

# TOP PHYSICS at the LINEAR COLLIDER

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## Outline

- Introduction: Past, Present and Future
- Linear Collider Specialties
- Top-Antitop at Threshold
- Top-Antitop at Higher Energies
- Summary

# The Top Quark in Numbers

- $m_t^? = 174.3 \pm 5.1 \text{ GeV}$  (CDF+D0)  
 $m_t^{\text{pole}} = 172 \pm 8 \text{ GeV}$  (LEP, ew.precision)
- $\sigma_{p\bar{p} \rightarrow t\bar{t}} = \begin{cases} 6.5^{+1.7}_{-1.4} \text{ pb} & \text{(CDF)} \\ 5.9 \pm 1.7 \text{ pb} & \text{(D0)} \end{cases}$  assumes SM
- $\left. \begin{array}{l} p_t \text{ spectrum} \\ \text{production process} \end{array} \right\}$  SM-like
- $\frac{B(t \rightarrow Wb)}{\Sigma B(t \rightarrow Wq)} \sim \frac{|V_{tb}|^2}{|V_{tb}|^2 + |V_{ts}|^2 + |V_{td}|^2} = 0.99 \pm 0.29$
- $B(t \rightarrow W_0 b) = 0.91 \pm 0.39$  (SM = 0.7)  
 $B(t \rightarrow W_+ b) = 0.11 \pm 0.15$  (SM = 0)
- $B(t \rightarrow Zq) < 0.33$   
 $B(t \rightarrow \gamma q) < 0.032$  → FCNC decays small

⇒ What else do we want?

More precise numbers? More quantities measured?

WHY?

⇒ Let's have a look into the past and the future.

# Pre-Top-Discovery Era < 1995

$$\Rightarrow \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, b + ? + \text{gauge symmetry}$$

- **Indirect hits toward  $(t, b)_L$  doublet flavor structure**

- ▷  $\mathcal{CP}$  in K physics,  $B - \bar{B}$  mixing, GIM mechanism

- ▷ bottom singlet causes FCNC:  $\Gamma_{B \rightarrow \mu\mu} \sim \Gamma_{B \rightarrow \mu\nu X}$

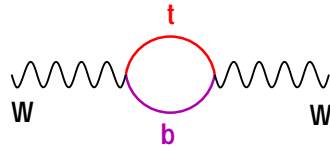
experiment:  $\Gamma_{B \rightarrow \mu\mu} < 10^{-5} \Gamma_{B \rightarrow \mu\nu X}$

- ▷ isospin of  $b$  from  $e^+ e^- \rightarrow b\bar{b}$  angular distr.

(PETRA, TRISTAN)

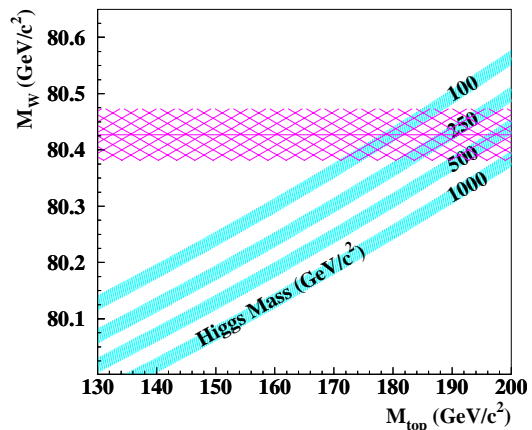
- **Ew precision tests predict  $m_t$  from isospin breaking**

→ top affects  $M_w$  by loop effects



- ▷ predicts  $m_t$  range

(Higgs model)



→ required precision of  $m_t$  for a given  $\delta M_w$

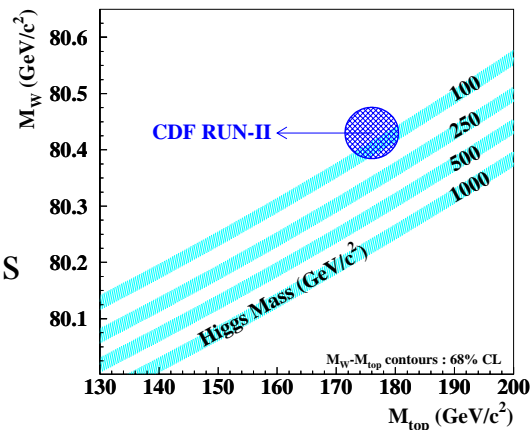
	Run I	Run II	LHC	LC(GIGAZ)
$\delta M_w$	40 MeV	20 MeV	20 MeV	6 MeV
$\delta m_t$	6 GeV	3 GeV	3 GeV	1 GeV
expected $\delta m_t$		2-3 GeV	1-2 GeV	< 0.1-0.2 GeV

# Post-Top-Discovery Era > 1995

- **Top precision measurements: top = top ?**
- **Top as unique QCD laboratory:**
  - ▷  $\Gamma_t \sim 1.5 \text{ GeV} \gg \Lambda_{\text{QCD}}$ : no toponia, T-mesons
    - dynamics dominated by perturbative QCD
    - top "almost real":  $m_t$ , spin, EDM, ... observables
- **Top as tool to uncover true "Next-SM"**
  - ▷ Higgs model:  $g_{tth} = \frac{\sqrt{2}m_t}{v}$  ("Goldberger-Treiman")
  - ▷ 4<sup>th</sup> generation:  $V_{tb} \ll 1, \Gamma_t \ll \Gamma_t^{\text{SM}}$
  - ▷ extended gauge groups: new gauge bosons affect top
  - ▷ SUSY: new decay & production mechanisms,  $\tilde{t}, \dots$
  - ▷ Large extra dimensions: KK gravitons  $\leftrightarrow$  top
  - ▷ etc., etc.

→ electroweak observables

- ▷  $M_w$  and  $m_t$ : predict and check  $m_h$  (Higgs model)
- ▷ constrains any new physics



## Next-SM Era > 200?

⇒ Example: SUSY (MSSM) ⇐

→ test consistency of Next-SM, constrain par. space

- Higgs mass measured, SUSY broken

$$m_{h,\text{light}}^2 = M_z^2 + G_F m_t^4 \ln\left(\frac{m_{\tilde{t}}}{m_t}\right) + \dots$$

→ required precision of  $m_t$  for a given  $\delta m_{h,\text{light}}$

	LHC	LC
$\delta m_{h,\text{light}}$	1 GeV	50 MeV
$\delta m_t$	4 GeV	200 MeV
expected $\delta m_t$	1-2 GeV	100-200 MeV

- SUSY top decays:

▷  $t \rightarrow H^+ b, \tilde{t} \tilde{\chi}^0$

▷ FCNC,  $\mathcal{CP}$  top couplings

- Light top squarks  $m_{\tilde{t}} \ll M_{\text{SUSY}}$

→ off-diagonal element  $\propto m_t M_{\text{SUSY}}$

in stop mass matrix

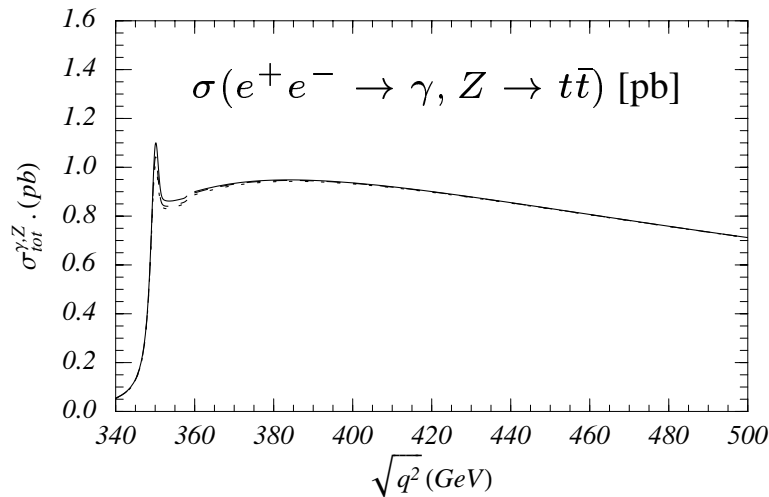
⇒ Aim of future colliders for top physics:

Measure all top properties as accurate as possible!

$m_t, V_{tb}, \Gamma_t, g_{tth}$ , couplings, spin,  
rare decays, FCNC, CP-properties,  
production mechanism, etc.

# Linear Collider and Top Physics

- $e^+e^-$  collider,  $E_{\text{cm}} = M_z, 350 \text{ GeV} - 5 \text{ TeV}$
- Lumi:  $10^{34} - 10^{35} \text{ cm}^{-2} \text{ s}^{-1} \rightarrow 100 - 1000 \text{ fb}^{-1} / \text{year}$
- Designs: TESLA: supercond.,  $E_{\text{cm}}^{\text{max}} \sim 1 \text{ TeV}$   
 N/JLC: nor. cond.,  $E_{\text{cm}}^{\text{max}} \lesssim 1 \text{ TeV}$   
 CLIC: nor. cond.,  $E_{\text{cm}}^{\text{max}} \lesssim 5 \text{ TeV}$
- $e^+e^- \xrightarrow{\gamma, Z} t\bar{t}$  main mechanism for  $E_{\text{cm}} < 1 \text{ TeV}$

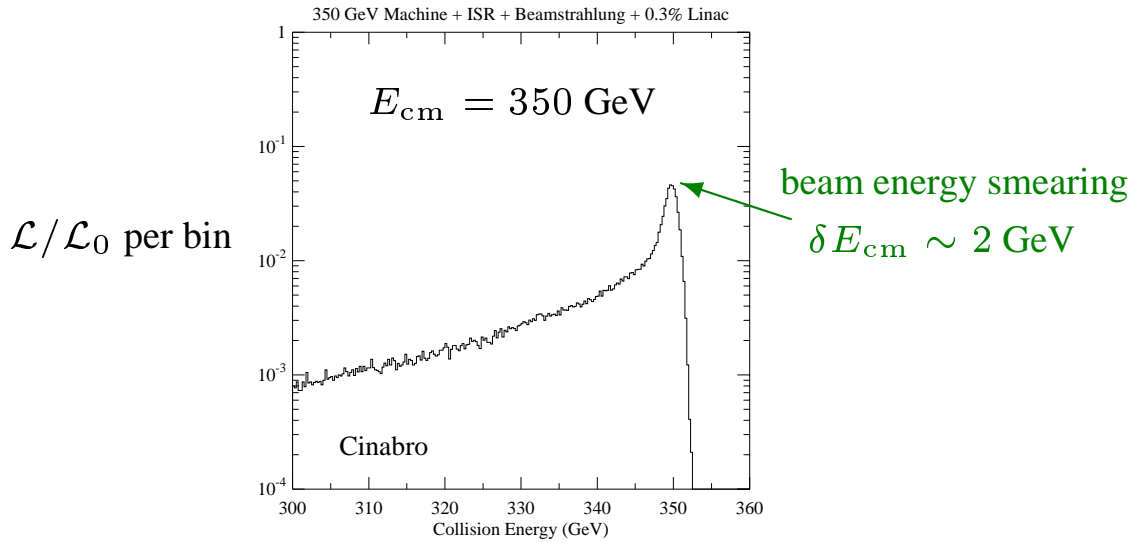


- $e^+e^- \rightarrow e\nu t b$  } competitive for  $E_{\text{cm}} > 1 \text{ TeV}$ ,  
 $e^+e^- \rightarrow \nu v t \bar{t}$  }  $W_L, Z_L$  scattering! single top!

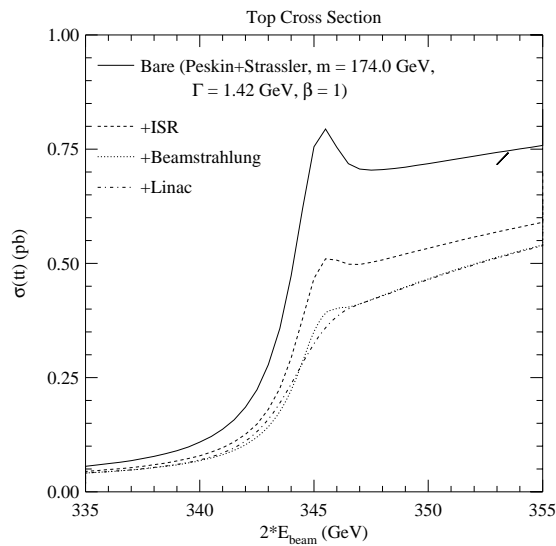
## LC Specialties

- **Statistics:** LC  $\sim 10^5 t\bar{t}$  pairs  $\rightarrow \sigma_{\text{tot}} < 1$  pb  
LHC  $\sim 10^8 t\bar{t}$  pairs  $\rightarrow \sigma_{\text{tot}} \approx 850$  pb
  - $E_{\text{cm}}$  **well known, tunable**  $\rightarrow$  **threshold & continuum**
    - ▷ ISR+beam strahlung+energy spread  $\Rightarrow$  lumi spect.
    - ▷ tune QCD phases at threshold  $\Rightarrow$  CP studies
  - **Electron beam polarizable:**  $\rightarrow e^-: 80\%$ 
    - ▷ reduction of background (e.g.  $W^+W^-$  with RH  $e^-$ )
    - ▷ tuning to enhance signals (non-SM) for spin obs.
  - **Clean environment:**
    - ▷ smaller background, more events used for physics
    - ▷ systematic uncertainties small
  - $\gamma\gamma, \gamma e$  **options:**
    - ▷ unique spin configurations
    - ▷ alternative production mechanisms, single top
- $\Rightarrow$  I'm going to talk about some interesting aspects of top physics at the LC that take advantage of the special LC features.

# Luminosity Spectrum



→ Lumi spectrum needs to be monitored (Bhabba)  
 important for  $t\bar{t}$  threshold measurements





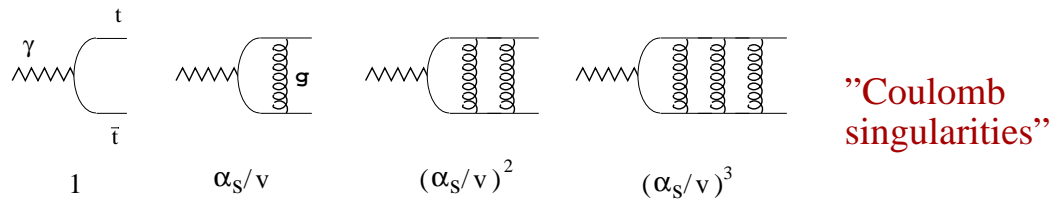
# Top Physics at the $t\bar{t}$ Threshold

$$E_{cm} \approx 350 \text{ GeV} \Rightarrow v_{\text{top}} = \sqrt{1 - \frac{4m_t^2}{E_{cm}^2}} \ll 1$$

→ top quarks move with **non-relativistic** speed

→ unique for the Linear Collider!

- **Top quarks have a lot of time to interact**



▷ perturbation theory in  $\alpha_s$  breaks down

▷ Bound state dynamics  $\Rightarrow$  Schrödinger equation

- **Top decay:  $\Gamma_t \approx 1.5 \text{ GeV} \gg \Lambda_{\text{QCD}}$**

▷ no formation of toponia

▷ smooth line-shape for  $\sigma(e^+e^- \rightarrow t\bar{t})$ , 1S peak

▷  $t\bar{t}$  dynamics described by perturbative QCD

- **Top rest system =  $t\bar{t}$  cm system +  $\mathcal{O}(v^2)$  corr.**

▷ facilitates spin, polarization measurements

- **QCD phases scale with binding energy  $E_{cm} - 2m_t$**

▷ QCD phases are tunable and calculable

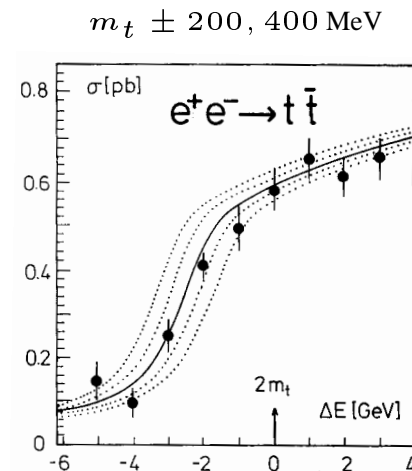
▷ QCD phases compete with new  $\mathcal{CP}$  phases

# Top Mass Measurement at Threshold

- Top mass reconstruction  $\Rightarrow$  standard
- Threshold Scan

$\rightarrow$  required: measurement of  $\sigma_{\text{tot}}$

- ▷ count number of  $t\bar{t}$  events
- ▷ no invariant mass reconstruction
- ▷  $t\bar{t}$  is in a color singlet state !  
 $\Rightarrow$  final state interactions, gluon radiation, Bose-Einstein correlation, etc. not relevant



$\rightarrow$  the rise of  $\sigma_{\text{tot}}$  defines  $2m_t$  !

- ▷ measures short-distance "threshold masses", adapted to low virtuality of top, e.g.  $1S$  mass,  $m_t^{1S}$
- ▷  $t\bar{t}$  is a color singlet  $\Rightarrow \delta m_t^{1S, \text{theory}} \ll \Lambda_{\text{QCD}}$
- ▷  $m_t^{1S}$  can be related precisely to  $\overline{\text{MS}}$  mass  $\overline{m}_t(\overline{m}_t)$
- ▷ experimental uncertainties:  
 $\mathcal{L} = 100 \text{ fb}^{-1}$  distributed to 10 scan points  
 $\Rightarrow \delta m_t^{1S, \text{exp}} < 100 \text{ MeV}$  (Peralta, Martinez, Miquel)

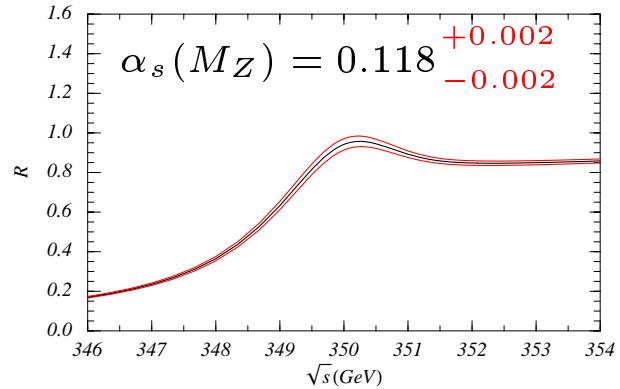
# Sensitivity of $\sigma_{\text{tot}}^{\text{theory}}$ to $\alpha_s, g_{tth}, \Gamma_t$

$\alpha_s$  larger

$\Rightarrow$  binding force larger

$$V_{\text{QCD}}(r) = -\frac{3\alpha_s}{4r}$$

$\Rightarrow \sigma_{\text{tot}}$  enhanced

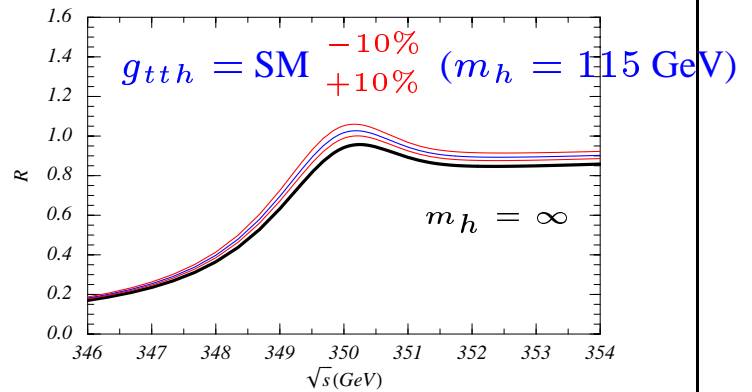


$g_{tth}$  larger

$\Rightarrow$  binding force larger

$$V_h(r) = -\frac{g_{tth}}{4r} e^{-m_h r}$$

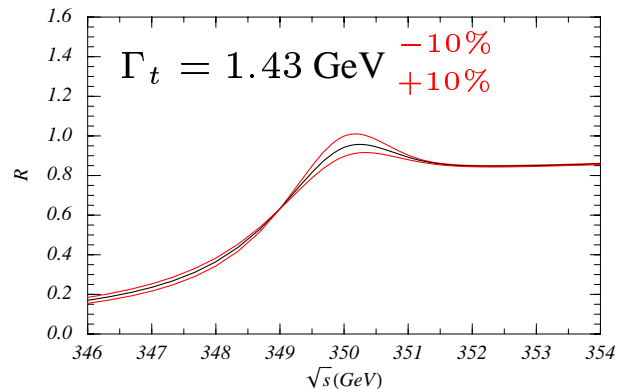
$\Rightarrow \sigma_{\text{tot}}$  enhanced



$\Gamma_t$  smaller

$\Rightarrow$  peak more pronounced

(see also F-B asymmetry)



- Comparison: experimental uncertainties

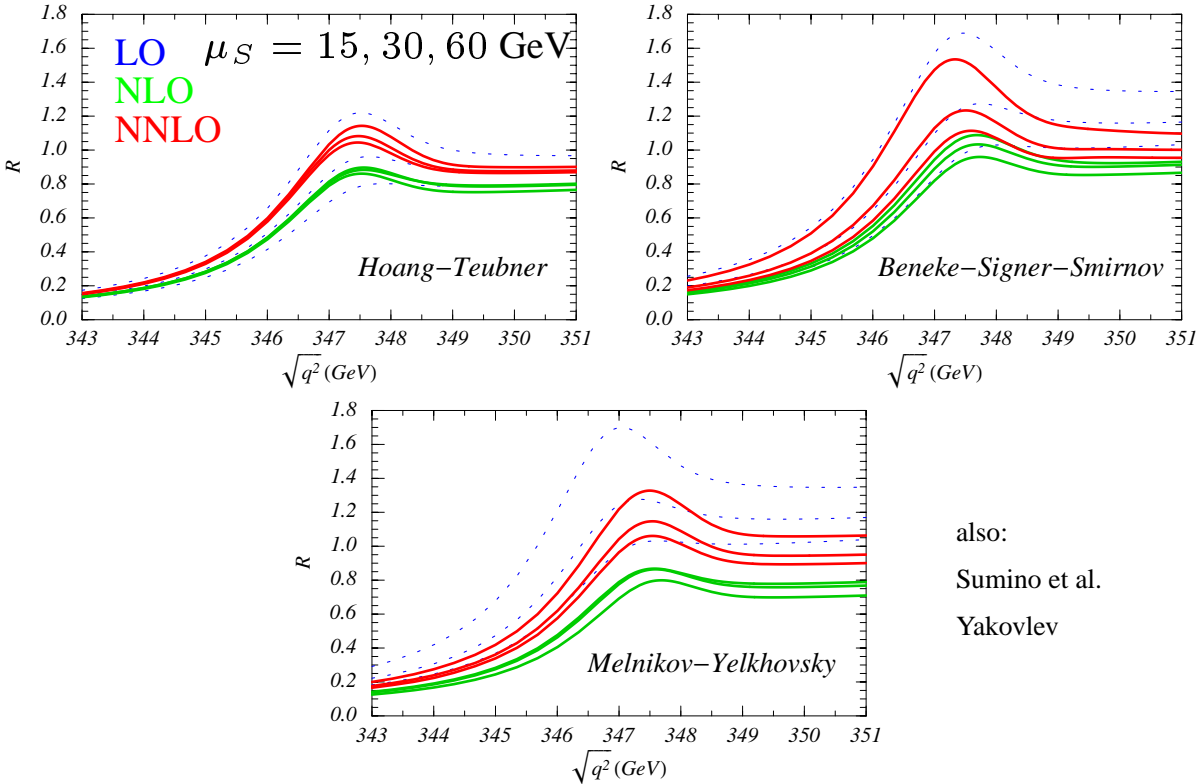
$$\mathcal{L} = 100 \text{ fb}^{-1} : \quad \delta\alpha_s(M_Z) \lesssim 0.002$$

$$(< 2000) \quad \delta\Gamma_t/\Gamma_t \lesssim 5 - 10\%$$

$$\delta g_{tth}/g_{tth} \lesssim 5 - 10\%$$

# Theoretical Situation for $\sigma_{\text{tot}}$

⇒ Until 2000 a number of NNLO calculations had been carried out



▷ peak position under control

$$\longrightarrow \delta m_t^{\text{theo}} < 100\text{--}200 \text{ MeV}$$

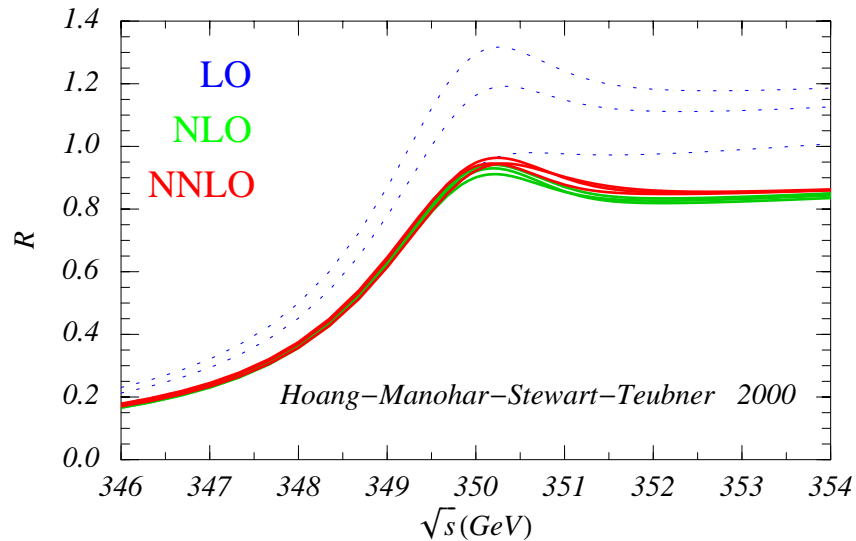
▷ large normalization uncertainties:  $\frac{\delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}} \sim 20\%$

→ measurements of  $\alpha_s$ ,  $\Gamma_t$ ,  $g_{ttH}$  in jeopardy  
old experimental studies appeared unrealistic

# New Calculation of the Cross Section

⇒ 2000: New renormalization group improved calculation at NNLL order

→ summation of  $(\alpha_s \ln v)^n$  to all orders



$$m_t^{1S} = 175 \text{ GeV}$$

$$\Gamma_t = 1.43 \text{ GeV} \quad \nu = 0.15-0.4$$

$$\alpha_s(M_Z) = 0.118$$

▷ peak position under control

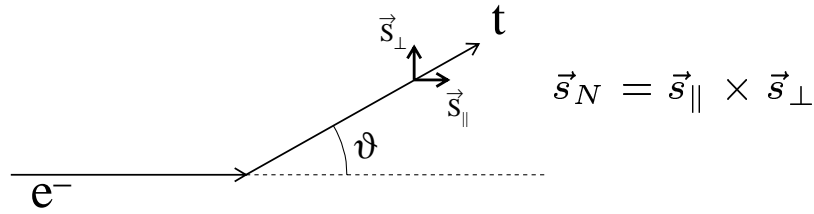
$$\longrightarrow \delta m_t^{\text{theo}} < 100-200 \text{ MeV}$$

▷ small normalization uncertainties:  $\frac{\delta \sigma_{t\bar{t}}}{\sigma_{t\bar{t}}} \sim 2\%$

→ measurements of  $\alpha_s$ ,  $\Gamma_t$ ,  $g_{tth}$  at the % level are feasible

# Top Polarization

⇒ **Top rest system =  $e^+e^-$  c.m. system**



$P_{\parallel}$ : dominated by S-wave

$P_{\perp}, P_N$ : sensitive to QCD phases

sensitive to T-odd  $\gamma t\bar{t}, Zt\bar{t}$  couplings

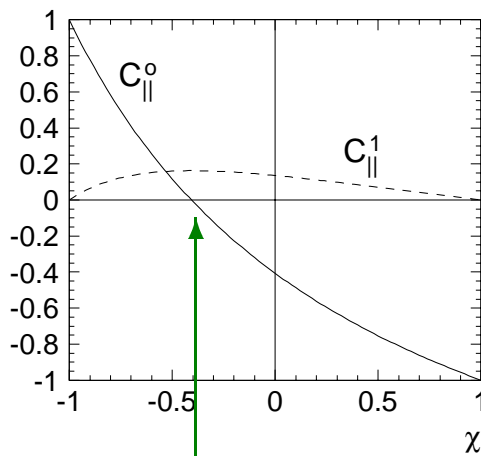
(EDM's)

- **Top polarization can be tuned by  $e^-, e^+$  polarization**

$$\mathcal{P}_{\parallel}(\mathbf{p}, E, \chi) = C_{\parallel}^0(\chi) + C_{\parallel}^1(\chi) \varphi_R(p, E) \cos \vartheta$$

$$\mathcal{P}_{\perp}(\mathbf{p}, E, \chi) = C_{\perp}(\chi) \varphi_R(p, E) \sin \vartheta$$

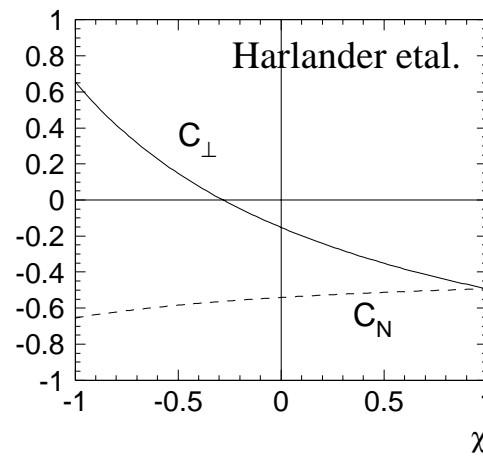
$$\mathcal{P}_N(\mathbf{p}, E, \chi) = C_N(\chi) \varphi_I(p, E) \sin \vartheta$$



$$C_{\parallel}^0 = 0$$

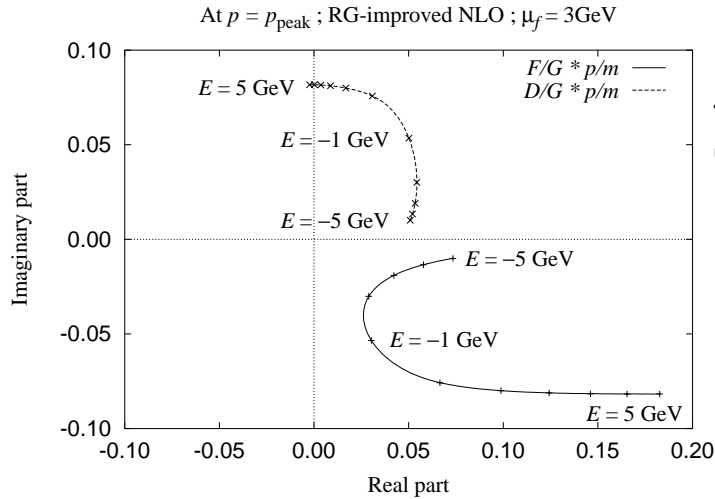
$$E_{\text{cm}} = 2m_t$$

QCD phases only



$$\chi = \frac{P_{e^+} - P_{e^-}}{1 - P_{e^+} P_{e^-}}$$

- **Top polarization can be tuned by changing  $E_{\text{cm}}$**   
 → QCD phases change with  $E = E_{\text{cm}} - 2m_t$



Jezebek, Nagano,  
Sumino

- **Sensitivities for  $\mathcal{CP}$  top EDM ( $t\bar{t}\gamma$ ), Z-EDM ( $t\bar{t}Z$ ), chromo-EDM ( $t\bar{t}G$ )**

$$\text{e.g. } \mathcal{L}_{\text{EDM}} = -\frac{ed_{t\gamma}}{2m_t} (\bar{t}i\sigma^{\mu\nu}\gamma_5 t)\partial_\mu A_\nu$$

	LHC ( $10 \text{ fb}^{-1}$ )	LC (500 GeV) ( $50 \text{ fb}^{-1}$ )	LC ( $t\bar{t}$ threshold) ( $50 \text{ fb}^{-1}$ )
EDM ( $d_{t\gamma}$ )	-	$10^{-1}$ – few $10^{-2}$	$10^{-1}$
Z-EDM ( $d_{tZ}$ )	-	$10^{-1}$ – few $10^{-2}$	$10^{-1}$
chr.-EDM ( $d_{tg}$ )	$10^{-2}$ – few $10^{-3}$	$\mathcal{O}(1)$	$10^{-1}$

# Top Physics at Higher Energies

$$E_{cm} \gtrsim 2m_t + 10 \text{ GeV}$$

- **Top mass**

- ▷ invariant mass reconstruction for single top
- ▷ top still slow for  $E_{cm} = 500 \text{ GeV}$
- ▷ limitation:  $\rightarrow$  top colored, on-shell pole unphysical (interconnection, jet energy, gluon radiation, ...)

$$\Rightarrow \delta m_t^{\text{sy st}} > \Lambda_{\text{QCD}} \sim 300 \text{ MeV}$$

$$\text{ECFA/DESY (1991): } \delta m_t = 500 \text{ MeV}$$

$$\text{G.P. Yeh (1999): } \delta m_t = 200 \text{ MeV (no systematics)}$$

- **Top Yukawa coupling**  $\rightarrow e^+ e^- \rightarrow t\bar{t}H$

$$\text{Juste et al. (1999): } E_{cm} = 800 \text{ GeV}, m_h = 120 \text{ GeV},$$

$$\mathcal{L} = 1000 \text{ fb}^{-1} (\sim 3 \text{ years á } 10^{34} \text{ cm}^{-2} \text{s}^{-1})$$

$$\Rightarrow \delta g_{tth}/g_{tth} \sim 5.5\% (q\bar{q}b\bar{b}b\bar{b}l\nu, q\bar{q}q\bar{q}b\bar{b}b\bar{b} \text{ channels})$$

$$\rightarrow \text{LHC: } \delta g_{tth}/g_{tth} < 16\%, \mathcal{L} = 100 \text{ fb}^{-1}$$

- **Strong coupling**

- ▷  $\sigma_{\text{tot}}$  maximal for  $E_{cm} \approx 400 \text{ GeV}$

- $\rightarrow \alpha_s$  from radiative corrections

- $\rightarrow$  systematic error dominated by luminosity spectrum

$$\text{Bernreuther (1999): } \delta\alpha_S(M_z) \sim 0.007$$



- **FCNC top-charm- $\gamma/Z$  coupling**

- ▷ consider  $e^+e^- \rightarrow t\bar{c} + \bar{t}c$  instead of rate top decay

- sensitivity kinematically enhanced

- Han,Hewett (1998):  $\mathcal{L} = 500 \text{ fb}^{-1}$ ,  $E_{\text{cm}} = 500 \text{ GeV}$

- sensitivities on FCNC couplings  $< 1\%$  achievable

- competitive to LHC from rare decay  $t \rightarrow c\gamma/Z$

- **Top decays in new light particles** (1991 studies)

- $t \rightarrow H^+b$ :  $3\sigma$  disc. for all  $\tan\beta$  (favored in MSSM)

$$\mathcal{L} = 10 \text{ fb}^{-1}, E_{\text{cm}} = 500 \text{ GeV},$$

$$m_t = 150 \text{ GeV}, m_{H^+} = 50 \text{ GeV}$$

- $t \rightarrow \tilde{t} + \tilde{\chi}^0$ : needs  $\mathcal{L} = 30 \text{ fb}^{-1}$  for  $3\sigma$

- **Spin correlations & anomalous couplings**

- ▷ top spin configuration via lepton angular distr.

- ▷ Off-diagonal basis: (Parke, Shadmi)

- spin basis in which non-SM are easy to detect

- SM:  $e_L^- e_R^+ \rightarrow t_\uparrow \bar{t}_\downarrow$  contains  $\sim 100\%$  of events  
for any energy

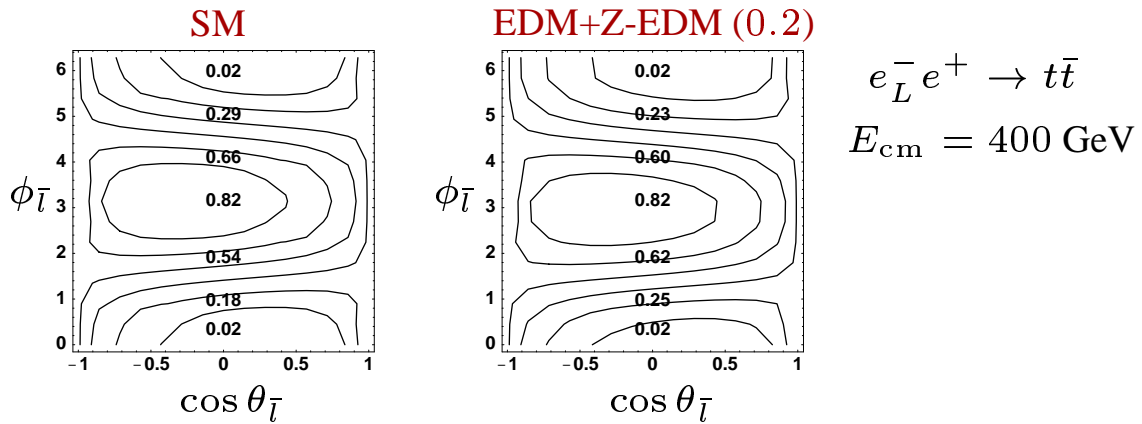
- not affected by QCD radiative corrections

- does not exist at the LHC due to gluon fusion!

# Double diff. angular distr. of leptons from top decay

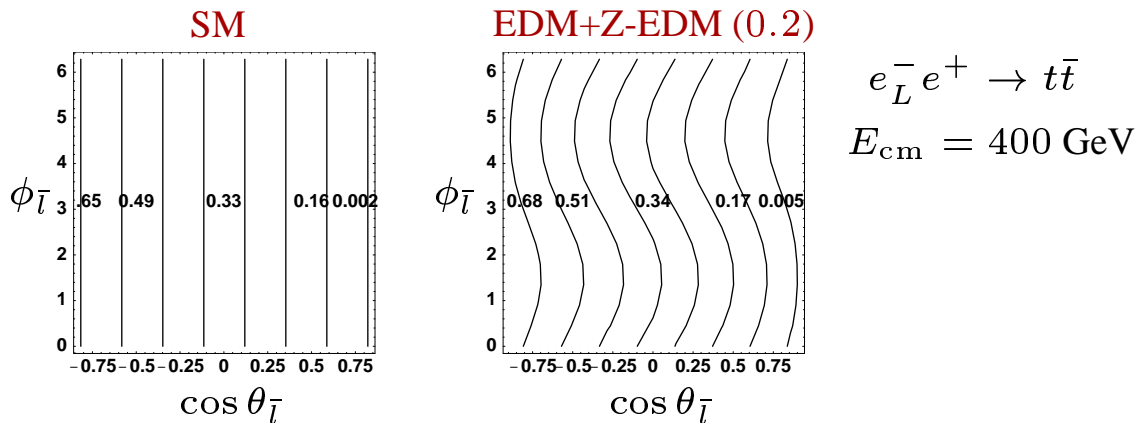
(Kiyo, Kodaira, Morii, Nasuno, Parke)

## • Helicity Basis



→ anomalous effects barely visible

## • Off-diagonal bases → $\phi_{\bar{l}}$ and $\cos \theta_{\bar{l}}$ in different system



→ anomalous effects clearly visible

→ asymmetry:  $A \sim \int_0^\pi d\phi_{\bar{l}} - \int_\pi^{2\pi} d\phi_{\bar{l}}$

→ QCD contributions to  $A$ :  $< 0.5\%$

→ anomalous coupling  $\mathcal{O}(0.1)$  detectable

## Conclusions & Outlook

- Top quark is a window to "New Physics"  
 $m_t$  large  $\rightarrow$  EWSB, non-SM effects
- Top quark lifetime  $\ll 1/\Lambda_{\text{QCD}}$   
 $\rightarrow$  unique QCD laboratory  
 $\rightarrow$  almost "real particle"  $\rightarrow$  ew physics
- Measure top quark as precise as possible  
 $\Rightarrow$  be prepared for surprises
- Linear Collider: versatile instrument for precision top studies (and much more !)  
 $\rightarrow$  complementary/competitive to hadron colliders
  - ▷  $E_{\text{cm}}$  tunable, well known  $\rightarrow t\bar{t}$  threshold
  - ▷ clean, background-low environment
  - ▷ LC:  $10^5$  tops  $\longleftrightarrow$  LHC:  $10^8$  tops
  - ▷ beam polarization
  - ▷  $\gamma\gamma, \gamma e$  options