Covariant treatment of neutrino oscillations in matter and electromagnetic fields

5/03/2001
La Thuile

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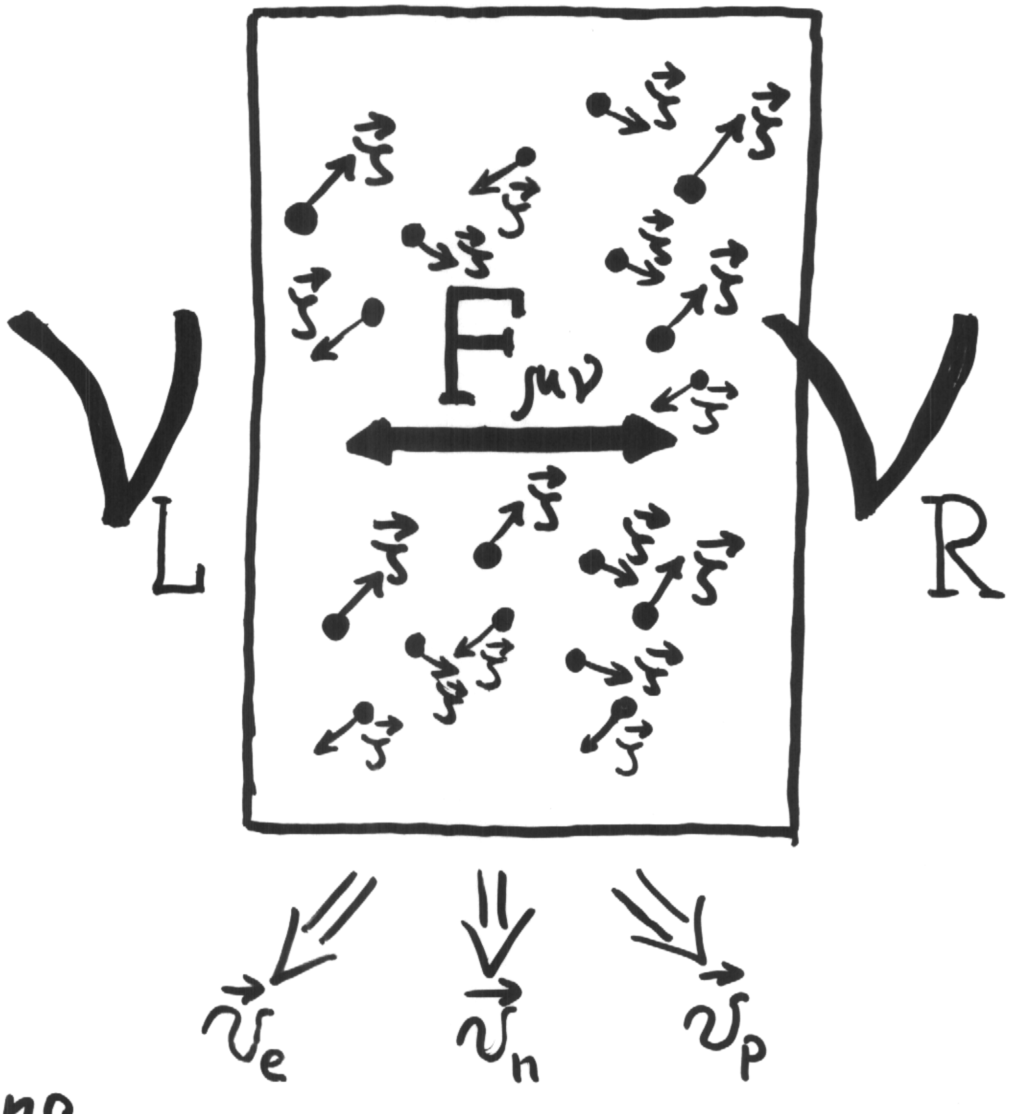

moving
matter components $f = e, n, p, \ldots$ with
polarizations $\vec{\xi}, \vec{\xi}, \vec{\xi}$

A. Egorov, A. Lobanov, A. Studenikin

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1. introduction

2. $\nu_L \leftrightarrow \nu_R$ in arbitrary e.m. field $F_{\mu\nu}$ in nonmoving and unpolarized matter

• 3. $\nu_L \leftrightarrow \nu_R$ in arbitrary $F_{\mu\nu}$ in moving and polarized matter

4. ν spin evolution equation

5. ν spin evolution within SM + SU(2) singlet ν_R in arbitrary $F_{\mu\nu}$ and matter

6. suppression of matter effects in the case of matter moving along propagation of ν

7. conclusion

[*Neutrino spin evolution in arbitrary electromagnetic field $F_{\mu\nu}$ and moving and polarized matter*

START

Bargmann-Michel-Telegdi equation for spin vector S_μ of neutral particle:

$$\frac{dS^\mu}{d\tau} = 2\mu\left[F^{\mu\nu}S_\nu - u^\mu(u_\nu F^{\nu\lambda}S_\lambda)\right] +$$

$$2\epsilon\underbrace{\left[\tilde{F}^{\mu\nu}S_\nu - u^\mu(u_\nu \tilde{F}^{\nu\lambda}S_\lambda)\right]}_{\text{T-}\cancel{\text{in}}\text{variance}}$$

magnetic
dipole moments
electric

- direct interaction of $\cancel{\nu}$ spin with $F_{\mu\nu}$
- P invariant theory

FINISH

Neutrino spin evolution equation for ν general interactions (e.g., P-~~in~~variant weak interactions) with moving and polarized matter

neutrino speed

$u_\mu = (\gamma, \gamma\vec{\beta})$, $\gamma = (1-\beta^2)^{-1/2}$, $\vec{\beta} = \text{const.}$, $S^2 = -1$, $u_\mu S^\mu = 0$

Lorentz invariant generalization of BMT eq. :

$$F_{\mu\nu} \longrightarrow F_{\mu\nu} + G_{\mu\nu}$$

ν interactions with moving and polarized matter

Evaluation of $G_{\mu\nu}$:

- ν evolution eq. has to be linear over S_μ , $F_{\mu\nu}$ and characteristics of matter

$$j_f^\mu = (n_f , n_f \vec{v}_f), \quad f = e, n, p, \ldots$$

fermions currents

$$\lambda_f^\mu = \left(n_f \vec{\zeta}_f \vec{v}_f , n_f \vec{\zeta}_f \sqrt{1-v_f^2} + \frac{n_f \vec{v}_f (\vec{\zeta}_f \vec{v}_f)}{1+\sqrt{1-v_f^2}}\right)$$

polarizations

$n_f \rightarrow$ number density of background f

$\vec{v}_f \rightarrow$ speed of reference frame in which mean momentum of fermions f is zero

$\vec{\zeta}_f \rightarrow$ mean value of polarization vectors of f in above mentioned ref. frame

In the reference frame $\vec{v}_f = 0 \Rightarrow$

$$\lambda_f^\mu = (0, n_f \vec{\zeta}_f),$$

$$(0 \leqslant |\vec{\zeta}_f|^2 \leqslant 1).$$

Mean value of polarization vectors:

$$\vec{\zeta}_f = \sum_{\{n\}} \langle \vec{O} \rangle \underbrace{\rho_f(\{n\})}_{} \Big/ \sum_{\{n\}} \underbrace{\rho_f(\{n\})}_{},$$

fermion distribution (Fermi-Dirac)

$$\langle \vec{O} \rangle = \int \Psi_f^+(x)\, \vec{O}\, \Psi_f(x)\, dx,$$

fermion quantum state in e.m. field

$$\vec{O} = \gamma_0 \vec{\Sigma} - \gamma_5 \frac{\vec{p}}{E} - \gamma_0 \frac{\vec{p}(\vec{p}\vec{\Sigma})}{E(E+m)},$$

fermion momentum energy mass

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}.$$

relativistic spin operator of fermion f

For each of fermions there are only u^f_μ, j^f_μ, λ^f_μ to constract $G_{\mu\nu}$.

speed

current

polaritation

If j^f_μ, λ^f_μ → slowly varying functions in space and time (similar to $F_{\mu\nu}$ in BMT eq.)

⟹ only four tensors (for each of f) linear in respect to matter charact.:

$$G_1^{\mu\nu} = \epsilon^{\mu\nu\rho\lambda} u_\lambda j_\rho \, ,$$

$$G_2^{\mu\nu} = \epsilon^{\mu\nu\rho\lambda} u_\lambda \lambda_\rho \, ,$$

$$G_3^{\mu\nu} = u^\mu j^\nu - j^\mu u^\nu \, ,$$

$$G_4^{\mu\nu} = u^\mu \lambda^\nu - \lambda^\mu u^\nu \, .$$

$u, j, \lambda \equiv$
$\equiv u^f, j^f, \lambda^f$

Thus, in general case of ν interaction with different background fermions f matter effects are described by antisymmetric tensor

$$G^{\mu\nu} = \epsilon^{\mu\nu\rho\lambda} g^{(1)}_{\rho} u_{\lambda} - \left(g^{(2)\mu} u^{\nu} - u^{\mu} g^{(2)\nu} \right),$$

where

$$g^{(1)\mu} = \sum_f \rho^{(1)}_f j^{\mu}_f + \rho^{(2)}_f \lambda^{\mu}_f ,$$

$$g^{(2)\mu} = \sum_f \xi^{(1)}_f j^{\mu}_f + \xi^{(2)}_f \lambda^{\mu}_f ,$$

- summation is performed over fermions f,
- coefficients $\rho^{(1),(2)}_f$, $\xi^{(1),(2)}_f$ are determined by ν interaction model.

In the usual notations

$$F_{\mu\nu} = (\vec{E}, \vec{B}) = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

$F_{\mu\nu}$ — e.m. field

$$\boxed{G_{\mu\nu} = (-\vec{P}, \vec{M})},$$

where

$$\vec{M} = \gamma \left\{ g_0^{(1)} \vec{\beta} - \vec{g}^{(1)} - [\vec{\beta} \times \vec{g}^{(2)}] \right\},$$

$$\vec{P} = -\gamma \left\{ g_0^{(2)} \vec{\beta} - \vec{g}^{(2)} + [\vec{\beta} \times \vec{g}^{(1)}] \right\}.$$

Substitution $\boxed{F_{\mu\nu} \rightarrow F_{\mu\nu} + G_{\mu\nu}}$

implies:

$$\boxed{\begin{array}{l} \vec{B} \rightarrow \vec{B} + \vec{M} \\ \vec{E} \rightarrow \vec{E} - \vec{P} \end{array}}$$

effects of ν interaction with moving and polarized matter

Finally:

three-dimentional ν spin vector

$$\boxed{\frac{d\vec{S}}{dt} = \frac{2\mu}{\gamma}\left[\vec{S}\times(\vec{B}_0+\vec{M}_0)\right] + \frac{2\epsilon}{\gamma}\left[\vec{S}\times(\vec{E}_0-\vec{P}_0)\right],}$$

effects of matter

laboratory frame

$$\vec{B}_0 = \gamma\left(\vec{B}_\perp + \frac{1}{\gamma}\vec{B}_\parallel + \sqrt{1-\frac{1}{\gamma^2}}\,[\vec{E}_\perp\times\vec{n}]\right), \quad \gamma = \frac{E}{m}$$

energy

mass

$$\vec{E}_0 = \gamma\left(\vec{E}_\perp + \frac{1}{\gamma}\vec{E}_\parallel - \sqrt{1-\frac{1}{\gamma^2}}\,[\vec{B}_\perp\times\vec{n}]\right),$$

in rest frame of ν $\qquad \vec{n} = \vec{\beta}/\beta$

$$\vec{M}_0 = \gamma\vec{\beta}\left(g_0^{(1)} - \frac{\vec{\beta}\vec{g}^{(1)}}{1+\gamma^{-1}}\right) - \vec{g}^{(1)},$$

$$\vec{P}_0 = -\gamma\vec{\beta}\left(g_0^{(2)} - \frac{\vec{\beta}\vec{g}^{(2)}}{1+\gamma^{-1}}\right) + \vec{g}^{(2)}.$$

$\vec{B}_0, \vec{E}_0, \vec{M}_0, \vec{P}_0$ in the rest frame of ν are expressed in terms of quantities determined in laboratory frame

How $\rho_f^{(i)}$ and $\xi_f^{(i)}$ are determined?

To fix ν interactions:

$$SM + SU(2)\text{-singlet } \nu_R \Rightarrow$$

$$L_{eff} = -f^{\mu}\left(\bar{\nu}\gamma_{\mu}\frac{1+\gamma_5}{2}\nu\right),$$

where

$$f^{\mu} = \frac{G_F}{\sqrt{2}}\left((1+4\sin^2\theta_W)j_e^{\mu} - \lambda_e^{\mu}\right)$$

In this case: $\epsilon = 0 \Rightarrow \boxed{\xi_e^{(i)} = 0,}$

ν electric dipole moment

and $(f_{\mu} = 2\mu g_{\mu}^{(1)}) \Rightarrow$

$$\boxed{\begin{aligned} \rho_e^{(1)} &= \frac{G_F}{2\mu\sqrt{2}}(1+4\sin^2\theta_W), \\ \rho_e^{(2)} &= -\frac{G_F}{2\mu\sqrt{2}}. \end{aligned}}$$

If :

$$\mu_\nu = \frac{3}{8\sqrt{2}\pi^2} e G_F m_\nu ,$$

B. Lee, R. Shrock, 1977
K. Fujikava, R. Shrock, 1980

$$\boxed{g^{(1)} = \frac{4\pi^2}{3em_\nu}\left(1+4\sin^2\theta_W\right), \quad g^{(2)} = -\frac{4\pi^2}{3em_\nu} .}$$

Neutrino ν_e spin evolution in relativistic flux of electrons ($f \equiv e$)

Effects of moving and polarized matter

$$\vec{M}_0 = n_e \gamma \vec{\beta} \Big\{ \big[\rho^{(1)} + \rho^{(2)} \vec{\zeta} \vec{v}_e \big] (1 - \vec{\beta} \vec{v}_e) + + \rho^{(2)} \sqrt{1 - v_e^2} \Big[\frac{\vec{\zeta} \vec{v}_e \; \vec{\beta} \vec{v}_e}{1 + \sqrt{1 - v_e^2}} - \vec{\zeta} \vec{\beta} \Big] + O(\gamma^{-1}) \Big\}$$

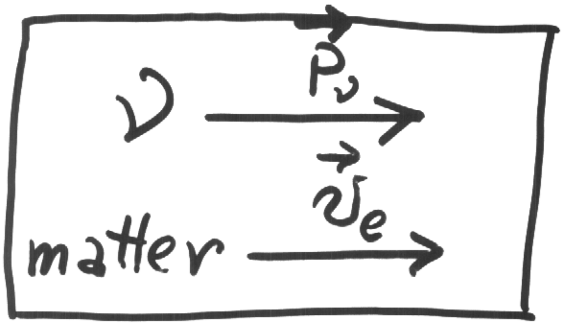
- slowly moving matter, $v_e \ll 1$:

$$\vec{M}_0 = \underbrace{n_e \gamma \vec{\beta} (\rho^{(1)}}_{\text{Wolfenstein term}} - \rho^{(2)} \vec{\zeta} \vec{\beta}) .$$

- relativistic flux of e, $v_e \sim 1$:

$$\vec{M}_0 = n_e \gamma \vec{\beta} \left(\rho^{(1)} + \rho^{(2)} \vec{\xi}\vec{v}_e\right)\left(1 - \vec{\beta}\vec{v}_e\right) \quad !$$

In case of



matter effect contributions to ν spin evolution equation is suppressed !

Conclusion

* Lorentz invariant formalizm for ν spin evolution in $F_{\mu\nu}$ and matter
* direct interaction of ν ($\mu \neq 0 \neq \epsilon$) with e.m. field $F_{\mu\nu}$
* nontrivial matter effects: motion and polarization ($f = e, n, p, \ldots$)

* $\nu_L \leftrightarrow \nu_R$ oscillation probability in $\vec{B} = \vec{B}_\perp + \vec{B}_\parallel \oplus$ matter

$$P_{\nu_L \leftrightarrow \nu_R}(x) = \sin^2 2\theta_{eff} \sin^2 \frac{\pi x}{L_{eff}},$$

$$\sin^2 2\theta_{eff} = \frac{E_{eff}^2}{E_{eff}^2 + \Delta_{eff}^2},$$

$$L_{eff} = 2\pi \left(E_{eff}^2 + \Delta_{eff}^2 \right)^{-1/2}.$$

for particular case $\nu = \nu_e$ and matter ($f = e$, $\zeta_e \approx 0$)

$$E_{eff} = 2\mu B_\perp \quad (\text{terms} \sim O(\gamma^{-1}) \text{ are omitted})$$

* $$\boxed{\Delta_{eff} = V(1 - \vec{\beta}\vec{v}_e) + \frac{2\mu B_\parallel}{\gamma}}, \quad V = \frac{G_F}{\sqrt{2}} n_e$$

matter effects can be eaten by matter motion

* Condition for maximal mixing of $\nu_L \leftrightarrow \nu_R$ in $\vec{B}$ ($\vec{B} \approx \vec{B}_\perp$, $\vec{B}_\parallel \approx 0$) are restored for ν propagating along moving ($v_e \sim 1$) matter.