

several allowed regions in the plane of oscillation parameters

matter enhanced MSW LMA. LOW, SMA,....

vacuum VO,...

After S-K measurement of recoil electron spectra during days and nights

large exclusion regions (no distortion)Fig

The most plausible allowed region is LMA

best-fit values

$$\Delta m^2_{\rm sol} = 3.7 \cdot 10^{-5}\,{\rm eV}^2; \quad \tan^2\theta_{\rm sol} = 3.7 \cdot 10^{-1}$$

SNO result strengthen this conclusion

Implications of SNO and Homestake for BOREXINO

Main contribution to the Homestake event rate

$$R_{Cl} = (2.56 \pm 0.16 \pm 0.16) \quad \text{SNU}$$

^8B and ^7Be neutrinos

according to SSM 5.9 SNU and 1.15 SNU

some contribution from CNO and pep neutrinos (0.7 SNU)

Using the SNO measured flux of ^8B neutrinos

Contribution of ^8B neutrinos

$$(R^{^8B}_{Cl})_{SNO} = \int_{E_{th}} \sigma_{\nu_e Cl}(E)\; X^{^8B}(E)\; \Phi^{SNO}_{\nu_e}\, dE = (2.00{\pm}0.17)\,\text{SN}$$

Contribution of ^7Be, CNO and pep neutrinos

$$R_{Cl}^{^7Be,CNO,pep} = (0.56 \pm 0.29)\ \text{SNU}$$

More that 4σ smaller than SSM prediction (1.8 SNU)

Flux of ^7Be neutrinos on the earth can be determineddetermined

for the calculation of the small contribution of CNO and pep neutrinos LMA (and LOW) solutions was used

The flux of ^7Be neutrinos on the earth from Homestake and SNO

$$\Phi_{\nu_e}^{^7Be} = (1.19 \pm 1.12) \cdot 10^9\,\text{cm}^{-2}\text{s}^{-1}$$

Much smaller than SSM BP00 prediction $((4.3 \pm 0.4)\ 10^9\,\text{cm}^{-2}\text{s}^{-1}\ )$

For the BOREXINO event rate from SNO and Homestake

$$R_{\text{Borexino}} = 24.4 \pm 8.9\ \text{events/day}$$

comparable with LMA prediction (30.7 events/day)

Oscillations in the range $\Delta m^2_{\mathrm{atm}}$ in the framework of three-neutrino mixing

From solar and atmospheric data

$$\Delta m^2_{21} \ll \Delta m^2_{31}$$

For $\frac{L}{E}$ in atmospheric and long baseline accelerator experiments

$$\Delta m^2_{21} \frac{L}{E} \ll 1$$

$$\mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) \simeq |\delta_{\alpha'\alpha} + U_{\alpha' 3} U^*_{\alpha 3} \;\; (e^{-i \Delta m^2_{31} \frac{L}{2E}} - 1)|^2$$

Transition probabilities have *two-neutrino form*

- describe oscillations $\nu_\mu \to \nu_\tau$, $\nu_\mu \to \nu_e$ and $\nu_e \to \nu_\tau$
- depend on three parameters Δm^2_{31}, $\tan^2\theta_{23}$, $|U_{e3}|^2$
- oscillation amplitudes

$$\mathrm{A}_{\tau;\mu} = (1 - |U_{e3}|^2)^2 \; \sin^2 2\theta_{23}$$

$$\mathrm{A}_{e;\mu} = 4\,|U_{e3}|^2\,(1 - |U_{e3}|^2)\; \sin^2\theta_{23}$$

The S-K data are well described if

$$|U_{e3}|^2 = 0$$

pure $\nu_\mu \to \nu_\tau$ oscillations $\mathrm{A}_{\tau;\mu} = \sin^2 2\theta_{23}$

From the three-neutrino analysis of the S-K data

$$|U_{e3}|^2 \leq 0.35$$

The most stringent bound on $|U_{e3}|^2$ from LBL reactor experiments CHOOZ and Palo Verde

No indications in favor oscillations were found in these experiments

From the exclusion plot

$\mathrm{B}_{e;e} = 4\,|U_{e3}|^2\,(1 - |U_{e3}|^2) \leq \mathrm{B}^0_{e;e}$

$$|U_{e3}|^2 \leq \frac{1}{2}\left(1 - \sqrt{1 - \mathrm{B}^0_{e;e}}\right) \simeq \frac{\mathrm{B}^0_{e;e}}{4}$$

or

$$|U_{e3}|^2 \geq \frac{1}{2}\left(1 + \sqrt{1 - \mathrm{B}^0_{e;e}}\right) \simeq 1 - \frac{\mathrm{B}^0_{e;e}}{4}$$

The second possibility is excluded by solar neutrino data ($\mathrm{P}(\nu_e \to \nu_e) \simeq 1$ in this case)

From CHOOZ exclusion curve at S-K best-fit value $\Delta m^2_{31} = 2.5\ \ 10^{-3}\text{eV}$

$$|U_{e3}|^2 \leq 3.7\ \ 10^{-2}$$

The value of $|U_{e3}|^2$ is very important for Super Beam and Neutrino Factory programs (CP violation in the lepton sector, spectrum mass of neutrinos etc)

A bound on $|U_{e3}|^2$ from three- neutrino analysis of the CHOOZ data

LMA is a wide region

$$2 \cdot 10^{-5} \lesssim \Delta m^2_{\rm sol} \lesssim 6 \cdot 10^{-4} {\rm eV}^2$$
$$0.4 \lesssim 1 \tan^2 \theta_{\rm sol} \lesssim 1$$

If $\Delta m^2_{\rm sol}$ in the upper part of the region the contribution of Δm^2_{21} term can be important

Three-neutrino ν_e survival probability

$$\begin{aligned} & P(\bar\nu_e \to \bar\nu_e) \\ = & \quad 1 - 2\,|U_{e3}|^2 \left(1 - |U_{e3}|^2\right) \left(1 - \cos\frac{\Delta m^2_{31}\, L}{2\,E}\right) \\ - & \quad \frac{1}{2}\,(1 - |U_{e3}|^2)^2 \sin^2 2\,\theta_{\rm sol} \left(1 - \cos\frac{\Delta m^2_{\rm sol}\, L}{2\,E}\right) \quad (1) \\ + & \quad 2\,|U_{e3}|^2\,(1 - |U_{e3}|^2)\,\sin^2\theta_{\rm sol}\,\times \\ \times & \left(\cos\left(\frac{\Delta m^2_{31}\, L}{2\,E} - \frac{\Delta m^2_{\rm sol}\, L}{2\,E}\right) - \cos\frac{\Delta m^2_{31}\, L}{2\,E}\right) \end{aligned}$$

(2)

At $\Delta m^2_{\rm sol} \leq 1 \cdot 10^{-4}{\rm eV}^2$ the CHOOZ bound on $|U_{e3}|^2$ does not depend on $\Delta m^2_{\rm sol}$

At larger $\Delta m^2_{\rm sol}$ more stringent bound

At $\Delta m^2_{\rm sol} = 6 \cdot 10^{-4}{\rm eV}^2$

$$|U_{e3}|^2 \leq 2 \;\; 10^{-2}$$

about two times smaller that two-neutrino bound

Fig

Neutrino oscillations in Δm^2_{sol} range in the framework of three-neutrino mixing

Probability of ν_e to survive

$$\mathrm{P}(\nu_e \to \nu_e) = \left| \sum_{i=1,2} |U_{ei}|^2 \, e^{-i\,\Delta m^2_{i1}\,\frac{L}{2E}} + |U_{e3}|^2 \, e^{-i\,\Delta m^2_{31}\,\frac{L}{2E}} \right|^2$$

because of hierarchy of mass squared differences

$$\Delta m^2_{21} \ll \Delta m^2_{31}$$

In averaged probability interference disappears

Survival probability in the three-neutrino case

$$\mathrm{P}(\nu_e \to \nu_e) = |U_{e3}|^4 + (1 - |U_{e3}|^2)^2 \;\; P^{(1,2)}(\nu_e \to \nu_e)$$

$P^{(1,2)}(\nu_e \to \nu_e)$ is the two-neutrino survival probability

(in vacuum or in matter)

depends on

$$\Delta m^2_{21} \quad \tan^2\theta_{12} \quad |U_{e3}|^2$$

Because

$$|U_{e3}|^2 \ll 1$$

$$\mathrm{P}(\nu_e \to \nu_e) \simeq P^{(1,2)}(\nu_e \to \nu_e)$$

Oscillations in $\Delta m^2_{\mathrm{sol}}$ *range are practically decoupled from oscillations in* $\Delta m^2_{\mathrm{atm}}$ range

If oscillation parameters are in LMA region the solar range of neutrino mass squared differences can be reached in reactor experiments with $L \simeq 100 km$

KamLAND started in January 2002

$\bar{\nu}_e$ from several reactors in Japan

the average distance 170 km

It is expected about 700 events/kt/year.

Fig

If LMA values will be confirmed

oscillation parameters will be measured in reactor experiments

New way of the investigation of the solar neutrino problem

In solar neutrino experiments solar fluxes from different reactions can be measured

CONCLUSION

Compelling evidences of neutrino oscillations

Atmospheric range of Δm^2

Will be checked in LBL experiments K2K, MINOS, OPERA, ICARUS,...

In more remote future Super Beams, Neutrino Factories

High accuracy in the determination of parameters $|U_{e3}|^2$,...

Investigation of the CP violation in the lepton sector, check of CPT, character of neutrino mass spectrum etc

Solar range of Δm^2

New solar experiments BOREXINO, LENS,...

KamLAND and other reactor experiments

- **How many massive neutrinos exist in nature?**

  If LSND will be confirmed, we need at least four

  MiniBooNE

- **What is the nature of the massive neutrinos. Are they Dirac or Majorana particles?**

  neutrinoless double β decay

  Effective Majorana mass

$$< m >= \sum_i U_{ei}^2 \, m_i$$

If neutrino mass hierarchy $| < m > | \leq 5 \cdot 10^{-3}\text{eV}$

Today's limit $| < m > | \leq (0.2 - 0.6)$ eV.

In future experiments (GENIUS, CUORE, MAJORANA, EXO,....)

$$| < m > | \simeq 10^{-2}\text{eV}$$

- **What is the value of the minimal neutrino mass m_1?**

  Today's bound from tritium experiments

  $$m_1 \leq (2.2 - 2.5)\,\text{eV}$$

  Future experiment KATRIN plan to reach

  $$m_1 \simeq (0.3 - 0.4)\ \text{eV}$$