# Physics of Hadron Production of Very High Multiplicites Events

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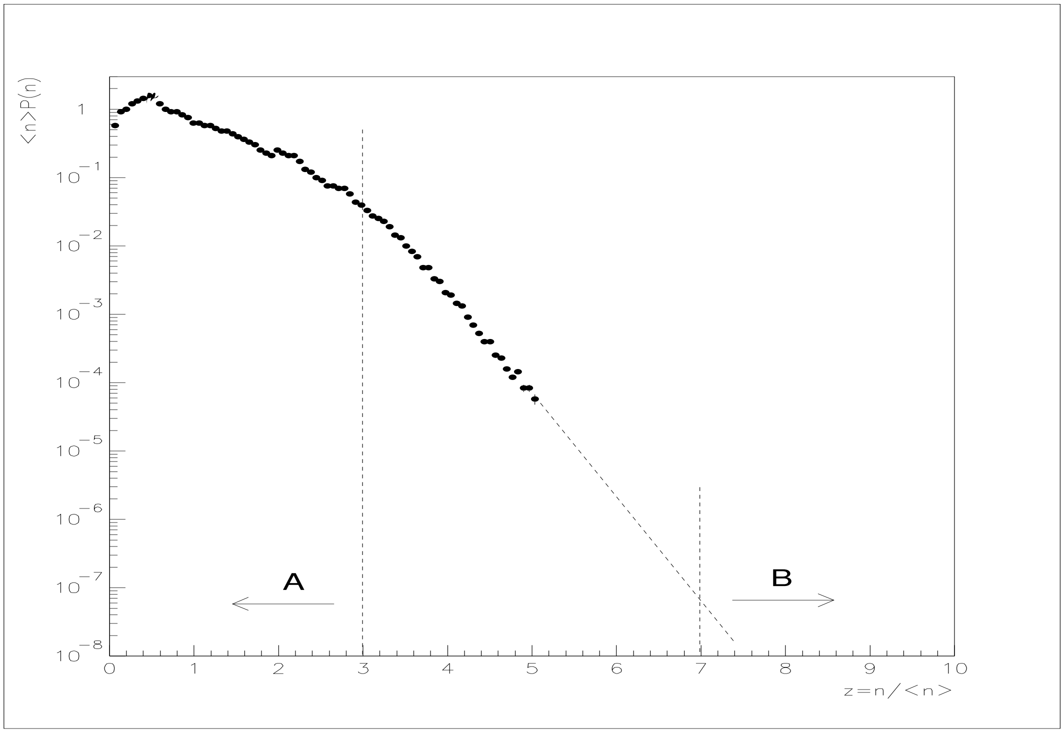


Figure 1: Multiplicity distribution
(Tevatron, the E-735 Group data).

— A: The multiperipheral domain of multiplicity
— B: The thermodynamical limit of multiplicity
— $A \Leftrightarrow B$: The VHM domain

- So, we expect that

$$n >> \bar{n}(s).$$

- But the cross section is noticeably small:

$$\sigma_n << 10^{-7}\sigma_{tot}.$$

# 1 Why Very High Multiplicity (VHM)?

• The hope to observe new dynamical phenomena or excitation of new degrees of freedom, *not developed in other hadron reactions*, was our first idea.

– We expect that in the VHM domain **all** degrees of freedom should be excited (interaction becomes "central").

– The VHM system is "cold" ("complete" annihilation of incident energy into particle masses).

• Intensive mixing would result in the "equilibrium" state.

– The "Gaussian" energy spectrum

– The system becomes "calm"

## 2 What was done in the VHM field?

First publications on the VHM physics:

1. J.Manjavidze and A.Sissakian, *JINR Rap. Comm.* 2/288 (1988) 13, *ibid.* 5/31 (1988) 5
The review paper

2. J.Manjavidze and A.Sissakian, Phys. Rep., *March issue* (2001)
accumulates more than 30 of our publications on the VHM theory.

3. J.Manjavidze and A.Sissakian, talk at Bogoliubov Memorial Conf., 2000, to be published in El. Part. At. and Nucl.

Semi-inclusive approach was considered in:

4. V.A.Matveev, A.N.Sissakian and L.Slepchenko, JINR Rep., P2-8670, 1973

The idea of different mechanisms of multiple production was offered in:

5. A.N.Sissakian and L.A.Slepchenko, Fizika, 10 (1978) 21

6. J.Manjavidze, Phys. Part. and Nucl., 30 (1999) 49

## 3 The VHM domain

General definition of the VHM region:

$$n >> \bar{n}(s): \quad \sigma_n << 10^{-7}\sigma_{tot}.$$

- If

– $\varepsilon_{max}$ - the energy of the fastest particle
– $(E - \varepsilon_{max})$ - the spent energy,
then the inelasticity coefficient

$$\kappa = \frac{E - \varepsilon_{max}}{E} = 1 - \frac{\varepsilon_{max}}{E} \leq 1. \tag{1}$$

In the VHM region:

$$\frac{\varepsilon_{max}}{E} = 1 - \kappa << 1. \tag{2}$$

- Using the energy conservation law ($n\varepsilon_{max} > E$),

$$n\frac{\varepsilon_{max}}{E} = n(1-\kappa) > 1. \tag{3}$$

As it follows from (2), the multiplicity

$$n > \frac{1}{1-\kappa} >> 1 \tag{4}$$

This is an 'ordinary' definition of the considered "VHM processes".

If the transverse momenta $p_{tr} = const.$, then

– Hadron interaction radii $\sim \sqrt{\ln s}$.

– Number of partons ~(disk area)~ $\ln s \sim \bar{n}(s)$.

– Each parton may produce $\sim \ln s$ particles.

Therefore,
**At $n > \bar{n}(s)^2$ we get out of the standard (*multiperipheral*) hadron kinematics.**
So,

$$n << \bar{n}(s)^2$$

is a standard hadron kinematics area.
For Tevatron energies:

$$n > \bar{n}(s)^2 \simeq 5\ 000$$

is the VHM domain.
At the same time,

$$n_{max} \simeq 60\ 000.$$

Therefore, the VHM domain for these energies is:

$$\bar{n}(s)^2 \simeq 5\ 000 \leq n << 60\ 000 \simeq n_{max}.$$

This is the second definition of VHM, based on the multiperipheral model.

## 4 Statistical method

• It is reasonable to depart from an exact definition of the final state kinematics and consider the following idea:

– In the VHM domain the entropy should tend to its maximum value (since the multiplicity $n$ measures it)

– The system should 'calm down' in the VHM domain.

– To describe the calm systems, a small number of "rough" parameters is necessary.

So,
• Nothing will happen if $n$ is measured with $\Delta n \neq 0$ accuracy, since $(\Delta n/n) << 1$ is easily attainable in the VHM region.

## 5 ‘Rough’ variable for energy spectrum

- By definition,

$$\sigma_n^{ab}(s) = \int d\omega_n(q)\delta(q_a + q_b - \sum_{i=1}^{n} q_i)|A_n^{ab}|^2, \quad (5)$$

where $A_n^{ab}$ is the amplitude of $n$ particle production in the interaction of particles $a$ and $b$.

- It can be written in the form:

$$\sigma_n(s) = \int_{-i\infty}^{+i\infty} \frac{d\beta}{2\pi} e^{\beta\sqrt{s}} \rho_n(\beta), \quad (6)$$

where

$$\rho_n(\beta) = \int \left\{ \prod_{i=1}^{n} \frac{d^3 q_i e^{-\beta\epsilon(q_i)}}{(2\pi)^3 2\epsilon(q_i)} \right\} |A_n^{ab}|^2. \quad (7)$$

- The most probable value $\beta_c$ in this integral is defined by the equation of state:

$$\sqrt{s} = -\frac{\partial}{\partial\beta} \ln \rho_n(\beta). \quad (8)$$

The solution of this equation will be $\beta_c(s, n)$.

Then $\beta_c$ may be considered as a ‘rough’ variable: instead of the $n$ energies $\epsilon(q_1),\ \epsilon(q_2), ..., \epsilon(q_n)$ we introduce one variable $\beta_c$ in such a way that (as it follows from (8)) $1/\beta_c$ is the mean energy, and the fluctuations of energies near $1/\beta_c$ are defined by the Boltzmann factor $e^{-\beta_c\epsilon}$, see (7).

**The "ideal gas" approximation**.

To find the physical meaning of $\beta_c$, one may consider an example of noninteracting particles, when $A_n = const$. The direct calculation gives

$$\rho_n(\beta) = |A_n|^2 \{2\pi m K_1(\beta m)/\beta\}^n ,$$

where $K_1$ is the Bessel function. Inserting this expression into (8), we can find that in the nonrelativistic case ($n \simeq n_{max}$)

$$\beta_c^0 = \frac{3}{2}\frac{(n-1)}{(\sqrt{s} - nm)}.$$

This means that the mean value

$$< E_{kin} >= \frac{3}{2}T, \tag{9}$$

where $E_{kin} = (\sqrt{s} - nm)$, is the kinetic energy and $T$ is the temperature. The eq.(9) is obvious for the 'ideal gas' approximation.

## 6 Relaxation of correlations

The $k$-th correction term is

$$\rho_{n,k} \sim \left\{ \frac{\partial^3 \ln \rho_n(\beta_c)/\partial\beta_c^3}{(\partial^2 \ln \rho_n(\beta_c)/\partial\beta_c^2)^{3/2}} \right\}^k \Gamma\left(\frac{3k+1}{2}\right). \tag{10}$$

Therefore, one should assume that:

$$\partial^3 \ln \rho_n(\beta_c)/\partial\beta_c^3 << (\partial^2 \ln \rho_n(\beta_c)/\partial\beta_c^2)^{3/2}. \tag{11}$$

to neglect it.

By definition, $K_3$ is the third energy correlator.

- Let

$$< \epsilon^l; n >= \frac{1}{\sigma_n} \int (\epsilon_1 d\epsilon_1)(\epsilon_2 d\epsilon_2) \cdots (\epsilon_l d\epsilon_l) \frac{d^l \sigma_n}{d\epsilon_1 d\epsilon_2 \cdots d\epsilon_l}$$

be the mean value of $l$ particle energies, if the total number of particles is $n$.

— $d^l\sigma_n/d\epsilon_1 d\epsilon_2 \cdots d\epsilon_l$ – the number of events

— $\epsilon_k$ – the energy of $k$-th particle

— $n$ – the total number of produced particles

— $\sigma_n$ – the total number of events with multiplicity $n$.

- Then:

$K_1(\epsilon, n) =< \epsilon^1; n >$ - mean energy,

$K_2(\epsilon, n) =< \epsilon^2; n > - < \epsilon^1; n >^2$ - dispersion,

$K_3(\epsilon, n) =< \epsilon^3; n > -3 < \epsilon^2; n >< \epsilon^1; n > +2 < \epsilon^1; n >^3$ - the third energy correlator,

...

Therefore, if, see (11),

$$|K_3(\epsilon, n)|^{2/3} << K_2(\epsilon_1),$$

then the mean energy of produced particles is a "good" variable.

## 7 "Thermodynamical" approximation

If $n \to n_{max}$ then $|p_i| << m,\ i = 1, 2, ..., n$. In this limit:

– The cross sections are rather small:

$$\sigma_n \sim (n_{max} - n)^n. \qquad (12)$$

– The inverse temperature

$$\beta_c(n, E)m = \frac{3}{2}\, \frac{n}{n_{max} - n}. \qquad (13)$$

Then:

$$K_1(\epsilon, n) = E$$

$$K_2(\epsilon, n) = \frac{3n}{2\beta_c^2} = \frac{2(n_{max} - n)^2}{3m^2 n} \to 0.$$

$$K_3(\epsilon, n) = -\frac{3n}{\beta_c^3} \sim (n_{max} - n)^3/m^3 n^2 \to 0.$$

Nevertheless,

$$\left(|K_3|^{2/3}/K_2\right) \sim n^{-1/3} \to 0.$$

So, *the arbitrary massive system should come to "equilibrium"*.

Remark: It's the first formal proof of this statement.

The Ehrenfest-Kac model describes the $random$ "production" and "absorption" of particles. The particles production in the 'event-by-event' experiments:

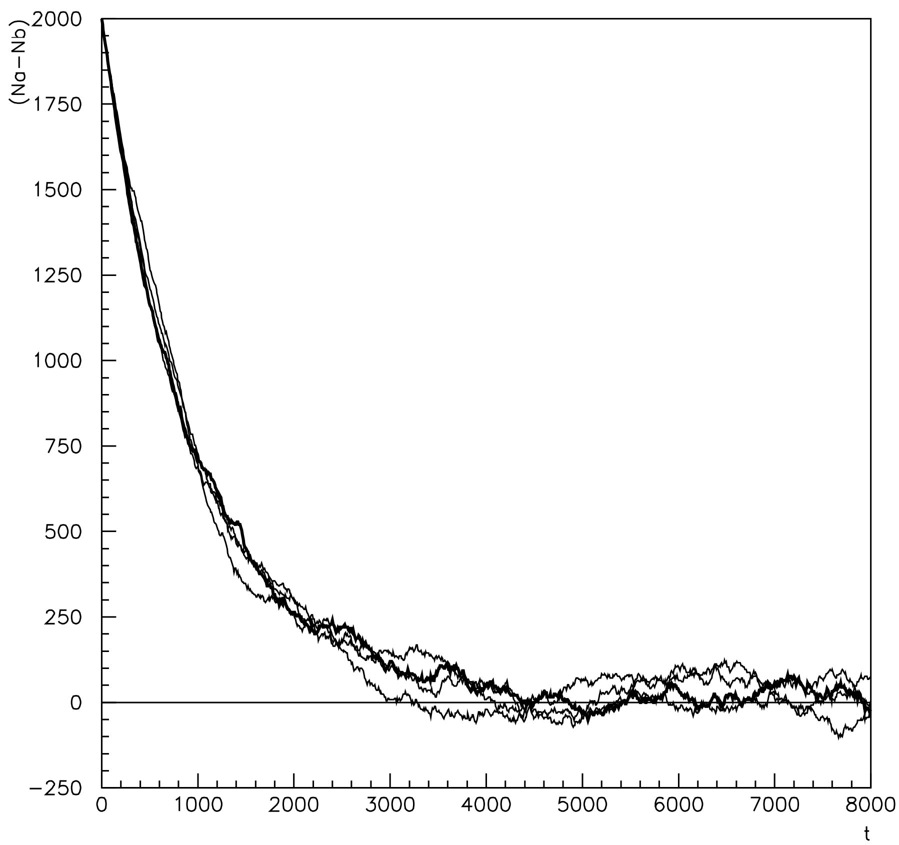


Figure 2: Particles production in the Markovian process.

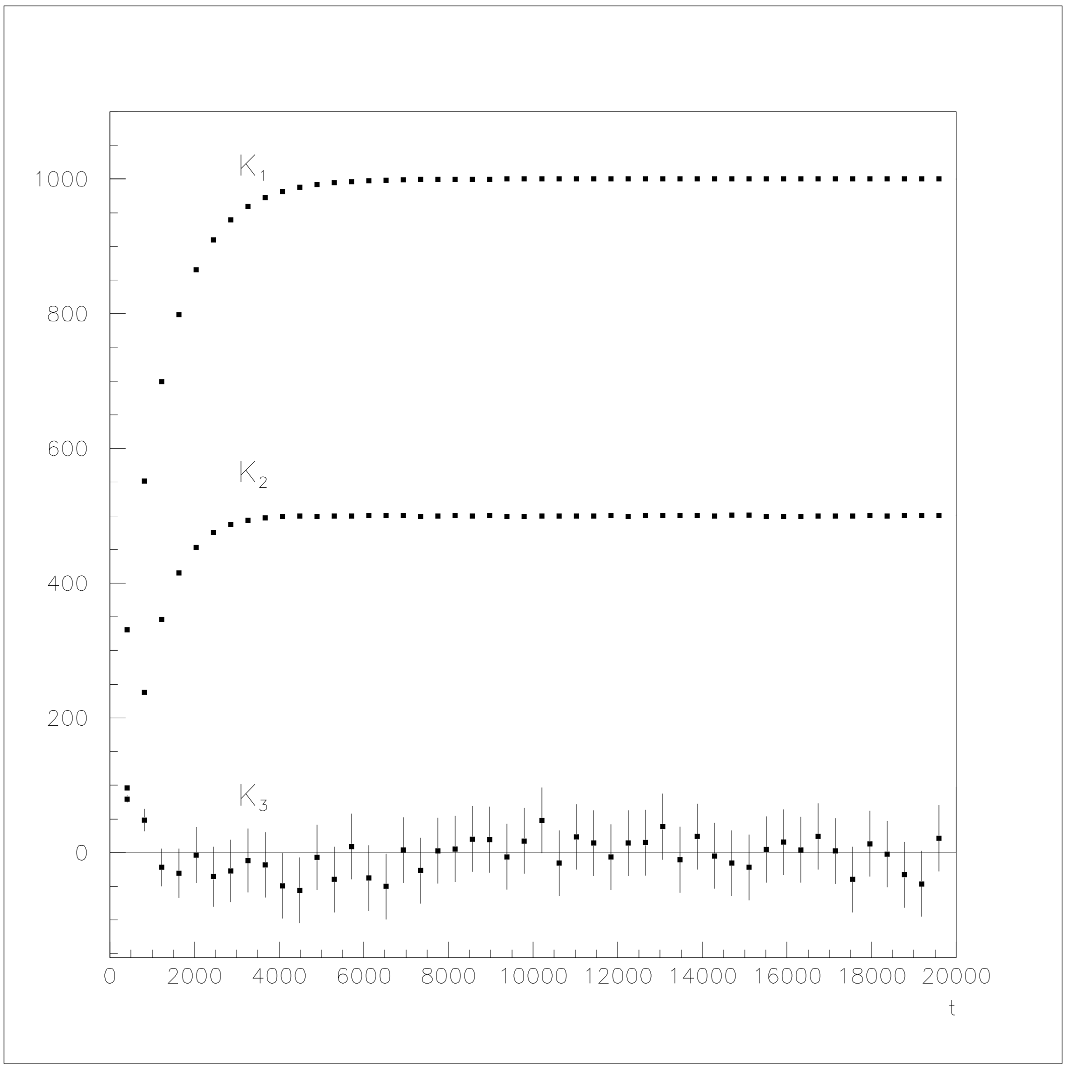


Figure 3: Equilibrium over particles number.

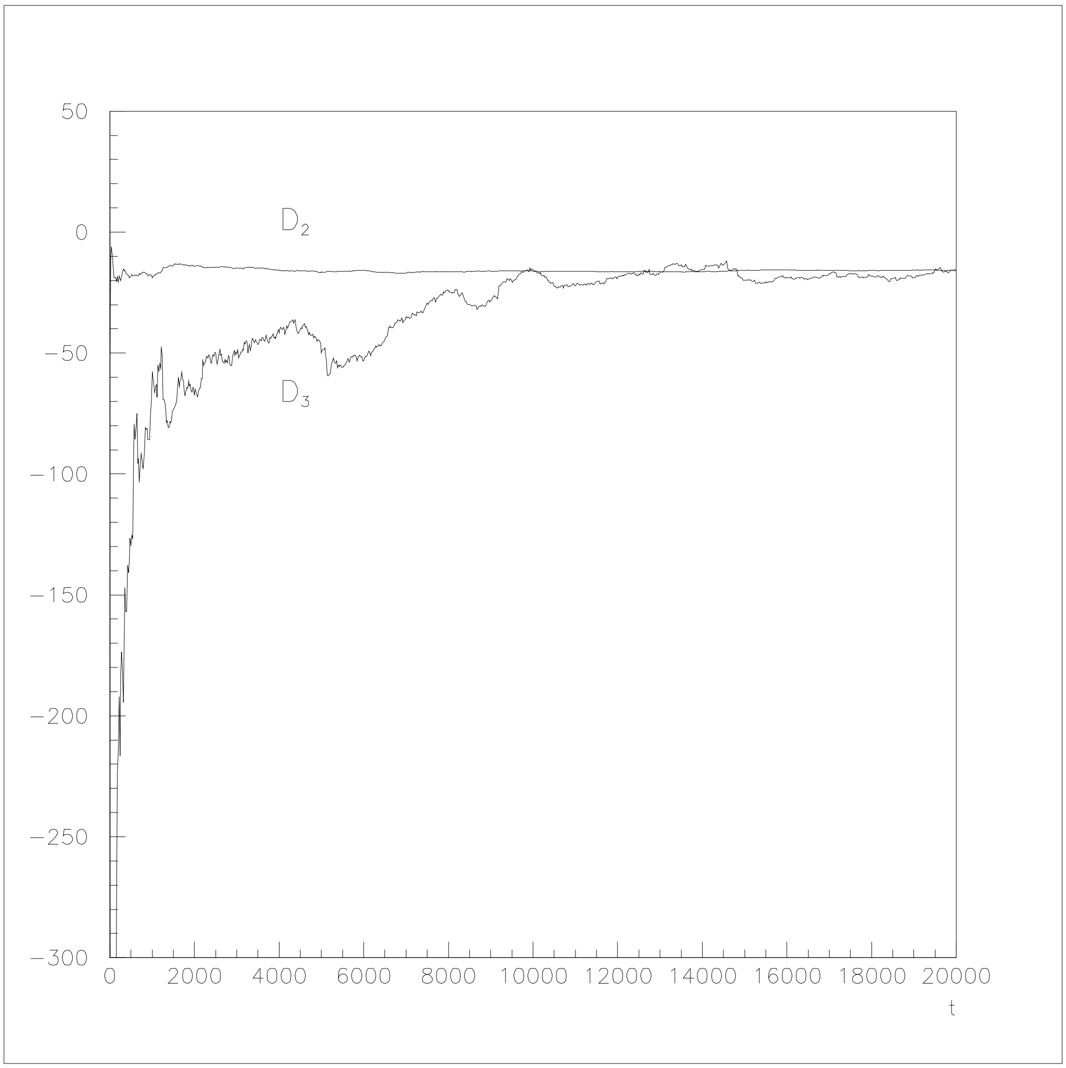


Figure 4: Label correlation functions.

# 8 PYTHIA predictions

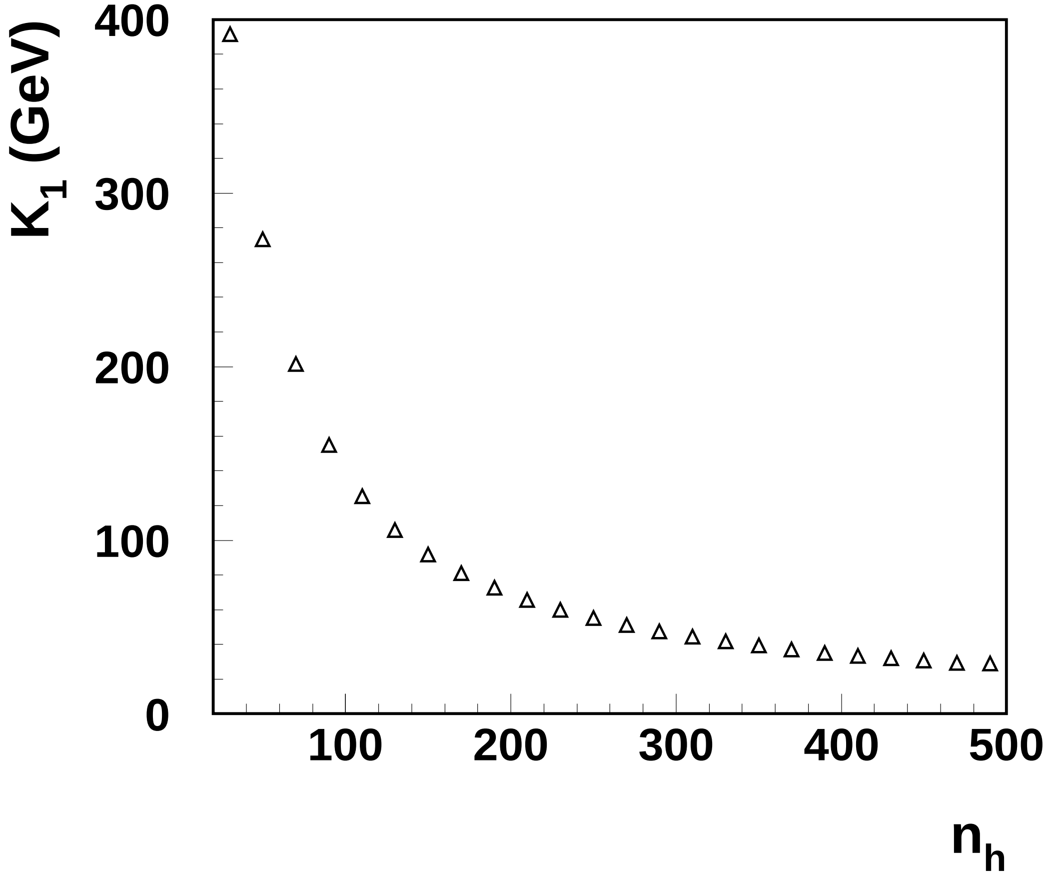


Figure 5: PYTHIA prediction for $K_1$.

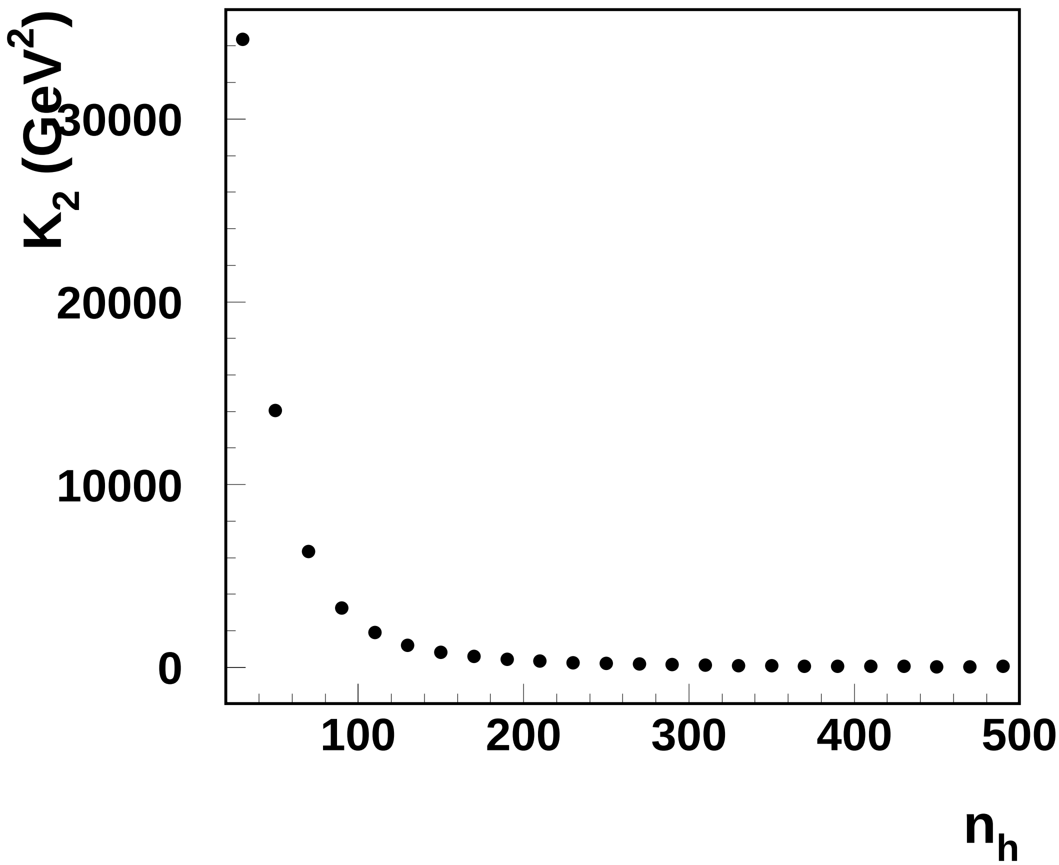


Figure 6: PYTHIA prediction for $K_2$.

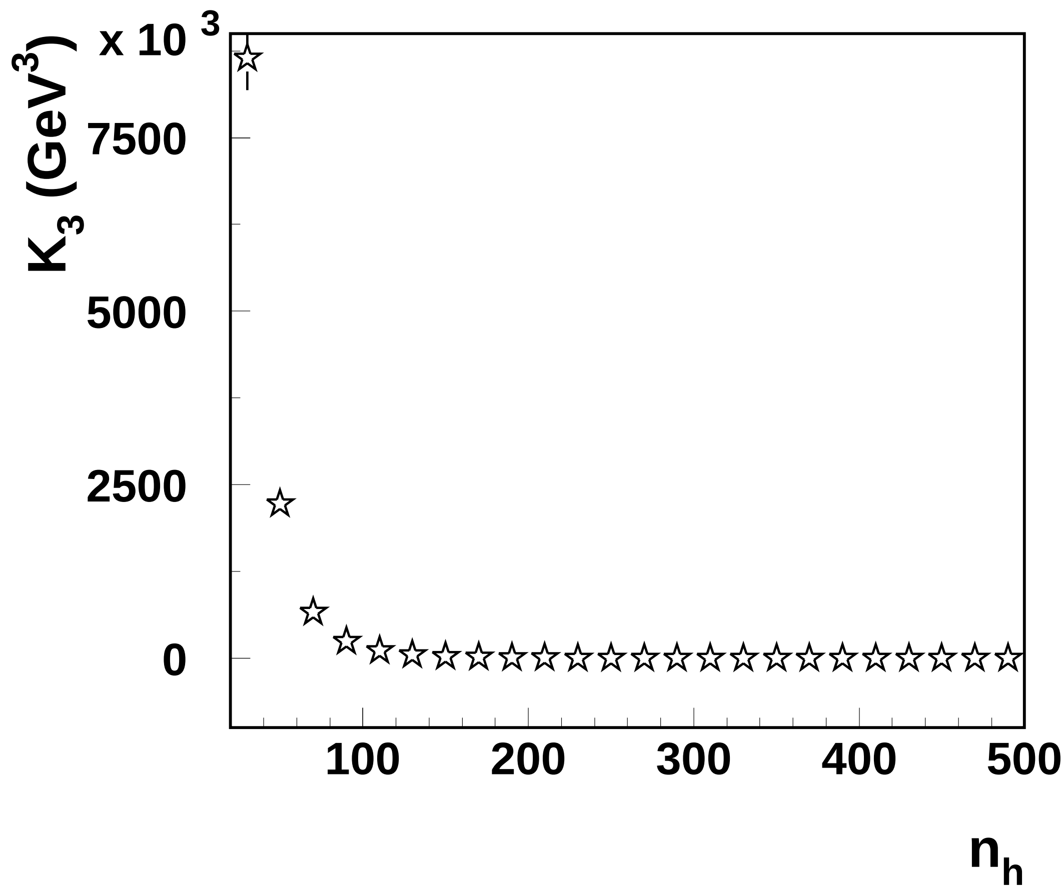


Figure 7: PYTHIA prediction for $K_3$.

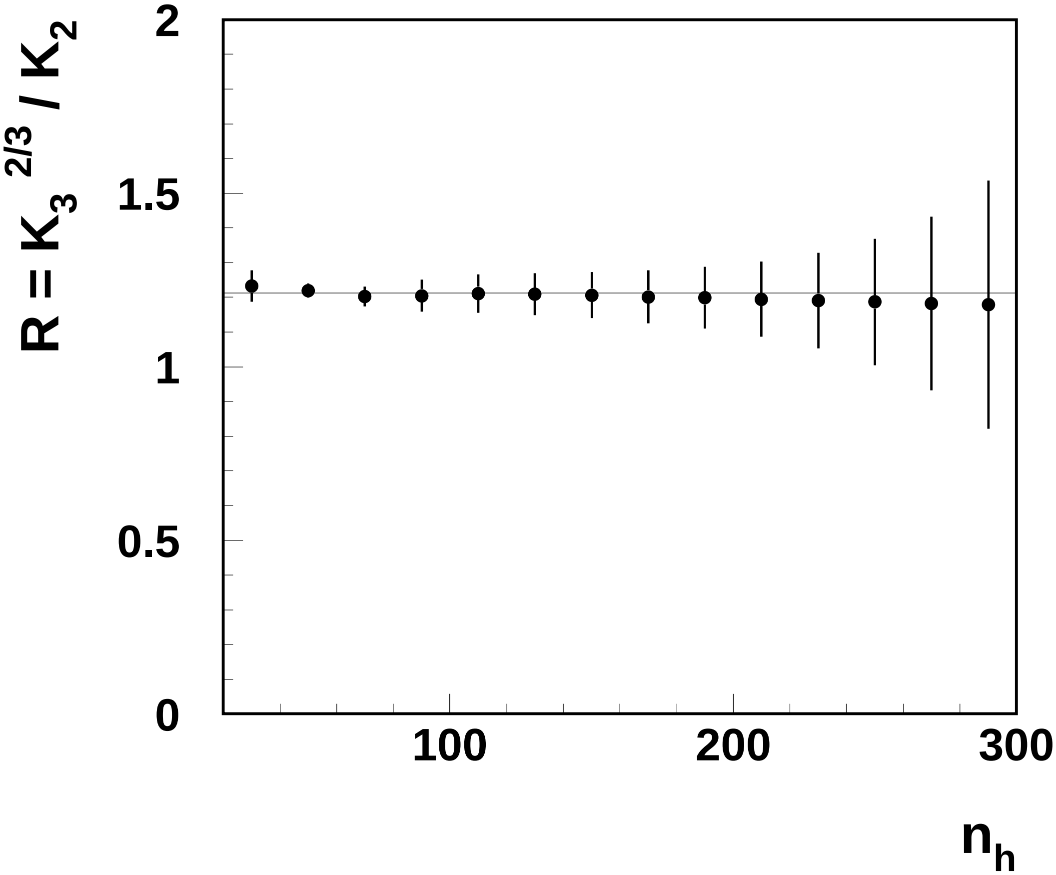


Figure 8: PYTHIA prediction for ratio $K_3^{2/3}/K_2$.

## 9 First questions to the experiment and perspectives

### — The mean value of energy correlation functions?

If

$$|K_l|^{2/l} << K_2, \quad l = 3, 4, ...,$$

it will allow to investigate:

(A) The collective phenomena (phase transition, etc.) at the given low temperature and high density system,

(B) The properties of the "calm" colour (quark) plasma *state*

### — Produced particles momentum spectra?

(C) The production process becomes hard: the ratio of the mean values

$$R(n,s) = \left\{ < p_\perp > / < p_\parallel > \right\} < \pi/2.$$

### — The vacuum structure?

(D)The isotopic spin fluctuations are "anomalous" over

$$C = n_c/n_0.$$

• For today we already have a theoretical basis for this generator. It is based on the new perturbation theory, published in several mathematical papers:

J.Manjavidze, Journ. Math. Phys., 41 (2000) 5710

J.Manjavidze and A.Sissakian, Theor. and Math. Phys., 123 (2000) 776

J.Manjavidze and A.Sissakian, Journ. Math. Phys., 42 (2001) 641

J.Manjavidze and A.Sissakian, Phys. Rep., in press (2001)

This formalism includes the perturbative QCD as the definite approximation only.

The generating functional:

$$\rho(\beta, z) = e^{-i\mathbf{K}} \int D(\xi, \eta) e^{-2iU(u,e)} e^{R(\beta,z;u)}$$

It contains:

- $\mathbf{K} = \int dt dt' \Theta(t-t') \left\{ \frac{\delta}{\delta \epsilon_\xi} \cdot \frac{\delta}{\delta \xi} + \frac{\delta}{\delta \epsilon_\eta} \cdot \frac{\delta}{\delta \eta} \right\}$
- $\Theta(t-t')$ – Green function
- $R(\beta, z; u) = \int (d^3q/\epsilon(q)) e^{-\beta\epsilon(q)} z(q) \left|\Gamma(q; u)\right|^2$
- $\Gamma(q, u) = \int dx e^{iqx} (\partial_\mu^2 + m^2) u(x; \xi, \eta)$
- $u(x; \xi, \eta)$ – solution of Lagrange equation
- $\xi, \eta$ – constants of integration
- $D(\xi, \eta) = \prod_t d\xi(t) \delta\eta(t) \delta(\dot{\xi} - v(\eta)) \delta(\dot{\eta})$
- $U(u, e)$ – describes interaction