NEUTRINOS, OSCILLATIONS, AND BIG BANG NUCLEOSYNTHESIS

1. Basics of BBN

2. Role of neutrinos in BBN

3. Restrictions on ν-oscillations from BBN (new and old)

4. Generation of lepton asymmetry by oscillations
   late-time leptogenesic,
   charge asymmetry (C, CP) in ν-sector

5. Sterile neutrinos and warm dark matter

6. Bounds on heavy (> MeV) neutrinos from BBN, astrophysics and direct experiments

BIG BANG NUCLEOSYNTHESIS.

(one of 3 pillars of the Standard Cosmological Model; the other two: CMBR and GR)

Observations: Hydrogen (H) 75% by mass
^4He – 25% —"—

other elements ⇒ the rest

^2H (deuterium) ~ $10^{-5}-10^{-4}$ by number

^3He —"— —"—

^7Li (a few) × 10^{-10} —"—

Span 10 orderds of magn.

very
(RATHER) well explained by big bang ~~cosmology~~ nucleosynthesis

→ **BBN**

Basics of BBN

Formation of D, ^3He, ^4He, ^7Li

$T = (\sim MeV) \to 0.07\ MeV$

$t = 0.1\ sec \to 10^3\ sec$

Particle content: $e^\pm$, γ, thermal equil. T_γ

ν_e, ν_μ, ν_τ, $T_\nu = T_\gamma$ down to $T \sim m_e$

$T_\nu \approx 0.7\, T_\gamma$ for $T \ll m_e$

Baryons $\underbrace{n_n \sim n_p \sim 10^{-10} \otimes (\text{a few})\, n_\gamma}$

known from observations with very poor accuracy,

in fact the <u>observed</u> baryons $\lesssim$ <u>0.1</u> of the necessary amount.

$\eta = \dfrac{n_B}{n_\gamma}$ is the only unknown parameter of calculations in <u>standard</u> model

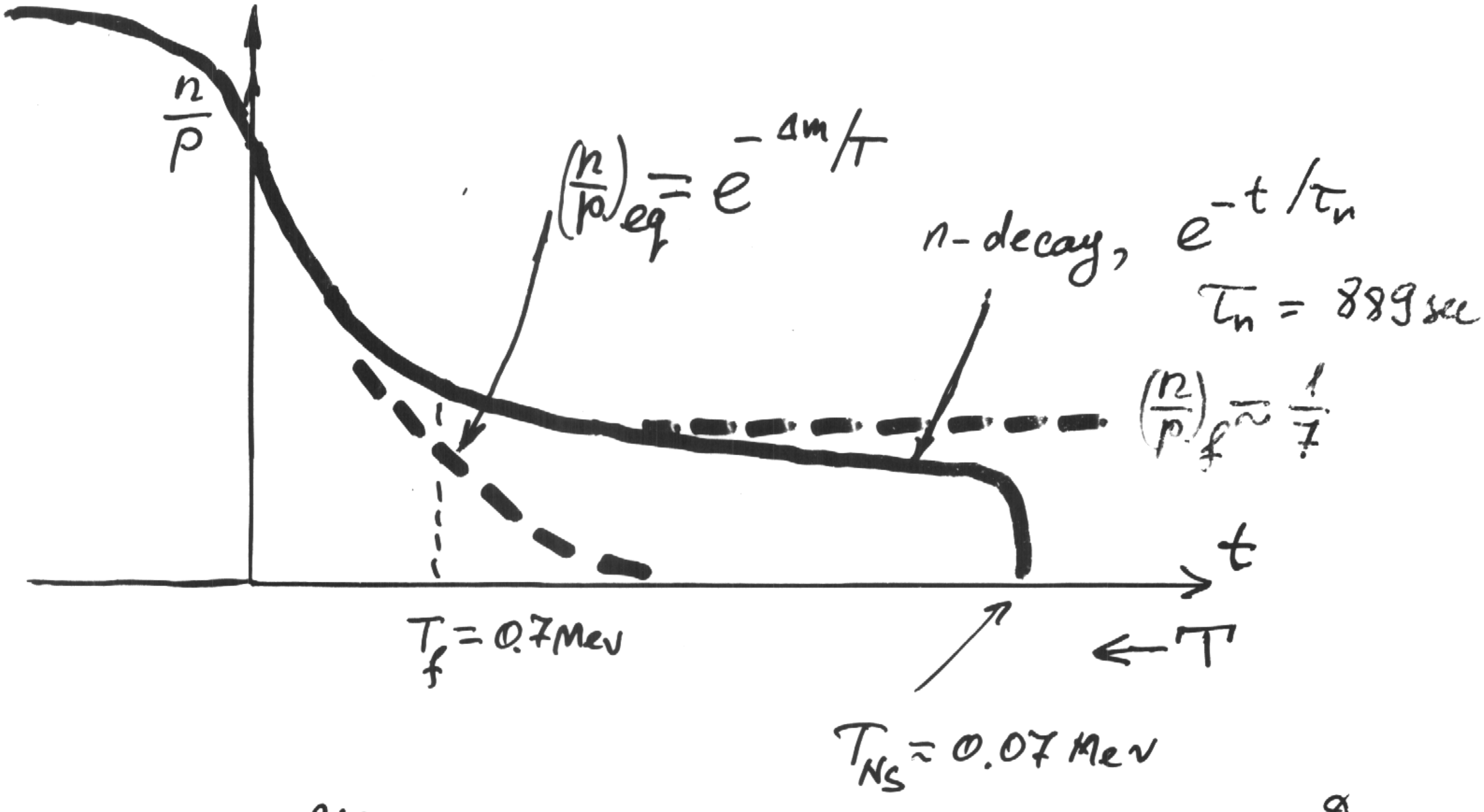
B = baryons = p + n.

$\underline{(n/p)\text{-freezing}}$ ← preparation of building blocks

BBN F3 E2.3

$$\left.\begin{array}{l} n + e^+ \leftrightarrow p + \bar{\nu} \\ n + \nu \leftrightarrow p + e^- \end{array}\right\} \quad \text{In equilibrium} \quad \left(\frac{n}{p}\right) = \exp(-\Delta m / T)$$

$\Gamma_{react} \sim T^5$, $H \sim T^2$ $\qquad T_{freez} = 0.7 - 0.6 \text{ MeV}$



$p + n \to \underline{{}^2H} + \gamma$ $\quad \sim \underline{(\text{a few}) \cdot 10^{-5}}$ by number $\quad E_B^{D} \simeq 2$ MeV

$N + D \to \underline{({}^3H, {}^3He)}$ $\quad$ — " —

$+ N \to \underline{{}^4He}$ $\quad \sim \underline{25\%}$ by mass $\quad E_B^{{}^4He} \simeq 28$ MeV

mass gap, no $A = 5$

$A_1 + A_2 \to A_6$ or $A_7 = {}^7Li$ $\quad \underline{(\text{a few}) \times 10^{-10}}$

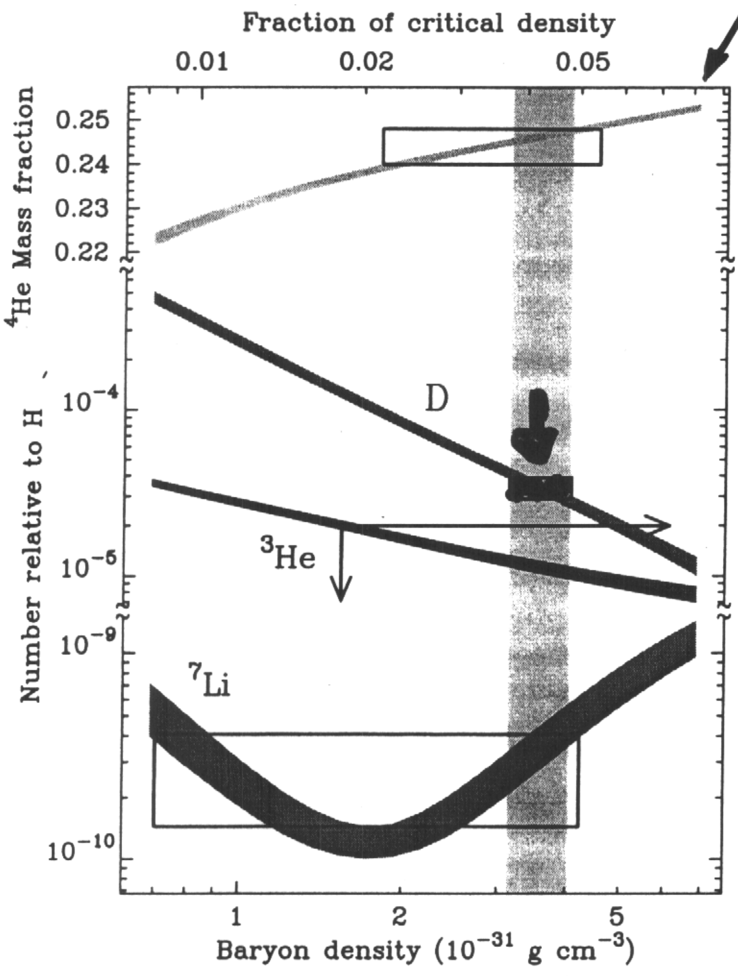
Tytler, O'Meara, Suzuki, Lubin
astro-ph/0001318

E2.4

→ $\Delta k_\nu < 0.2$

larger n_B, larger T_{NS}
more n survive
$He^4 \nearrow$

span 9-10 orders of magnitude



} larger $\frac{n_B}{n_\gamma}$ more probable
$D, He^3 \to He^4$
$D\downarrow, He^3\downarrow$

← the only unknown parameter in the standard model

Figure 2: Abundances expected for the light nuclei ^{4}He, D, ^{3}He and ^{7}Li (top to bottom) calculated in standard BBN. New estimates of the nuclear cross-section errors from Burles et al. (1999a) and Nollet & Burles (1999) were used to estimate the 95% confidence intervals which are shown by the vertical widths of the abundance predictions. The horizontal scale, η, is the one free parameter in the calculations. It is expressed in units of the baryon density or critical density for a Hubble constant of 65 kms^{-1}Mpc^{-1}. The 95% confidence intervals for data, shown by the rectangles, are from Izotov and Thuan 1998a (^{4}He); Burles & Tytler 1998a (D); Gloeckler & Geiss 1996 (^{3}He); Bonifacio and Molaro 1997 (^{7}Li extended upwards by a factor of two to allow for possible depletion).

} Relation between primordial and present-day abundances? ?
** Wishful thinking?

E2.5 [17
C8

Tytler et al: "High D either does not exist or extremely rare"

Is 15% excluded?

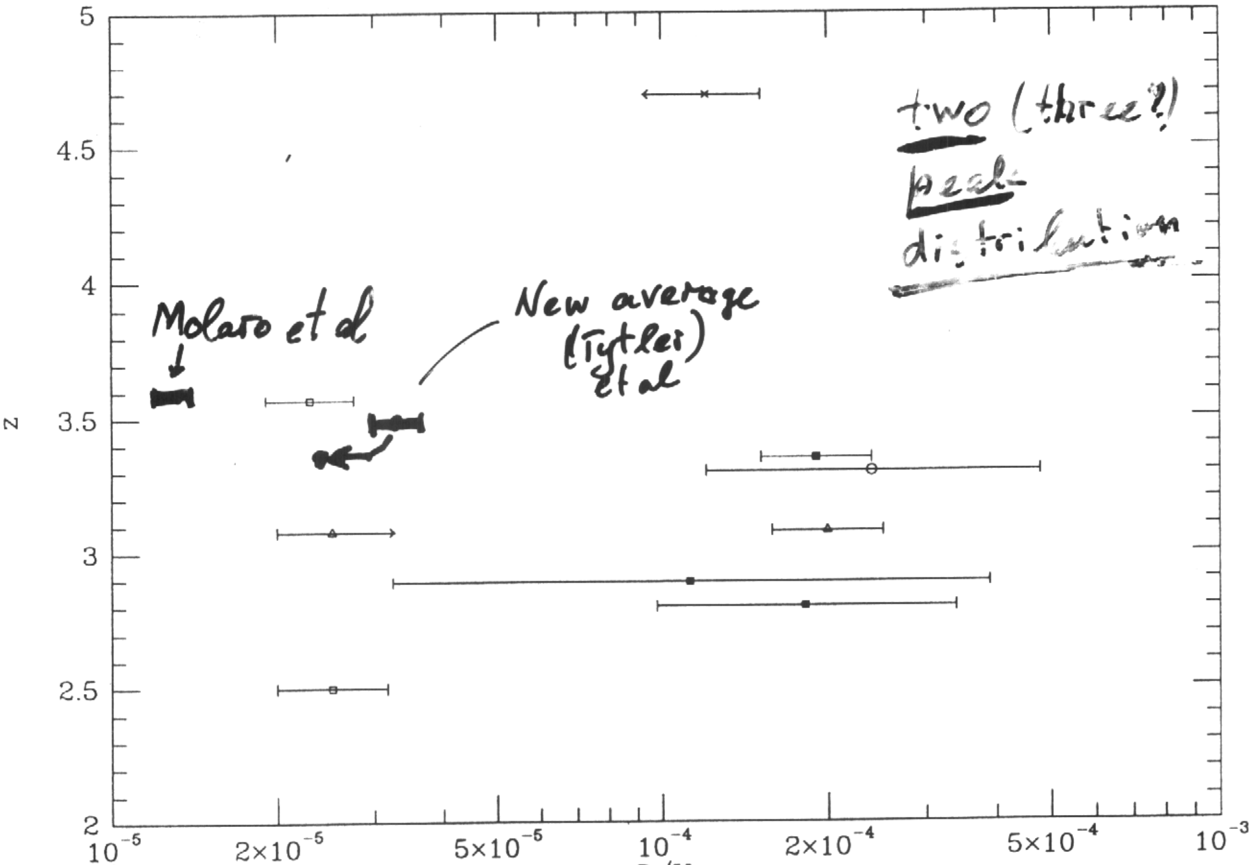


Figure : A summary of the D/H measurements in quasar absorption systems. The observations were made by (○) Carswell *et al.* (1994) and Songaila *et al.* (1994), (■) Rugers & Hogan (1996a,b,c), (△) Carswell *et al.* (1996), (×) Wampler *et al.* (1996), and (□) Tytler, Fan, & Burles (1996) and Burles & Tytler (1996). The Rugers & Hogan (1996a) (■) point and the (○) point have been separated in redshift for clarity. See the text for general details.

Z=0.5

Webb

Tytler

Role of neutrinos in BBN

1. Cooling rate: $\rho = \frac{3}{32\pi} \frac{m_{pl}^2}{t^2} = \frac{\pi^2}{30} g_* T^4$,

$$g_* = \underset{\gamma}{2} + \underset{e^\pm}{\frac{7}{8} \cdot 4} + \underset{\nu}{N_\nu \frac{7}{8} \cdot 2} = 10.75$$

for $N_\nu = 3$
$(\nu_e, \nu_\mu, \nu_\tau)$

a) $\left(\frac{n}{p}\right)_{equilibrium} \sim e^{-\Delta m/T}$, $T_f \sim g_*^{-1/6}$

b) time when $T = T_{NS} \simeq 60\text{-}70$ keV, $t_{NS} \sim g_*^{-1/2}$

T_{NS} is fixed by n_B/n_γ and nuclear binding energy

More N_ν → smaller T_f ⇒ larger $\left(\frac{n}{p}\right)_f$

→ smaller t_{NS} ⇒ less n decayed to $T = T_{NS}$

⇓

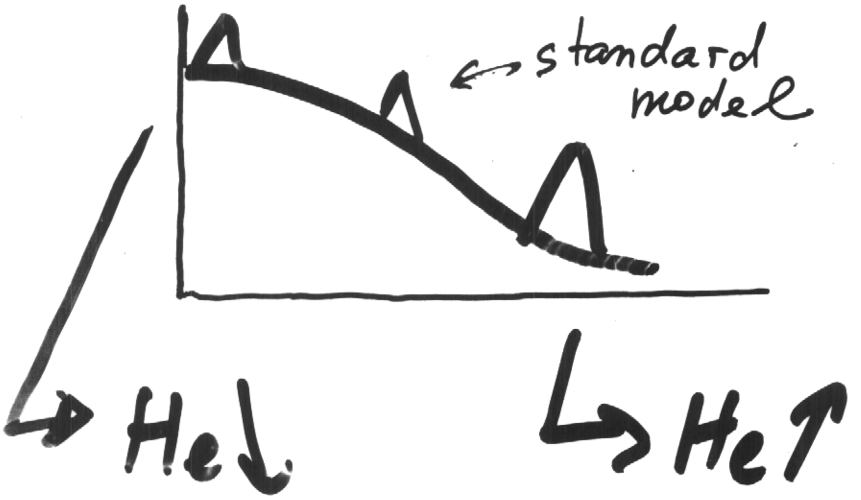
both effects give larger $\frac{n}{p}$ at NS ⇒

⇒ more He^4

$\Delta N_\nu = 1 \Rightarrow \Delta He^4 \simeq 5\%$

2. Spectrum of ν_e (normally $f_{\nu_e} = \frac{1}{e^{E/T}+1}$)

$$\begin{cases} n+\nu_e \leftrightarrow p+e \\ n+e^+ \leftrightarrow p+\bar{\nu}_e \end{cases}$$



$\frac{n}{p} = \frac{1}{6} - \frac{1}{7}$.

$\frac{n}{p} \sim e^{+\xi}$

3. Lepton asymmetry: $f_{\nu_j} = [e^{E/T - \xi} + 1]^{-1}$

$\frac{\mu}{T} \equiv \xi$, $\quad |\xi_{\nu_\mu, \nu_\tau}| < 2-3$

$|\xi_e| <$ (a few) $\cdot 10^{-2}$

⇓

(d) Differential neutrino heating – distorts spectrum

$T_\nu = T_\gamma = T_e$ at $T > m_e$

$T_\nu < T_\gamma = T_e$ at $T < m_e$, $e^+e^- \to 2\gamma$ after ν decoupling at 2-3 MeV

Residual $e^+e^- \to \nu\bar{\nu}$ distorts spectrum (exact solution of kin. eqn.)

$\underline{\delta\rho/\rho \sim 10^{-2}}$, $\underline{T_e > T_\nu}$ $\quad \delta He/He \sim 10^{-4}$.

Effects of $\nu_{active} \leftrightarrow \nu_{sterile}$ in BBN

1. Creation of new relativistic species
   $\underline{K_\nu > 3}$

2. Generation of lepton asymmetry in active sector $(N_{\nu_e} \neq N_{\bar{\nu}_e})$

3. Distortion of ν_e spectrum

Light sterile neutrinos

$$\begin{cases} \nu_1 = \nu_a \cos\vartheta_{vac} + \nu_s \sin\vartheta_{vac}\,; \\ \nu_2 = -\nu_a \sin\vartheta_{vac} + \nu_s \cos\vartheta_{vac}. \end{cases}$$

$\nu_1 \approx \nu_a$, $a = e, \mu, \tau$

$\nu_2 \approx \nu_s$ if ϑ is small

$\vartheta \equiv \vartheta_{vac}$ = vacuum mixing angle

→ ρ = density matrix of neutrinos;

[wave function description is not applicable (as e.g. in the Sun) because of fast loss of coherence]

$\mathrm{Im} A \neq 0$

$$\circledast \quad \frac{d\rho}{dt} = i\big[\underset{\substack{vac \\ \oplus\,\text{refraction} \\ \text{index}}}{H}, \rho\big] + \underline{\{H_{int}, \rho\}} \in \begin{cases} \text{AD}, \text{Stodolsky}, \\ \text{Raffelt} \oplus \text{Sigl} \end{cases}$$

non-hermittean Hamiltonian, because the system is open

usully mimicked by $\underline{-\{\gamma, (\rho - \rho_{eq})\}}$,

$$\gamma = \begin{pmatrix} \gamma & 0 \\ 0 & 0 \end{pmatrix}, \quad \rho_{eq} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} f_{eq}, \quad f_{eq} = \left(1 + e^{E/T}\right)^{-1}$$

Refraction index or effective potential

$$i\not\partial\psi_a = \frac{1}{\sqrt{2}}G\left(\bar\psi_d O_\alpha^{db}\psi_b\right) O_\alpha \psi_a$$

$$\left\langle \frac{G}{\sqrt{2}}\bar\psi O_\alpha \psi \right\rangle = \delta_{\alpha 0} V_{eff}$$

$\langle J_j \rangle = 0$ because of isotropy,

$\langle J_0 \rangle \neq 0$, $\langle J_0 \rangle \sim$ (charge asymmetry)

Nonlocality: $\bar\psi_d O_\alpha^{dc} \psi_a \psi_c$

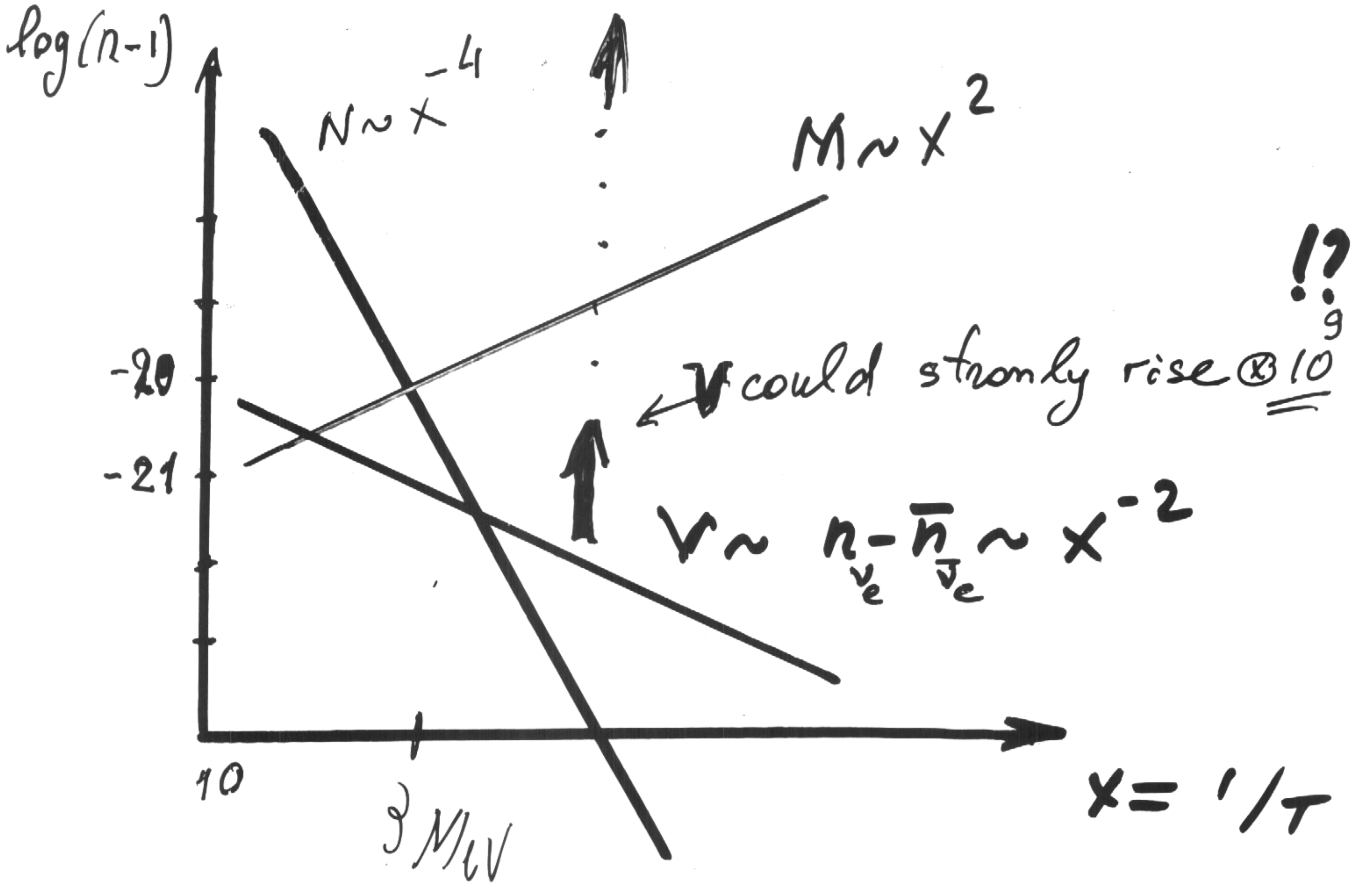
$$\frac{g^2}{M^2}$$

not "current × current"

$$V_{eff} = \pm C_1 \eta G_F T^3 + C_2^a \frac{G_F^2 T^4 E}{\alpha}$$

$C_1 = 0.95$; $C_2^e = 0.61$; $C_2^{\mu,\tau} = 0.17$

Refraction index (log / log) - scale



$$\delta m^2 \sim 1\,eV^2$$

Exact expressions for coherence breaking terms: perturb. solution to second order $(i\not{\partial}+m)\psi = G\bar{\psi}\,\gamma\psi\psi$

$$\frac{d\rho}{dt} = \ldots - \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\,\frac{d^3p_3}{\ldots}\,\frac{d^3p_4}{\ldots}\,(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)$$

$$\Big\{ |A_{ann}|^2 \Big[\{g,\rho_1 g\bar{\rho}_2\} + \{g,\bar{\rho}_2 g\rho_1\} - f_3 f_4 \Big]$$

$$+ |A_{el}|^2 \Big[f_2\{g^2,\rho_1\} - f_4\{g^2\rho_3\} \Big] \Big\}. \qquad \rho = \rho(t,p)$$

$$g = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad \frac{\partial\rho}{\partial t} + \frac{\partial\rho}{\partial p}\,\dot{p}$$

In cosmology: $\underset{\sim\sim\sim}{\dfrac{d\rho}{dt} = \dfrac{\partial\rho}{\partial t} - Hp\dfrac{\partial\rho}{\partial p}}$ $\quad -\dot{p} = Hp$

$x = \dfrac{1\,\mathrm{MeV}}{T}$, $y = \dfrac{p}{T}$; if $\dot{T} = -HT$

$\hookrightarrow \dfrac{d\rho}{dt} \Rightarrow Hx\dfrac{\partial\rho}{\partial x}$

Problem: solve kinetic eqs. numerically with correct coherence breaking terms, even in non-resonant case

Exact description of coherence loss

↳ integrals over phase space ⇒ AD

technically very difficult

(without oscillations, exact integro-differential kinetic eqs. AD + S. Hansen + S. Pastor + D. Semikoz)

$$i\partial_t \rho = \ldots - i\{\gamma, \rho - \rho_{eq}\}$$

What is γ: $\gamma = \gamma_{elastic}$?

$\gamma_{annihilation}$ (AD + R. Barbieri)

- or $\gamma_{tot} = \gamma_{el} + \gamma_{ann}$ (?) (many others)
- $\gamma_{el} \sim 10\,\gamma_{ann}$, but conserves number density,
- while, $\{\gamma, \rho - \rho_{eq}\}$ does not.

no new ν_a are produced

Standard (old) approach:

Rate of ν_s production

$$\Gamma_s = \left\langle \sin^2 2\theta_{matter} \sin^2 \frac{\delta m^2 t}{2E} \Gamma_{active} \right\rangle$$

$$\sin 2\theta_m = \frac{a \sin 2\theta_{vac}}{a \cos 2\theta_{vac} - E(n-1)} , \quad a = \frac{\delta m^2}{2E}$$

$$E(n-1) = V_{eff}$$

$$\left(\sin^4 2\theta_{vac}\right) \delta m^2 < 6 \cdot 10^{-3} eV^2 \left(\frac{T_a}{3 MeV}\right)^6 (\Delta K_\nu)^2$$

If $\Gamma_a = \Gamma_{ann}$, $T_e \simeq 3 MeV$, $T_\mu \simeq 5 MeV$ (R. Barbieri, AD)

If $\Gamma_a = \Gamma_{tot}$, $T_e = 1.8 MeV$, $T_\mu \simeq 2.7 MeV$ (K. Kainulainen + many others)

Truth (exact treatment of coherence breakin) is in between:

$$\delta m^2_{\nu_e} \sin^4 2\theta_{vac} < 5 \cdot 10^{-4} (\Delta N_\nu)^2 \quad \text{weaker}$$

$$\delta m_{\nu_{\mu,\tau}} \sin^4 2\theta_{vac} < 3.3 \cdot 10^{-4} (\Delta N_\nu)^2 \leftarrow \text{stronger limit}$$

(if $\Delta N_\nu < 1$)

No limit if $\Delta N \geq 1$ is allowed ($\nu_{\mu,\tau}$)

For ν_e there is a limit due to spectrum dist.

(NP?

$$\rho = \begin{bmatrix} \rho_{aa} & R+iI \\ R-iI & \rho_{ss} \end{bmatrix}$$

$$F = \frac{\delta m^2 \sin 2\vartheta}{2E}$$

$$W = \frac{\delta m^2 \cos 2\vartheta}{2E} + C_e \frac{G_F^2 T^4 E}{\alpha}$$

$$\begin{aligned}
\dot\rho_{aa}(p_1) &= \underline{-FI} - A_{el}^2\left[\rho_{aa}(p_1)f_l(p_2) - \rho_{aa}(p_3)f_l(p_4)\right] \\
\dot\rho_{ss}(p_1) &= \underline{FI}, && (8) \\
\dot R(p_1) &= \underline{WI} - (A_{el}^2/2)\left[R(p_1)f_l(p_2) - R(p_3)f_l(p_4)\right] \\
&\quad -(A_{ann}^2/4)\left[\rho_{aa}(p_1)\bar R(p_2) + \bar\rho_{aa}(p_2)R(p_1)\right], && (9) \\
\dot I(p_1) &= \underline{-WR - (F/2)(\rho_{ss} - \rho_{aa})} - (A_{el}^2/2)\left[I(p_1)f_l(p_2)\right. \\
&\quad \left.-I(p_3)f_l(p_4)\right] - (A_{ann}^2/4)\left[\rho_{aa}(p_1)\bar I(p_2) + \bar\rho_{aa}(p_2)I(p_1)\right]. && (10)
\end{aligned}$$

$$\rho_{as} = R + iI$$

$$A^2 \to \int A^2 \cdot \frac{1}{2E_1} \frac{d^3p_2}{(2\pi)^3 \cdot 2E_2} \cdot \frac{d^3p_3\, d^3p_4}{(2\pi)^6 \cdot 4E_3E_4} \cdot (2\pi)^4 \delta^4(\Sigma p)$$

Assumption: <u>kinetic</u> equilibrium of active ν's $\Rightarrow$ (<u>high T are effective</u> <u>ν_a in equil.</u>

!!! $\underline{\rho_{aa} = C(x)\, e^{-y}}$; $x = \frac{m_0}{T}$, $y = \frac{p}{T}$

$$\delta m^2 \sin^4 2\vartheta\Big|_{\nu_e} < 5 \cdot 10^{-4} \Delta N_\nu^2$$

$$\delta m^2 \sin^4 2\vartheta\Big|_{\nu_{\mu,\tau}} < 3.3 \cdot 10^{-4} \Delta N_\nu^2$$

$\Delta N_\nu = k_{eff}^{-3}$

Solution

NP9

1. Oscillation frequency is typically much larger than reaction rates and "large" y_2 (if $\rho_{ss} < \rho_{aa}$)

$\hookrightarrow R \simeq \frac{F \rho_{aa}}{2W}$

stationary point

2. From (9) $I \approx \frac{\dot{R}}{W} \approx \frac{F}{2W^2} \dot{\rho}_{aa}$ ← fast oscillations in comparison with $\dot{T}$ (expansion rate)

$+ \int |A|^2_{ee} \dots + \int |A|^2_{ann}$

$\hookrightarrow$ closed eq. for $\underline{\rho_{aa} = C(x) e^{-y}}$

3. $\int d^3y \; [\dot{\rho} = \dots]$

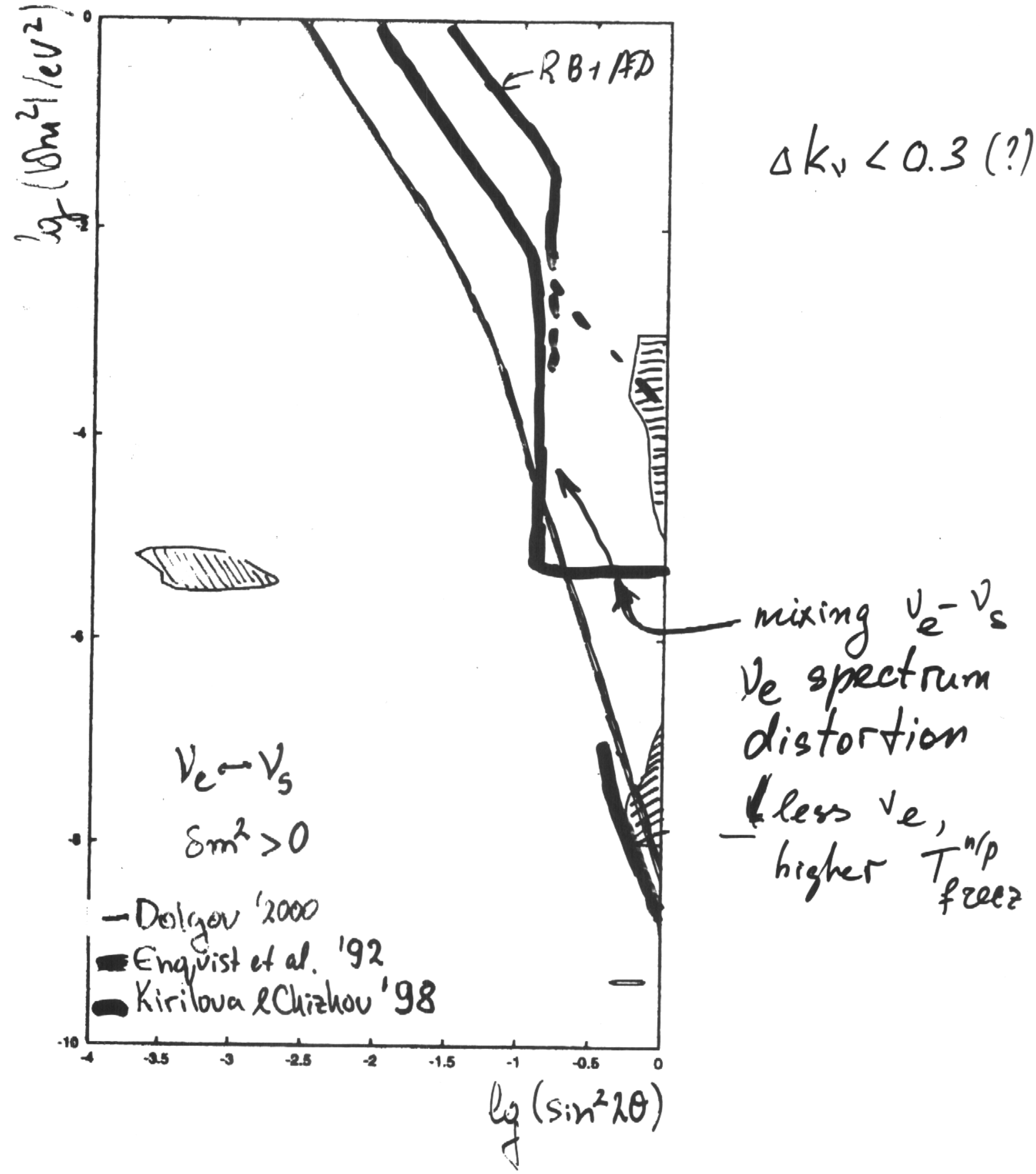
$$\hookrightarrow C'_x = \frac{K_e}{x^4}\Big[C^2 - 1 + \underbrace{\frac{C^2}{288}(6I_2 + I_1^2) + C(\dots)}_{\text{this is } \sim A^2_{ann}}\Big]$$

$$I_n = \int dy\, y^3 e^{-y} \left(\frac{F}{W}\right)^n$$

If $|C-1| < 1$, then $C \approx 1 + \int_0^\infty dx_1\, e^{-\frac{2k}{3}\left(\frac{1}{x_1^3} - \frac{1}{x}\right)} \Phi(x)$

known function

All integrals can be taken analytically



VO, LMA and LOW from SK

SMA from four-neutrino mixing scheme
Guinti et al. '2000 hep-ph/0007154

From Daniela Kirilova
(Dubna, 2000)

RESONANT CASE.

↳ generation of lepton asymmetry in active ν sector

Conditions for large asymmetry: $\delta m^2 > 10^{-6}\, eV^2$

$\sin^2 2\vartheta_{vac} < 10^{-3}$

Refraction index charge dependent:

$$n - 1 = \dots \pm c\eta G_F T^3 \dots \quad , \quad \eta = \frac{N_\nu - \bar{N}_\nu}{N_\nu + \bar{N}_\nu \text{ or } N_\gamma}$$

resonance conditions are different for ν and $\bar{\nu}$

↳ positive feedback ⇒ $\eta \nearrow$

1. R. Barbieri + AD ⇒ found the effect but (1990 or 91?) concluded: it was small
2. R. Foot, R. Volkas, M. Thomson (1996) strong asymmetry rise, up to 0.2–0.3
3. X. Shi, X. Shi, G. Fuller (1996) strong and chaotic

Numerical solution: difficult to achieve stability; Australian group, P. Di Bari + R. Foot ⇒ OK ⊕ analytic analysis

Kinetic equations:

$$\begin{cases} i\,(\partial_t - Hp\partial_p)\,\rho_{aa} = F\,\dfrac{\rho_{sa}-\rho_{as}}{2} - i\Gamma\,(\rho_{aa}-f_{eq}) \\ i\,(\partial_t - Hp\partial_p)\,\rho_{ss} = -F\,\dfrac{\rho_{sa}-\rho_{as}}{2} \\ i\,(\partial_t - Hp\partial_p)\,\rho_{as} = W\rho_{as} + F\,\dfrac{\rho_{ss}-\rho_{aa}}{2} - i\dfrac{\Gamma}{2}\,\rho_{as} \\ i\,(\partial_t - Hp\partial_p)\,\rho_{sa} = -W\rho_{sa} - F\,\dfrac{\rho_{ss}-\rho_{aa}}{2} - i\dfrac{\Gamma}{2}\,\rho_{sa} \end{cases}$$

$$F = \frac{\delta m^2 \sin 2\theta}{2E}, \qquad W = \frac{\delta m^2 \cos 2\theta}{2E} + V_{eff}$$

$$V_{eff}^a = C_2^a \frac{G_F^2 T^4 E}{\alpha} \pm C_1 G_F T^3 \eta,$$

$$\eta = \frac{n_\nu - \bar{n}_\nu}{n_\gamma}, \qquad n_\nu - \bar{n}_\nu \sim \int d^3p\,(\rho_{aa} - \rho_{\bar{a}\bar{a}})$$

$\hookrightarrow$ integro-differential equation, very difficult for numerical treatment

(fast oscillations under $\int d$

Single mode approximation $p \to \langle p \rangle = 3.15\,T$

possibly oversimplification

accurate

Approximate analytic solution (agrees with FTV; ν_μ; ν_e? DB,F

(based on a large parameter: oscillation frequency with respect to expansion)

1) $x = m/T$, $y = p/T$ (for $\dot{T}/T = -H$)

$$(\partial_t - Hp\,\partial_p) \Longrightarrow Hx\,\partial_x$$

2) $q = \xi x^3$, $\xi_e = 6.63 \cdot 10^3 (\delta m^2 \cos 2\vartheta)^{1/2}$

$\xi_{\mu,\tau} = 1.257 \cdot 10^4 (\delta m^2 \cos 2\vartheta)^{1/2}$

→ makes resonance (in the limit of vanishing asymmetry) at $\underline{\underline{q = y}}$

$$\frac{1}{Hx} V_{eff} = 1.12 \cdot 10^9 \cos 2\delta\, (\delta m^2)^{1/2} \frac{x^2}{y} \left[\left(\frac{y}{q}\right)^2 - 1 \right]$$

(for $\eta = 0$)

R.1

$$\rho_{aa} = f_{eq}(1+a), \quad \rho_{ss} = f_{eq}(y)(1+s)$$

$$\rho_{as} = \rho^*_{sa} = f_{eq}(h+il)$$

$$s'_q = -\frac{K s_2}{y} l,$$

$$a' = \frac{K}{y}(s_2 l - 2\gamma a)$$

$$h' = \frac{K}{y}(Wl - \gamma h)$$

$$l' = \frac{K}{y}\left[s_2 \frac{a-s}{2} - Wh - \gamma l\right]$$

$$q = \frac{1}{T_s}$$

deceptibly simple !

very difficult to treat numerically $\hookrightarrow W \Rightarrow \int d^3p\,(a-\bar{a})$

$$K_e = 5.63\cdot 10^4 (\delta m^2 \cos 2\theta)^{1/2} \gg 1$$

$$K_\mu = 2.97\cdot 10^4 (\delta m^2 \cos 2\theta)^{1/2} \gg 1$$

$$W = U \pm yVZ, \quad \underline{U = \frac{y^2}{q^2} - 1}$$

$$V = b\,q^{-4/3}, \quad \gamma = \epsilon \frac{y^2}{q^2}, \quad \epsilon \sim 10^{-2} \text{ (known)}$$

$$b_e = 3.31\cdot 10^{-3} (\delta m^2 c_2)^{-1/3}, \quad b_{\mu,\tau} = 7.77\cdot 10^{-3} (\delta m^2 c_2)^{-1/3}$$

$$Z = \frac{10^{10}}{8\pi^2}\left[(\sim 8)\eta_{other} + \underline{\int_0^\infty dy\, y^2 f_{eq}(y)(a-\bar{a})}\right]$$

(K.5)

I. Formal solution of eqs: $\quad l' = \ldots$
$\quad h' = \ldots$

$$l(q,y) = -\frac{K \sin 2\vartheta}{2y} \int_{q_{in}}^{q} dq_1 \left[a(q_1) - s(q_1)\right] \cdot$$

$$\Delta\Gamma = \Gamma - \Gamma_1, \quad \Delta\phi = \phi - \phi_1 \qquad \cdot\, e^{-\Delta\Gamma} \cos \Delta\phi$$

$$\Gamma_q' = \frac{K\gamma}{y}, \quad \phi_q' = \frac{K}{y} W, \quad W = u \pm v z$$

II. The same formal solution of eq. for a', s'

$$\Delta = a - s - (\bar{a} - \bar{s}), \quad \Sigma = a - s + (\bar{a} - \bar{s})$$

$$\left[\begin{aligned} \Delta' + \frac{K\gamma}{y}\Delta &= \frac{K^2\gamma}{y_2} \int_{q_{in}}^{q} dq_1 \gamma_1 \Delta_1 e^{-\Delta\Gamma} - \\ &- \left(\frac{K \sin 2\vartheta}{y}\right)^2 \int dq_1 e^{-\Delta\Gamma} \left(\Sigma_1 \frac{c - \bar{c}}{2} + \Delta_1 \frac{c + \bar{c}}{2}\right) \\ \Sigma' + \frac{K\gamma}{y}\Sigma &= \frac{K^2\gamma}{y_2} \int dq_1 \gamma_1 \Delta_1 e^{-\Delta\Gamma} - \\ &- \left(\frac{K \sin 2\vartheta}{y}\right)^2 \int dq_1 e^{-\Delta\Gamma} \left(\Sigma_1 \frac{c + \bar{c}}{2} + \Delta_1 \frac{c - \bar{c}}{2}\right) \end{aligned}\right.$$

quickly ocs. function

Integrating by parts $\Rightarrow$ γ-terms $\Rightarrow$
$\varkappa \Sigma'$, $\quad \underline{\varkappa = 1 \div 2}$

$$\frac{c-\bar{c}}{2} = \sin\left[K(q-q_1)\left(\frac{1}{y}+\frac{y}{qq_1}\right)\right] \underline{\sin\int_{q_1}^{q} dq_2\, V_2 Z_2}$$

$$\frac{c+\bar{c}}{2} = \cos(\ldots)\cos(\ldots)$$

does not depend on y

Initially: $\Sigma_{in} = 2,\ \Delta_{in} \lesssim 10^{-9}$

$\hookrightarrow \underline{\underline{\Sigma - \Delta \approx 2}}$

III ⊗ $\int dy\, y^2 f_{eq}(y) \Rightarrow \underline{\underline{K(q-q_1) \sim 1}}$

$$\hookrightarrow K\int_{q_1}^{q} dq_2\, V(q_2)Z(q_2) \approx \underline{\underline{\xi\, V(q)\, Z(q)}}$$

$\oplus\ \underline{\underline{(VZ)' \frac{\xi^2}{2K}}}$

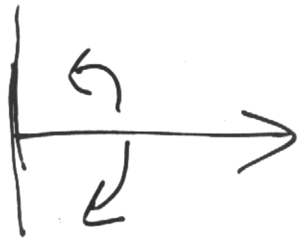
$$\int dy\, y^2 f_{eq}(y)\,(a+s-\bar{a}-\bar{s}) = \text{const}$$

$\uparrow$ total leptonic charge

$$\hookrightarrow \frac{d}{dq}\int dy\, y^2 f_{eq}(y)\,\Delta = Z'_q \cdot 16\pi^2$$

RZ

(x) $\int dy\, y^2 f_{eq}(y)$ both sides of eq. $\Delta' = \dots$
and rotate contour in complex y-plane



$$\sin\left[\xi\left(\frac{1}{y}-\frac{y}{q_2}\right)\right]$$

$$\to \exp\left\{-\xi\left(\frac{1}{y}+\frac{y}{q_2}\right)\right\}$$

$$\varkappa Z \doteq \frac{10^{10} K (\sin 2\vartheta)^2}{16\pi}\,\frac{q^2}{\sqrt{x^2+4}}\left[t_2 f_{eq}(q t_2) - t_1 f_{eq}(q t_1)\right]$$

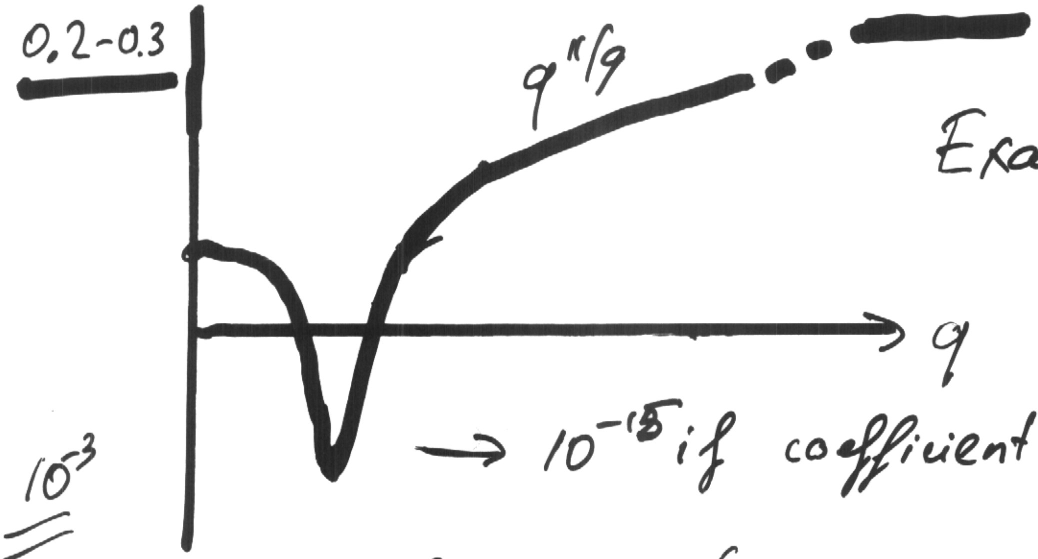
$$x = VqZ = 6q^{-1/3}Z$$

$$t_{1,2} = \frac{1}{2}\left(\sqrt{x^2+4} \pm x\right)$$

With known $Z(q)$ one can find $\Delta(q,y)$

Exact in the limit $\Delta \ll \Sigma$

$\Sigma \simeq \Sigma_{in} \simeq 2$



$\to 10^{-15}$ if coefficient is $\otimes 5$

$\to$ But $\Delta(q \simeq 6000, y_{aver}) \simeq 200$!

$T \simeq 1$

$\Delta < \Sigma$

R. 8

Large $q > 1$ $(q \gtrsim 5)$

$\Sigma + \Delta = \text{const}$, (only $\bar{\nu}$-resonance is effective)
(ν)

$\tau = 1/\nu z$, $q = q(\tau)$

Calculate all integrals in saddle point approximation (OK for large k)

$$\frac{d}{d\tau}\left[\frac{q^{4/3}(\tau)}{6\tau}\right] = \frac{10^{10}}{16\pi^2}\,\frac{2\lambda}{2+\lambda}\,\tau^2 f_{eq}(\tau)$$

$\lambda = \pi K (\sin 2\theta)^2 (dq/d\tau)$

$T = 1\,\text{MeV}$ $L_e = \frac{n_{\nu_e} - n_{\bar{\nu}_e}}{n_\gamma} \approx 0.0435$

$T = 0.5\,\text{MeV}$ $L_e = 0.25$

(*) * spectrum far from $(1 + e^{y-\mu})^{-1}$

Chaoticity – not observed. • (R.3)

For large $(K \sin 2\vartheta)$ – numerical chaos,

since $L \sim \exp(-K s^2 N) \to$ very small
$N \gg 1$

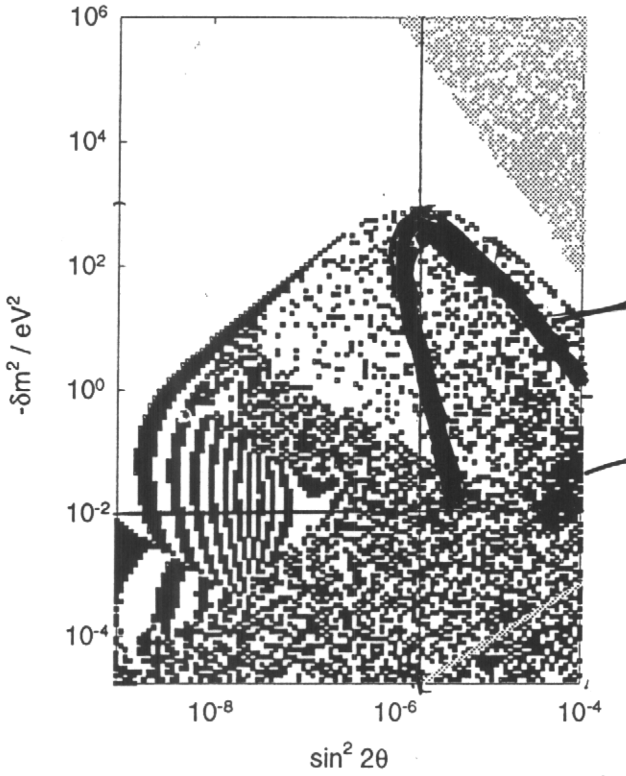
Possibly $\Delta(\vartheta, y)$ quickly oscillates if saddle point is a bad approximation

but $L = \int_0^\infty dy\, y^2 f_{eq}(y) \Delta$ is seemingly smooth (?)

Single mode approximation shows chaoticity

Numerical solution (Di Bari, Foot) – either no chaoticity or numerically uncertain

Enqvist, Kainulainen, Sorri



Di Bari, Foot

no osc

Figure 2: The distribution of the final sign of the neutrino asymmetry in the mixing parameter space. Negative $sign(L)$ is plotted in black, positive $sign(L)$ in white. The initial asymmetry was chosen to be $L^{\rm in} = 10^{-10}$.

L with the initial value $L^{\rm in} = 10^{-10}$. As can be seen, the structure of $sign(L)$ is highly complex. In the upper left hand corner, extending downwards to large θ, there is a regular region with no change in the asymmetry. Its existence is relatively easy to understand: this is the region where only the very first oscillation is carried out before the sign of the asymmetry is fixed. Since the direction of the first oscillation is determined by the sign of the initial asymmetry $L^{\rm in}$ (not necessarily the initial ν_τ-asymmetry), the sign of final neutrino asymmetry in this part of the parameter space should indeed be regular and fully determined. The bands seen in the left hand side of Fig. 2 are formed as the system goes through two or more oscillations. In this region the number of oscillations is slowly increased as θ grows leading to less determined $sign(L)$ but it still can hardly be described as chaotic yet.

In addition to the two more or less regular regions there are regions where $sign(L)$ appears

Heavy ν_s. $m > 10\,\mathrm{MeV}$

KARMEN

DEAD

$$\nu_a = \cos\vartheta\,\nu_1 + \sin\vartheta\,\nu_2$$

$$\nu_s = -\sin\vartheta\,\nu_1 + \cos\vartheta\,\nu_2$$

$$\nu_2 \to \nu_1 + l + \bar{l}\,, \qquad \tau_2 = \frac{1\,\mathrm{sec}}{(m_s/10\,\mathrm{MeV})^5 \sin^2 2\vartheta}$$

($\nu_s \leftrightarrow \nu_{\mu,\tau}$ mixing)

$1\,\mathrm{sec} \to 0.7\,\mathrm{sec}$ for $\nu_s \leftrightarrow \nu_e$ mixing

$m > 140\,\mathrm{MeV}$ $\quad \nu_2 \to \pi^0 \nu_1$

$$\tau = 5.8 \cdot 10^{-9}\,\mathrm{sec} \left[\frac{m_\pi^3}{\sin^2\theta\; m_s (m_s^2 - m_\pi^2)} \right]$$

strongly dominant mode

Cosmological production:

$$\frac{\Gamma_S}{H} \simeq \frac{\sin^2 2\vartheta_M}{2} \left(\frac{T}{T_W}\right)^3$$ ← good approximation for large masses

T_W? (controversy)

(?) $T_W \simeq 3\,\mathrm{MeV}$ ← decoupling of normal ν

$$\sin 2\vartheta_M = \frac{\sin 2\vartheta}{1 + 0.76 \cdot 10^{-19} (T_\gamma/\mathrm{MeV})^6 (\delta m^2/\mathrm{MeV})^{-1}}$$

Essential effects:

1. Total number / energy density of ν_s at BBN; ~ (production rate)
2. Spectrum of ν_e produced by the decay, deviation from equilibrium
3. Nonstandard ratio of T_ν / T_γ; $\nu_s \to \nu_a + e^+e^-$ has a strong effect
4. Non-conservation of entropy (non-adiabatic expansion)

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consideration permits one to impose a somewhat better limit, but we will not go into such detail here.

Keeping these restrictions in mind we proceed to derive bounds on M_s and the mixing angle θ demanding that the influence of heavy neutrinos on BBN should not be too strong.

4 Big-Bang Nucleosynthesis

In general, if a neutrino has a mass exceeding an MeV then it may influence the light element abundances unless it decays much before the onset of nucleosynthesis (see e.g. [14, 15] and references therein). A heavy unstable sterile neutrino will affect BBN in several ways. The main effect is through the increased energy density which leads to a faster expansion and hence an earlier freeze-out of the n/p-ratio. However, also the decay products must be taken into account since the electron neutrinos directly enter the n-p reactions. Moreover, if the ν_s decays into the e^+e^- channel the temperature evolution is altered (see [3, 16] for discussion).

The calculations are described in detail in Ref. [3]; here we only briefly describe the basic steps. First we introduce the dimensionless variables

$$x = m_0 a(t) \quad \text{and} \quad y = p\,a(t)\,, \tag{9}$$

where $a(t)$ is the cosmic scale factor and m_0 is an arbitrary normalisation factor that we have chosen as $m_0 = 1$ MeV. We also introduce a dimensionless temperature $T = T_\gamma/m_0$ and measure all masses in units of m_0. In terms of these variables the Boltzmann equations describing the evolution of the sterile neutrino, ν_s, and the active neutrinos, ν_a, can be written as

$$\partial_x f_{\nu_s}(x,y) = \frac{\left(f^{eq}_{\nu_s} - f_{\nu_s}\right)}{\tau_{\nu_s}/\mathrm{sec}} \frac{1.48\,x}{(Tx)^2} \left(\frac{10.75}{g_*(T)}\right)^{1/2} \left[\frac{M_s}{E_{\nu_s}} + \frac{3\times 2^7 T^3}{M_s^3}\left(\frac{3\zeta(3)}{4} + \frac{7\pi^4}{144}\left(\frac{E_{\nu_s}T}{M_s^2} + \frac{p^2_{\nu_s}T}{3E_{\nu_s}M_s^2}\right)\right)\right], \tag{10}$$

non rel. decay

$$\partial_x \delta f_{\nu_a} = \mathrm{D}_a(x,y,M_s) + \mathrm{S}_a(x,y)\,, \tag{11}$$

decay scatterings

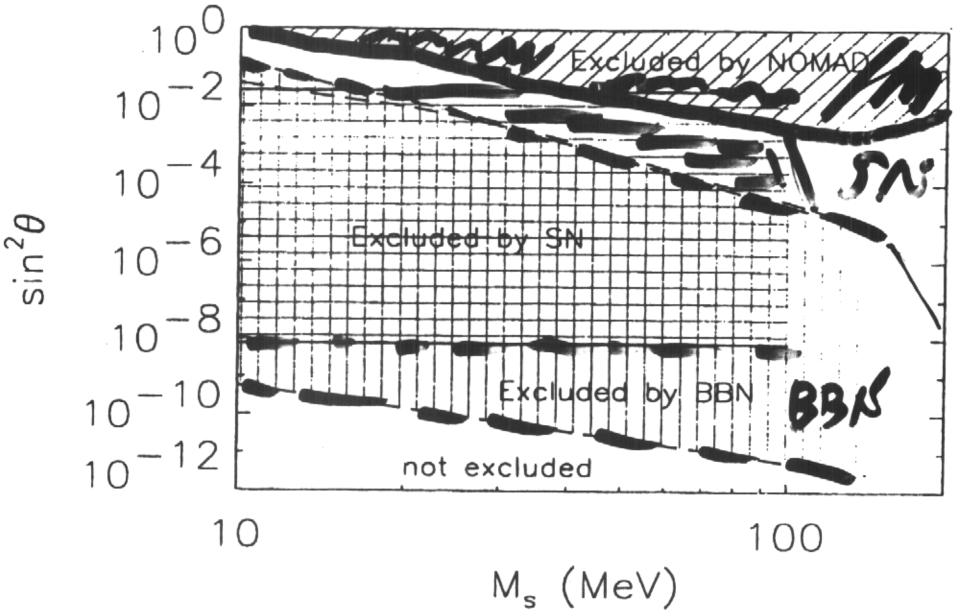
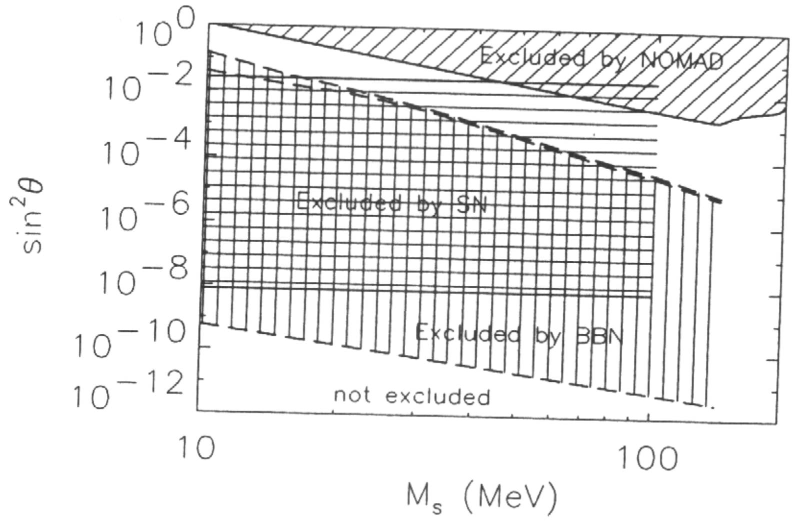


Figure 3: Summary of our exclusion regions in the $\sin^2\theta$-M_s-plane. SN 1987A excludes all mixing angles between two solid horizontal lines. BBN excludes the area below the two upper dashed line, if the heavy neutrinos were abundant in the early universe. These two upper dashed lines both correspond to the conservative limit of one extra light neutrino species permitted by the primordial ^{4}He-abundance. The higher of the two is for $\nu_{\mu,\tau}$ mixing, and the slightly lower curve is for ν_e mixing. In the region below the lowest dashed curve the heavy neutrinos are not efficiently produced in the early universe and their impact on BBN is weak. For comparison we have also presented here the region excluded by NOMAD Collaboration [2] for the case of $\nu_s \leftrightarrow \nu_\tau$ mixing.

kinetic equilibrium is assumed. This assumption very much simplifies the calculations because the integro-differential kinetic equations are reduced to simple ordinary differential equations. However, we have found that this approximation deviates substantially from the more exact treatment presented here. Even more accurate calculations are feasible with the technique developed in our previous works [14, 15, 16] where an accuracy at the sub-percent level was achieved, but this is a difficult and time-consuming problem. We would like to postpone it until the existence of heavy sterile neutrinos becomes more plausible.



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Decay $\nu_2 \to \pi \nu_a$

① $\rho_{em} = \frac{11}{2} \frac{\pi^2}{30} T^4 + r_s n_0 \frac{m_s}{2} \left(1 + \frac{m_\pi^2}{M_s^2}\right)$

$\rho_\nu = \frac{21}{4} \frac{\pi^2}{30} T^4 + r_s n_0 \frac{m_s}{2} \left(1 - \frac{m_\pi^2}{M_s^2}\right)$

$K_\nu^{eff} = 0.6$ for $m_s = 150$ MeV

$K_\nu^{eff} = 1.3$ for $m_s = 200$ MeV

^4He

(instead of $K_\nu^{eff} = 3$)

$\boxed{T_{\nu e} = T_\gamma}$

Is true, if ν_e are thermalized, and

entropy dilution of ν_s is small

② ν_e are not thermalized (late decay)

$\Rightarrow$ deficite of $\nu_e \Rightarrow$ He$\uparrow$

$\sin^2\theta < 5.8 \cdot 10^{-8} \frac{m_\pi}{m_s} \frac{1}{\left(\frac{m_s}{m_\pi}\right)^2 - 1}$ is excluded

$\tau_{\nu_2} > 0.1$ sec (except for a small region near 0.2 sec where 2 effects are cancelled)

KeV neutrinos, may be WDM

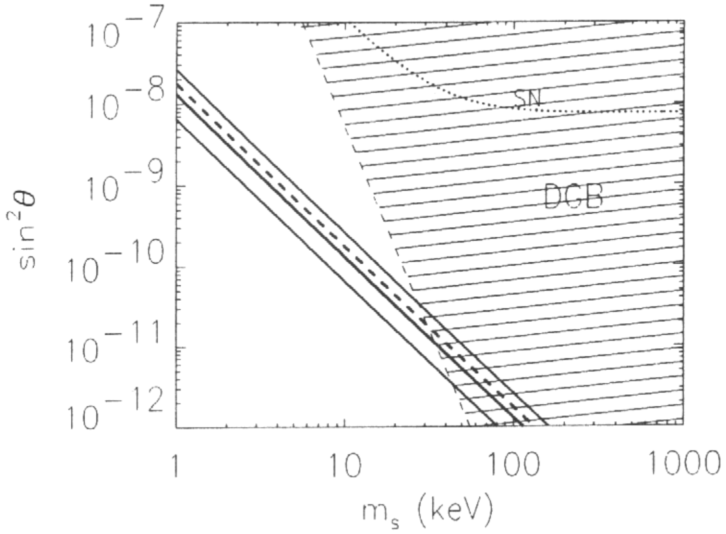


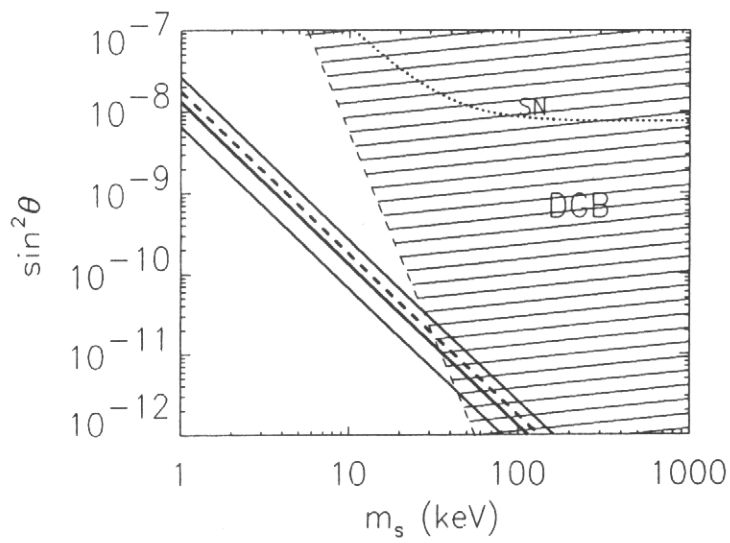
Figure 1: Bounds from $(\nu_\alpha - \nu_s)$-mixing. The middle full line describes the mass-mixing relationship if sterile neutrinos are the dark matter for $(\nu_\tau - \nu_s)$-mixing. The two other full lines allow a factor 2 uncertainty in the amount of dark matter, $\Omega_{DM} = 0.15 - 0.6$. The dashed line is for $(\nu_e - \nu_s)$-mixing. The hatched region for big masses is excluded by the Diffuse Gamma Background. The region above the dotted line is excluded by the duration of SN 1987A for $(\nu_\tau - \nu_s)$-mixing.

seems that the sterile neutrinos follow an equilibrium distribution function. This is not quite the case, because the small momentum neutrinos are produced first, and hence their relative importance is increased by the subsequent entropy release (which dilutes the active neutrinos). A different non-thermal effect can appear for $(\nu_\mu - \nu_s)$-mixing, since the factor c_2 is 0.17 when the μ's are absent (for $T \ll m_\mu$), whereas it grows to $c_2 = 0.61$ when the muons are fully present in the plasma. This means that bigger momenta will be produced with a factor 1.9 more efficiently [25].

avoid Gerstein-Zeldovich limit, $n_{\nu_s} < n_{\nu_a} \approx 55/cm3$

$\otimes 2$

$m_\nu < 94 eV \Omega h^2$



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about 1.3 GeV for $m = 1$MeV, and about 0.13 GeV for $m = 1$keV. Further, the dilution factor is somewhere between 1 and 4 depending upon whether the production happens before or after the QCD transition, and can thus enlarge the allowed region slightly compared to the figures, where we for simplicity used $g_* = 10.75$. Looking at eq. (9) it seems that the sterile neutrinos follow an equilibrium distribution function. This is not quite the case, because the small momentum neutrinos are produced first, and hence their relative importance is increased by the subsequent entropy release (which dilutes the active neutrinos). A different non-thermal effect can appear for $(\nu_\mu - \nu_s)$-mixing, since the factor c_2 is 0.17 when the μ's are absent (for $T \ll m_\mu$), whereas it grows to $c_2 = 0.61$ when the muons are fully present in the plasma. This means that bigger momenta will be produced with a factor 1.9 more efficiently [26].