Search for the Higgs boson: Theoretical perspectives

XV Rencontres de Physique de la Vallée d'Aoste
La Thuile, March 6^{th}, 2001

Giovanni Ridolfi - CERN-TH and INFN
(with many thanks to Gino Isidori and Alessandro Strumia)

1. **A reminder (with something new)**
2. **Two questions (and possible answers)**
3. **Outlook**

A brief reminder:

- The $SU(2) \times U(1)$ gauge symmetry of electroweak interactions is spontaneously broken to $U(1)_{\rm em}$; we observe the (4-1=3) Goldstone modes as longitudinal polarization states of $W^\pm$ and Z^0.

- The mechanism that induces SSB is still unexplored.

- Simplest solution: a perturbative SSB induced by a scalar $\phi \sim (\mathbf{2}, 1)$ and a potential

$$V(\phi) = m^2 |\phi|^2 + \lambda |\phi|^4 , \quad m^2 < 0.$$

  Minimum at $|\phi|^2 = -m^2/(2\lambda) \equiv v^2/2$. Only one physical degree of freedom H with tree-level mass $m_H = 2\lambda v^2$; $v \simeq 246$ GeV from β decay.

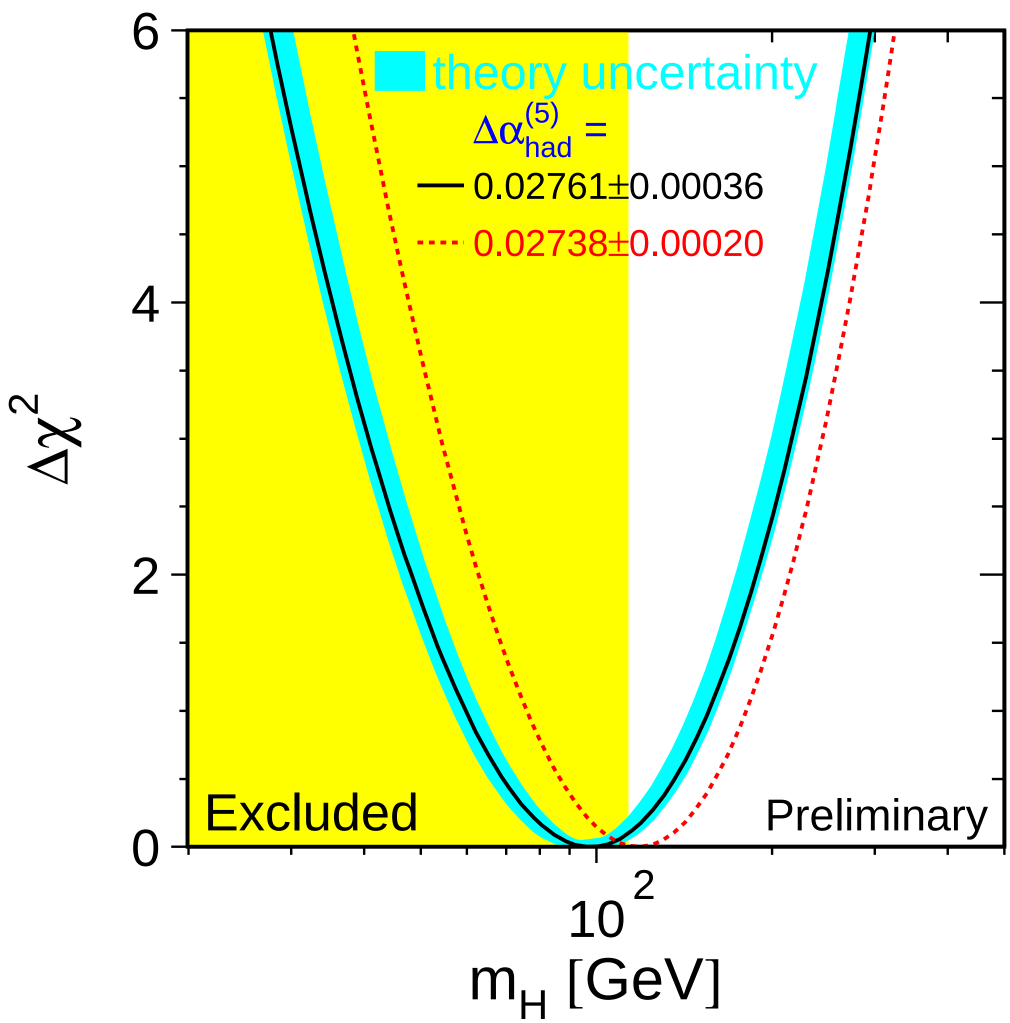
- Large $m_H \leftrightarrow$ strong interactions in the SSB sector:

$$\Gamma(H \to VV) = \frac{3}{32\pi} \frac{m_H^3}{v^2}$$

  becomes approximately equal to m_H for $m_H \sim 1.4$ TeV. A qualitative change in the analysis.

The Higgs boson of the minimal standard model is a light particle.

Fits to precision data:



Direct search: $m_H > 113$ GeV

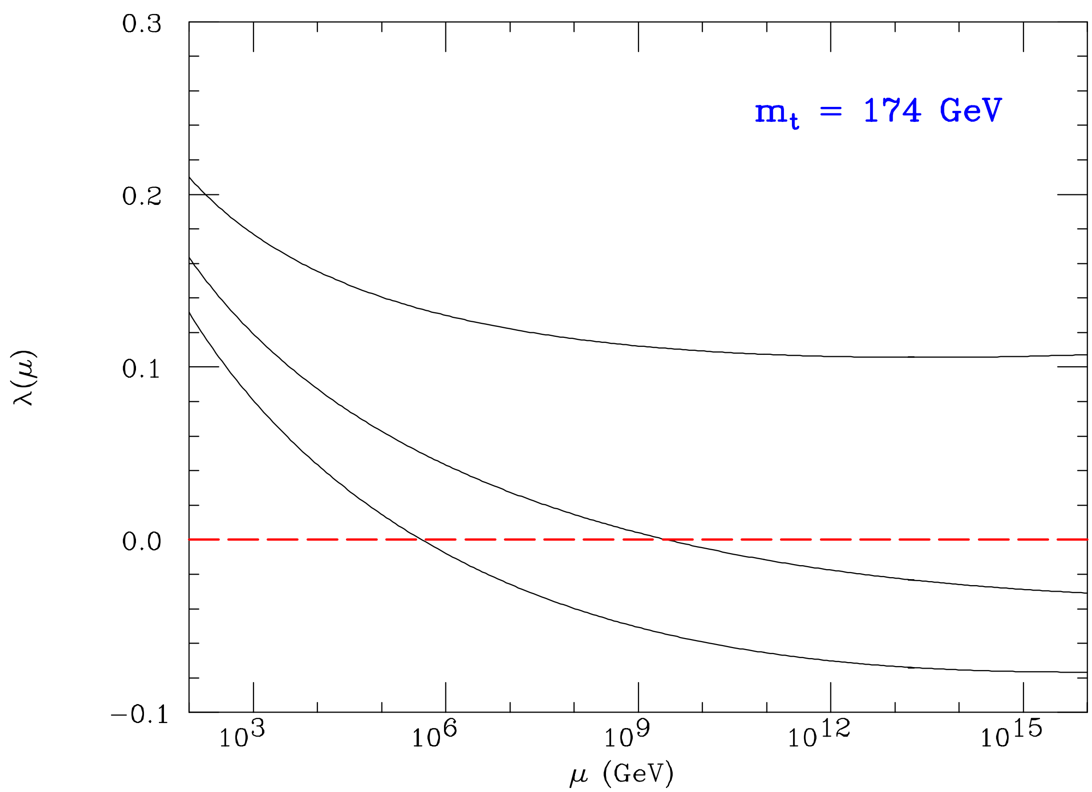
Very unlikely that m_H is larger that ~ 250 GeV

Theoretical constraints

Vacuum stability beyond leading order:

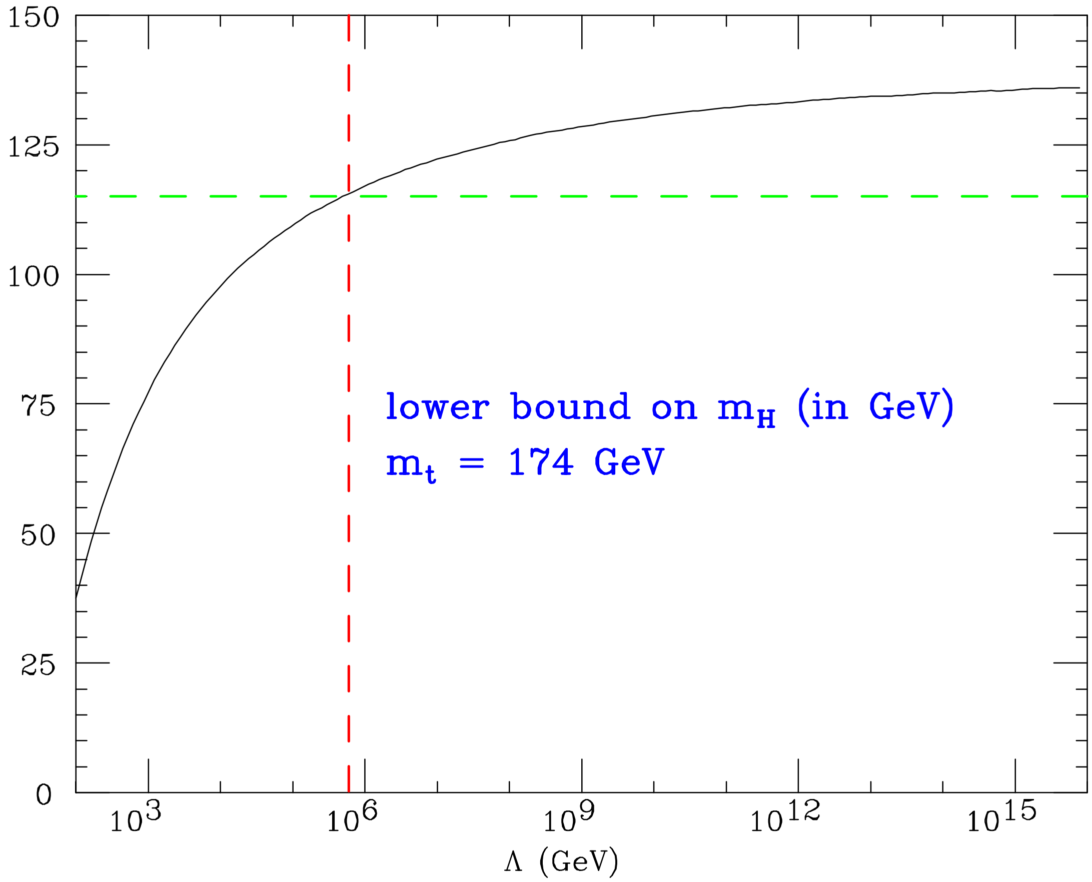
$$V_{eff}(\phi) \sim m^2(\phi)\phi^2 + \lambda(\phi)\phi^4$$

bounded from below provided $\lambda(\phi) > 0$. Not generally true: λ becomes negative at a scale Λ, where new phenomena should become relevant and restore vacuum stability.



smaller $\lambda(m_Z) \Rightarrow$ smaller Λ

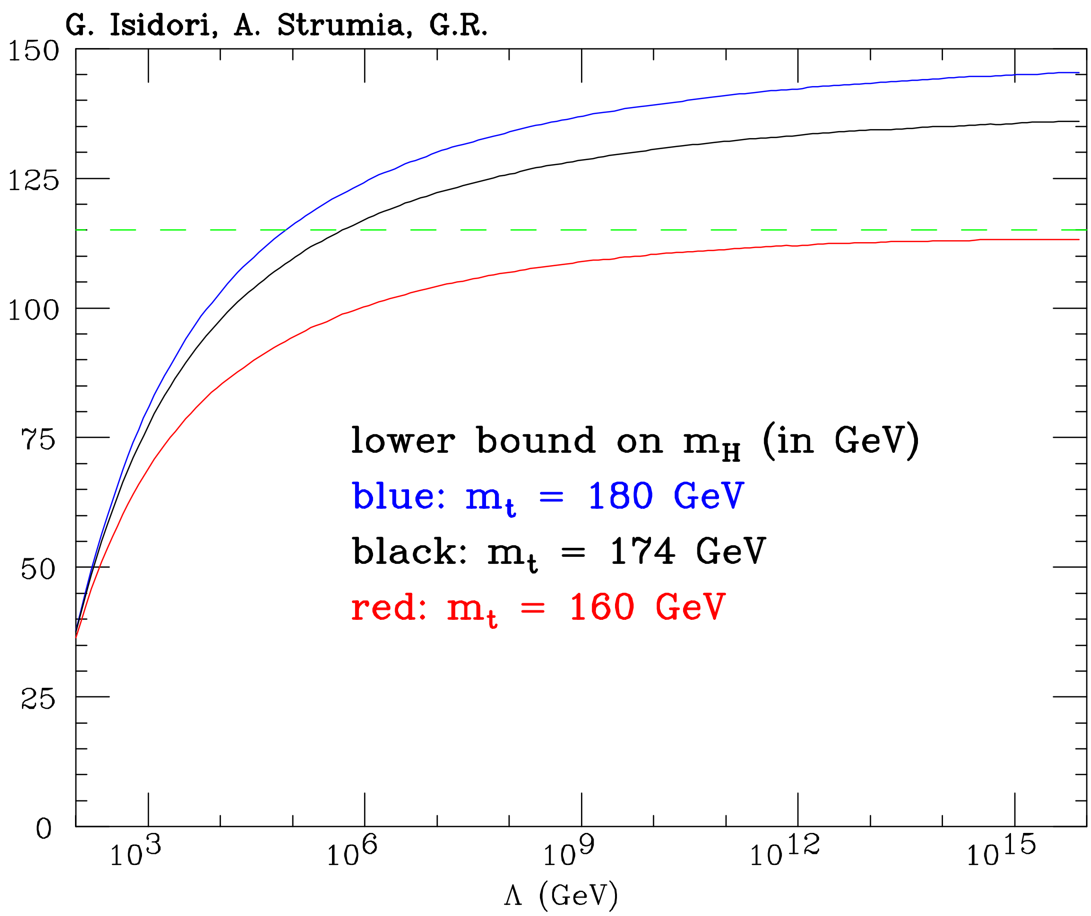
Since $m_H^2 \simeq \frac{\sqrt{2}}{G_F}\,\lambda(m_Z)$, this implies a Λ-dependent lower bound on m_H:



$m_H \simeq 115$ GeV $\to$ new physics at $\Lambda \lesssim 10^6$ GeV?

Not really ...

The lower bound is very sensitive to the value of the top quark mass:

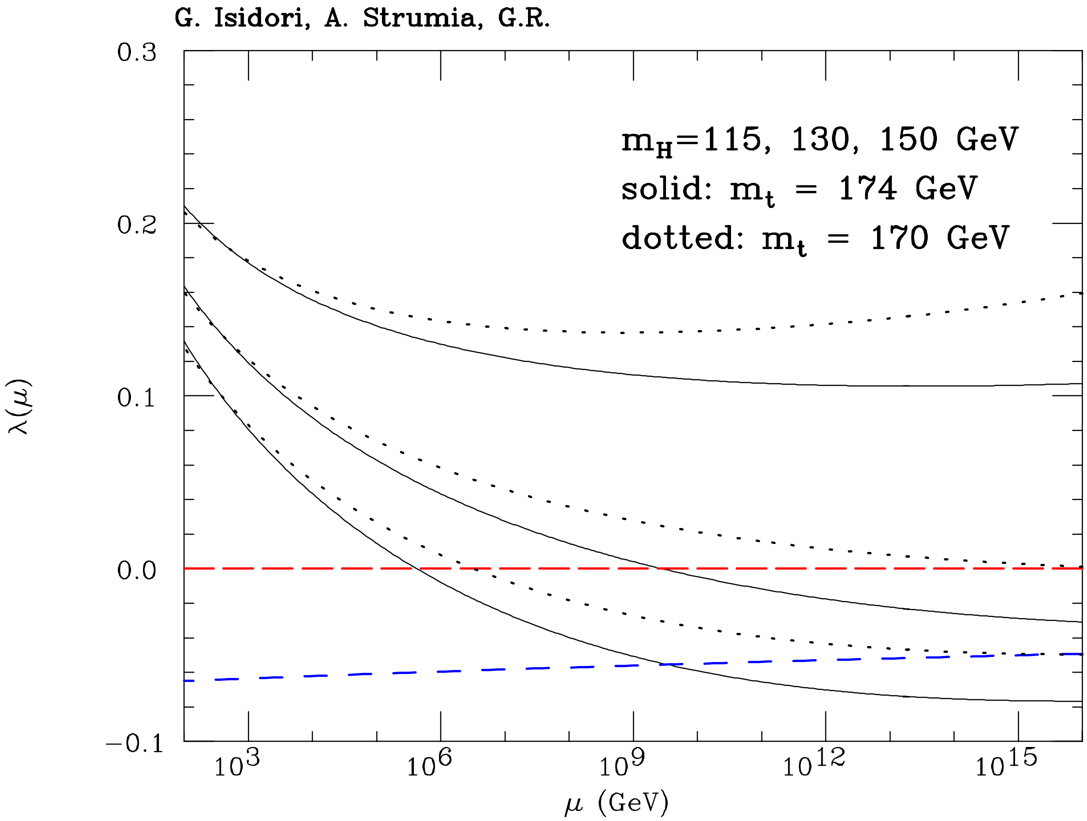


Furthermore, one could release the lower bound by allowing *metastability* of the ground state (instead of requiring absolute stability), provided the lifetime of the metastable vacuum is larger than the age of the Universe, $T \sim 10^{10}$ yrs.

The decay probability of the false vacuum per unit volume and per unit time is given, to one loop accuracy, by

$$\frac{\Gamma}{V} = \frac{S_0^2[h]}{4\pi^2} e^{-S_0[h]} \left| \frac{Det'(S_0''[h])}{Det(S_0''[0])} \right|^{-1/2}$$

where $h(x)$ – the *bounce* – is the solution of the classical field equations that interpolates between the true and the metastable vacuum state, and $S_0[h]$ ($S_1[h]$) the corresponding value of the tree-level (one-loop) euclidean action.



Assume $m_H = 115$ GeV.

- For m_t at the central value, the metastability bound is violated at a much higher Λ than the absolute stability bound;

- If $m_t = 170$ GeV ($\sim 1\,\sigma$ away), no lower bound up to the grand unification scale.

Upper bounds

Consider elastic scattering of longitudinally polarized Z bosons:

$$Z_L Z_L \to Z_L Z_L$$

The corresponding amplitude, in the limit $s \gg m_Z^2$, can be computed using the equivalence theorem:

$$\mathcal{M} = -\frac{m_H^2}{v^2}\left[\frac{s}{s-m_H^2} + \frac{t}{t-m_H^2} + \frac{u}{u-m_H^2}\right]$$

Unitarity bound on the $J = 0$ partial amplitude:

$$|\mathcal{M}_0|^2 \to \left[\frac{3}{16\pi}\frac{m_H^2}{v^2}\right]^2 < \frac{s}{s-4m_Z^2}.$$

For $s \gg m_Z^2$

$$m_H < \sqrt{\frac{16\pi}{3}}\, v \sim 1\ \mathrm{TeV}$$

Slightly more restrictive bounds (~ 800 GeV) are obtained considering different processes.

A more severe constraint: triviality

Neglecting gauge and Yukawa couplings,

$$\lambda(\mu^2) = \frac{\lambda(m_Z^2)}{1 - \frac{3}{4\pi^2}\lambda(m_Z^2)\log\frac{\mu^2}{m_Z^2}}$$

$\Rightarrow$ The scalar quartic coupling has a Landau pole at

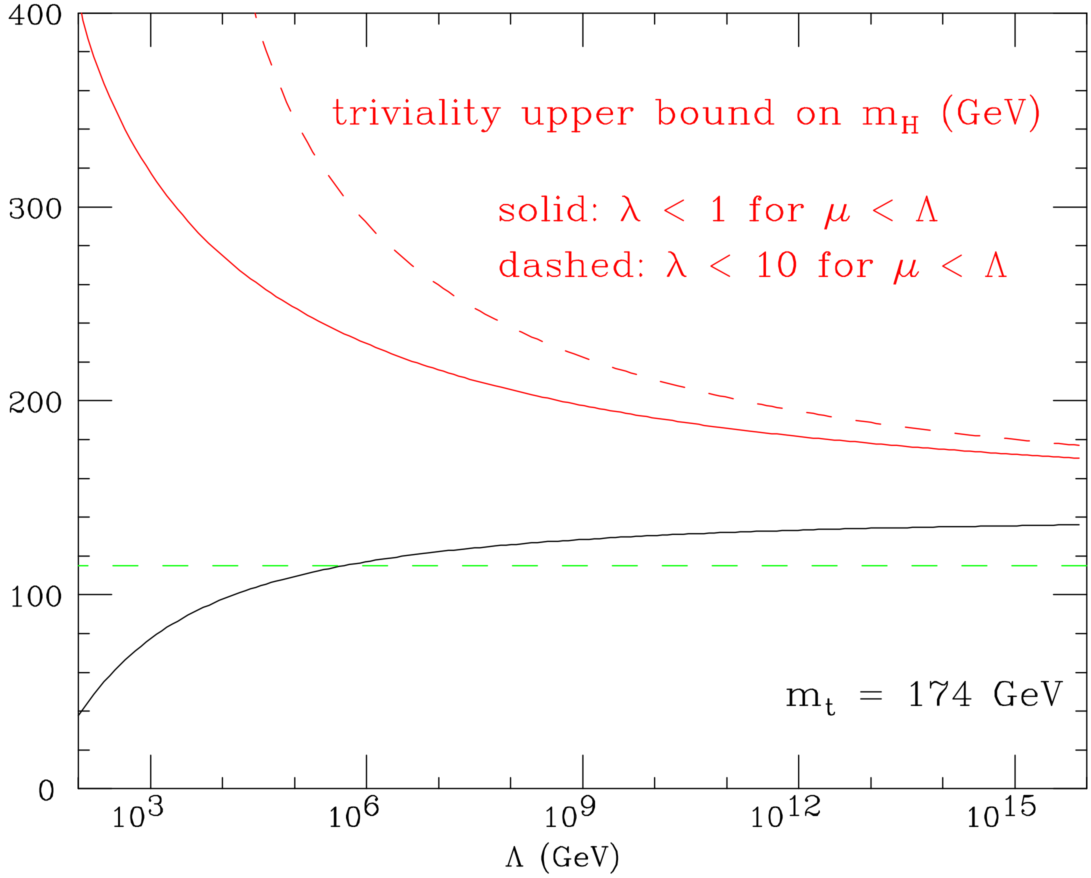
$$\mu^2 = m_Z^2 \exp\left[\frac{4\pi^2}{2\lambda(m_Z^2)}\right]$$

$\Rightarrow$ the theory is (probably) consistent at arbitrarily large energy scales only if $\lambda = 0$ (triviality).

Unacceptable: we are forced to admit that the SM is only an effective theory, valid up to some energy scale Λ.

larger $\lambda(m_Z) \Rightarrow$ smaller Λ

Requiring that λ stay within the perturbative domain for all scales $\mu < \Lambda$, we get an upper bound on m_H as a function of Λ.



The triviality upper bound is always much more stringent than the unitarity one, even if $\lambda < 10$ is allowed.

SSB induced by the Higgs mechanism with one scalar doublet and perturbative coupling very appealing:

1. relatively simple;
2. experimental information (m_H in the range 100-200 GeV) nicely consistent with theoretical constraints;
3. can accommodate a consistent description (even though not an explanation) of the observed pattern of flavor violation (GIM suppression, FCNC phenomena, CP violation).

Two questions:

1. Can we build a reasonable* extension of the standard model, with a heavy* Higgs?
2. How can the Higgs boson be so light? (the problem of naturalness)

**reasonable = consistent with precision data*

**heavy = close to the unitarity bound*

Question #1.

A model-independent approach ($Barbieri,\ Strumia$): fit precision data with

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i}{\Lambda^p} \mathcal{O}_i^{(4+p)}$$

where $\mathcal{O}_i^{(4+p)}$ are all the operators of dimension $4+p$, $p \geq 1$, consistent with the classical symmetries of $\mathcal{L}_{\text{SM}}$ (flavor-universal, B, L, CP-conserving).

No attempt to investigate the origin of the second term. The lowest (6) dimension operators are

$$\mathcal{O}_{WB} = (H^\dagger \tau^a H) W^a_{\mu\nu} B_{\mu\nu}$$
$$\mathcal{O}_H = |H^\dagger D_\mu H|^2$$
$$\mathcal{O}_{LL} = \frac{1}{2}(\bar{L}\gamma_\mu \tau^a L)^2$$
$$\mathcal{O}'_{HL} = i(H^\dagger D_\mu \tau^a H)(\bar{L}\gamma_\mu \tau^a L)$$
$$\mathcal{O}'_{HQ} = i(H^\dagger D_\mu \tau^a H)(\bar{Q}\gamma_\mu \tau^a Q)$$
$$\mathcal{O}_{HL} = i(H^\dagger D_\mu H)(\bar{L}\gamma_\mu L)$$
$$\mathcal{O}_{HQ} = i(H^\dagger D_\mu H)(\bar{Q}\gamma_\mu Q)$$
$$\mathcal{O}_{HE} = i(H^\dagger D_\mu H)(\bar{E}\gamma_\mu E)$$
$$\mathcal{O}_{HU} = i(H^\dagger D_\mu H)(\bar{U}\gamma_\mu U)$$
$$\mathcal{O}_{HD} = i(H^\dagger D_\mu H)(\bar{D}\gamma_\mu D)$$

Results: fitted values of Λ (in TeV) for each operator for different values of m_H at 95% C.L. ($\chi^2 < \chi^2_{SM} + 3.85$):

m_h	$115\,\mathrm{GeV}$		$300\,\mathrm{GeV}$		$800\,\mathrm{GeV}$	
c_i	-1	$+1$	-1	$+1$	-1	$+1$
$\mathcal{O}_{WB}$	9.7	10	7.5	—	—	—
$\mathcal{O}_{H}$	4.6	5.6	3.4	—	2.8	—
$\mathcal{O}_{LL}$	7.9	6.1	—	—	—	—
$\mathcal{O}'_{HL}$	8.4	8.8	7.5	—	—	—
$\mathcal{O}'_{HQ}$	6.6	6.8	—	—	—	—
$\mathcal{O}_{HL}$	7.3	9.2	—	—	—	—
$\mathcal{O}_{HQ}$	5.8	3.4	—	—	—	—
$\mathcal{O}_{HE}$	8.2	7.7	—	—	—	—
$\mathcal{O}_{HU}$	2.4	3.3	—	—	—	—
$\mathcal{O}_{HD}$	2.2	2.5	—	—	—	—

Interpretation:

- Values of Λ are generally quite large;
- Fit to data increasingly difficult with increasing m_H;
- For most operators, no value of Λ can be found that allows a Higgs boson mass close to the unitarity bound;
- A fit is possible, for m_H in the range 300–500 GeV, with suitable operators and Λ larger than ~ 10 TeV. However, a coincidence is needed.

A different approach: build explicit models in which non-standard physics compensate the effect of a heavy Higgs on fits to precision data.

It can be shown ($Peskin,\ Wells$) that this is indeed possible in many different contexts (models with extra gauge bosons, extra dimensions, ...). In some cases, they lead to observable effects at the next generation of high energy experiments.

These models show that one cannot close off the idea that m_H could be substantially larger than indicated by standard model fits.

Question #2: Naturalness and fine tuning

The mass of a scalar is not naturally small: it is not protected by any (ordinary) symmetry. As a consequence, it naturally tends (via radiative corrections) to become as heavy as the heaviest degree of freedom of the theory, unless the parameters are accurately chosen.

Directly seen in a simple example: consider a theory of two scalars interacting through the potential

$$V_0(\phi, \Phi) = \frac{m^2}{2}\phi^2 + \frac{M^2}{2}\Phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{\sigma}{4!}\Phi^4 + \frac{\delta}{4}\phi^2\Phi^2$$

with $M^2 \gg m^2 > 0$, and λ, σ, δ positive.

Is the mass hierarchy conserved at the quantum level?

Compute one-loop radiative corrections to m^2 by taking the second derivatives of the effective potential at the minimum $\phi = \Phi = 0$:

$$m^2_{\text{one loop}} = m^2(\mu^2) + \frac{\lambda m^2}{32\pi^2}\left(\log\frac{m^2}{\mu^2} - 1\right) + \frac{\delta M^2}{32\pi^2}\left(\log\frac{M^2}{\mu^2} - 1\right)$$

$$\mu^2 \frac{\partial m^2}{\partial \mu^2} = \frac{1}{32\pi^2}\left(\lambda m^2 + \delta M^2\right)$$

Corrections proportional to M^2 appear at one loop. One can choose $\mu^2 \sim M^2$ in order to get rid of them, but they reappear through the running of $m^2(\mu^2)$.

Only way out: choose the parameter so that

$$\lambda m^2 + \delta M^2 \sim 0 \to \frac{\delta}{\lambda} \sim \frac{m^2}{M^2}$$

This is what we usually call *fine tuning* of the parameters.

The same thing happens if $m^2 < 0$, $M^2 \gg \left|m^2\right| > 0$. In this case the the tree-level potential has a minimum at

$$\Phi = 0, \quad \phi^2 = -6m^2/\lambda \equiv v^2$$

and the symmetry $\phi \to -\phi$ is spontaneously broken. The degrees of freedom in this case are

$$\begin{aligned} &\Phi; && m_\Phi^2 = M^2 \\ &\phi' \equiv \phi - v; && m_{\phi'}^2 = -2m^2 = \lambda v^2/3 \end{aligned}$$

At one loop, v^2 is given by the minimization condition

$$\begin{aligned} m^2 + \frac{\lambda}{6}v^2 + \frac{1}{32\pi^2}\Bigg[\lambda\left(m^2 + \frac{\lambda}{2}v^2\right)\left(\log\frac{m^2 + \frac{\lambda}{2}v^2}{\mu^2} - 1\right) \\ + \delta\left(M^2 + \frac{\delta}{2}v^2\right)\left(\log\frac{M^2 + \frac{\delta}{2}v^2}{\mu^2} - 1\right)\Bigg] = 0. \end{aligned}$$

Following the same procedure as in the unbroken case one finds

$$m_{\phi'}^2 = \frac{\lambda v^2}{3} + \frac{v^2}{32\pi^2}\left[\lambda^2\log\frac{m^2 + \frac{\lambda}{2}v^2}{\mu^2} + \delta^2\log\frac{M^2 + \frac{\delta}{2}v^2}{\mu^2}\right]$$

with $v \sim M$ without a suitable tuning of the parameters.

In the case of the standard model Higgs, we are already faced with this problem. The correction to m_H^2 due to a loop of top quarks is given by

$$\delta m_H^2(top) = \frac{3G_F m_t^2}{\sqrt{2}\pi^2}\Lambda^2 \simeq (0.27\,\Lambda)^2$$

where we are assuming that the scale Λ that characterized non-standard physics acts a cut-off for the loop momentum.

With $\Lambda \sim 5$ TeV, as indicated by fits to precision data,

$$\delta m_H^2(top) \sim (1.5 \text{ TeV})^2$$

which is two orders of magnitude larger than the indirect measure of m_H.

A paradox: precision tests indicate small m_H and large Λ at the same time!

Supersymmetry offers a solution to the hierarchy problem, provided the mass splittings within supermultiplets are not much larger than the Fermi scale. Scalar masses are protected by a fermion-boson symmetry. No quadratic divergences.

In particular, the contribution to m_H^2 of a loop of s-top $\tilde{t}$ has the effect of replacing

$$\Lambda^2 \;\rightarrow\; m_{\tilde{t}}^2 \log \frac{\Lambda^2}{m_{\tilde{t}}^2}$$

without affecting fits to precision observables.

Supersymmetric grand unification of gauge couplings further supports this argument.

Supersymmetry at the Fermi scale is an interesting way out of the hierarchy problem.

(Are there others? extra dimensions?)

General features of supersymmetric models in the Higgs sector:

- At least two doublets
  $\Rightarrow$ two vev's v_1, v_2 (usually $v_2/v_1 = \tan\beta$)
  $\Rightarrow$ at least five physical degrees of freedom (usually h, H, A, $H^{\pm}$).
- the quartic scalar coupling λ is replaced by a combination of (squared) weak gauge couplings g, g'.
  $\Rightarrow$ no stability lower bound;
  $\Rightarrow$ the lightest scalar in the Higgs sector has a mass $\sim gv$, close to the weak vector boson masses.

Including loop corrections,

$$m_h^2 \simeq m_Z^2 + \frac{3}{\sqrt{2}\pi^2} G_F m_t^4 \log \frac{m_{\tilde{t}}^2}{v^2} \quad (\tan\beta \gtrsim 4)$$

Outlook

- Indications of a Higgs boson with mass in the range 100–200 GeV are definitely quite strong;
- The possibility of a heavier Higgs, whose effects are compensated by some kind of non-standard physics, is not ruled out, but seems quite unnatural (a debatable question, of course...)
- The hierarchy problem has now become so compelling that it can be cast in the form of a paradox. Supersymmetry at the Fermi scale is still the most appealing candidate for its solution.