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MUON g-2, DARK MATTER DETECTION AND ACCELERATOR PHYSICS

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1. INTRODUCTION

Recently, the Brookhaven E821 experiment has measured $a_\mu = \frac{1}{2}(g-2)$ for μ^+ with remarkable accuracy:

$$a_\mu = 11\,659\,202\,(14)(6) \times 10^{-10}$$ [hep-ex/0102017]

This has led to a 2.6σ deviation from the prediction of the Standard Model:

$$a_\mu^{exp} - a_\mu^{SM} = 43\,(16) \times 10^{-10}$$

Supersymmetry offers a possible explanation and the initial calculations were made using global supersymmetry [Fayet; Grifols, Mendes; Ellis, Hagelin, Nanopoulos; Barbieri, Maiani]

However in global supersymmetry there is theorem:

$$a_\mu^{SUSY} = 0$$ [Ferrara, Remiddi]

One needs broken supersymmetry to get a non-zero result, and how to do this in global supersymmetry is problematical.

In supergravity (SUGRA) models spontaneous breaking of SUSY occurs and in SUGRA GUT models led to first calculations of a_μ^{SUGRA}:

Kosower, Krauss, Sakai [1983]

Yuan, Arnowitt, Chamseddine, Nath [1984]

(of which Yuan et al. was the first complete calculation).

Here SUSY breaking triggers electroweak breaking

$$M_{SUSY} \simeq M_{Electroweak} \simeq \langle H \rangle$$

setting the scale of SUSY masses to be $\simeq 100\,\mathrm{GeV} - 1\,\mathrm{TeV}$, and hence the scale of a_μ^{SUGRA}

This mass scale is supported by:

* LEP data consistent with grand unification if SUSY masses lie $\simeq 100\,\mathrm{GeV} - 1\,\mathrm{TeV}$

* SUGRA models with R-parity invariance have a dark matter candidate, lightest neutralino, $\tilde{\chi}_1^0$, with astronomically observed amount of relic density when SUSY masses in this range

We consider then a_μ^{SUGRA} for SUGRA GUT models with R-parity invariance:

* Models with universal soft breaking at M_G (mSUGRA).

* Models with non-universal scalar masses at M_G in the Higgs and 3rd generation of squarks and sleptons.

Find there is interaction between

* Accelerator bounds

Consider (i) $m_h > 114$ GeV

(ii) $m_h > 120$ GeV

$b \to s + \gamma$ bounds

$1.8 \times 10^{-4} < BR(b \to s + \gamma) < 4.5 \times 10^{-4}$

* Relic density bounds

$0.025 \le \Omega_{\tilde{\chi}_1^0} h^2 \le 0.25$

* a_μ^{SUGRA} bounds (2σ)

$11 \times 10^{-10} \le a_\mu^{SUGRA} \le 75 \times 10^{-10}$

To produce strong constraints on the SUSY parameter space to produce:

* Predictions on dark matter detection cross sections $\sigma_{\tilde{\chi}_1^0 - p}$

* SUSY mass spectra to be seen at accelerators (Tevatron, LHC, NLC).

2. Technical Details (Ref: hep-ph/0102181)

In order to get accurate results, need to include a number of corrections:

* Relic density calculation
  co-annihilation $\tilde{\tau}_1 - \tilde{\chi}_1^0$ effects
  large $\tan\beta$
* Large $\tan\beta$ NLO corrections to $b \to s + \gamma$
* Loop corrections to m_b, m_τ (important at large $\tan\beta$)
* Two loop and pole mass corrections to m_h
* QCD RGE corrections below M_{SUSY}

There still remains some error in theoretical calculation of m_h and to be conservative we assume 3 GeV over estimate i.e.

$$m_h^{exp} = 114 \text{ GeV} \Rightarrow m_h^{th} = 111 \text{ GeV}$$
$$m_h^{exp} = 120 \text{ GeV} \Rightarrow m_h^{th} = 117 \text{ GeV}$$

We do not assume any GUT group relations (such as Yukawa unification or proton decay constraints) except grand unification of gauge coupling constants.

3. mSUGRA Model

mSUGRA depends on 4 parameters and 1 sign:

m_0 : scalar masses at M_G

$m_{1/2}$: gaugino masses at M_G ($m_{\tilde{\chi}_1^0} \cong 0.4\, m_{1/2}$)

A_0: cubic soft breaking mass at M_G

$\tan\beta$: $\langle H_2\rangle / \langle H_1\rangle$ at M_{EW}

$\frac{\mu}{|\mu|}$: sign of Higgs mixing parameter ($W^{(2)} = \mu H_1 H_2$)

Parameter range:

$$m_0,\, m_{1/2} \leq 1 \text{ TeV} \quad (m_{\tilde{g}} \lesssim 2.5 \text{ TeV})$$

$$2 \leq \tan\beta \leq 40$$

$$|A_0| \leq 4\, m_{1/2}$$

It's been known from the beginning [Kosower et al., 1983] that a_μ^{SUGRA} increases with $\tan\beta$. The leading term comes from light chargino, $\tilde{\chi}_1^{\pm}$.

Expanding for $(\mu \pm \tilde{m}_2)^2 \ll M_W^2$:

$$a_\mu^{SUGRA} \cong \frac{\alpha}{4\pi} \frac{1}{\sin^2\theta_W} \left(\frac{m_\mu^2}{m_{\tilde{\chi}_1^\pm}\, \mu} \right) \frac{\tan\beta}{1 - \frac{\tilde{m}_2^2}{\mu^2}} \left[1 - \dots \right] F(x)$$

$$m_{\tilde{\chi}_1^\pm} \cong 0.8\, m_{1/2}\,; \quad x = \frac{m_{\tilde{\nu}}^2}{m_{\tilde{\chi}_1^\pm}^2}$$

One has

$$F(x) \cong 0.6\,, \quad \frac{m_{1/2}^2}{m_0^2} \gg 1\,; \quad F(x) \simeq \frac{\ln m_0^2}{m_0^2}\,, \quad \frac{m_0^2}{m_{1/2}^2} \gg 1$$

which immediately says two things:

(1) Sign of μ is sign of a_μ and from $g_\mu - 2$:

[Nath, Lopez et al.]

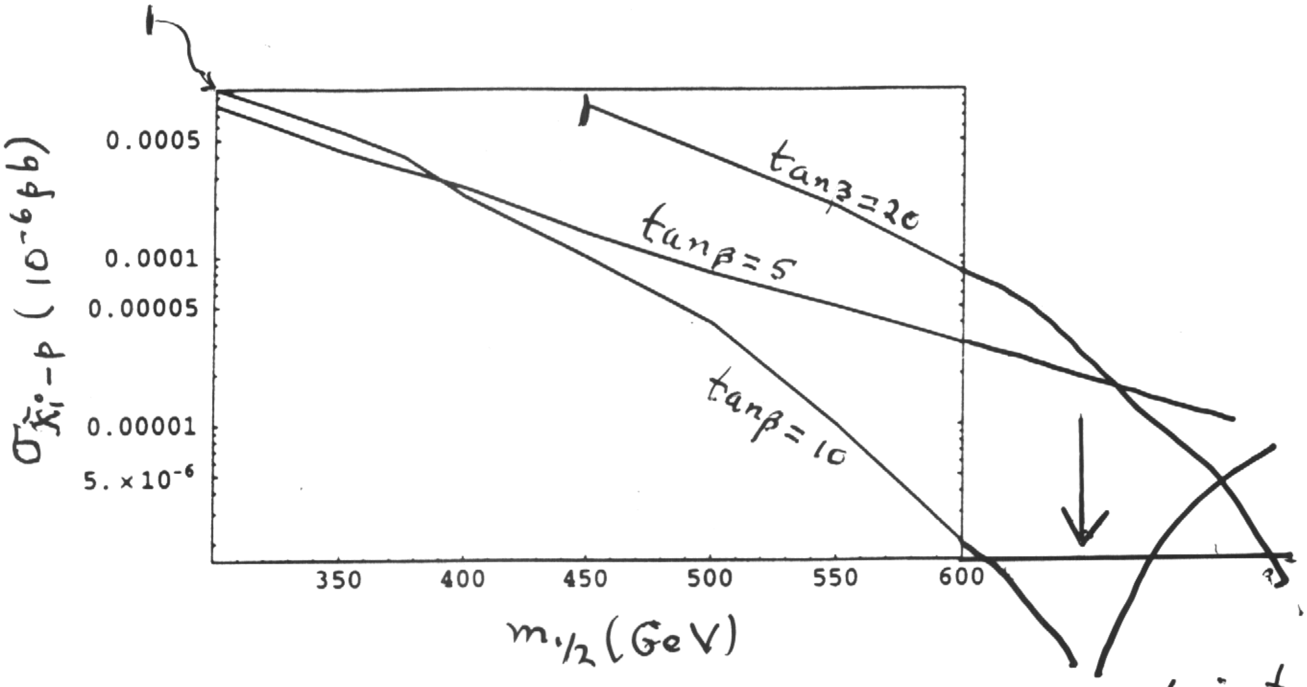
$$\mu > 0$$

Good news for dark matter detection since for $\mu < 0$ cancellations can occur reducing cross section of $\sigma_{\tilde{\chi}_1^0 - p} < 10^{-12}$ pb which is unaccessible to all future planned detectors!

(2) a_μ^{SUGRA} decreases with increasing $m_{1/2}$, which implies two things:

$\sigma_{\tilde{\chi}_1^0 - p}$ for $\tan\beta = 5, 10, 20$; $\mu < 0$

$m_{1/2} > 300$ GeV



| = lower bound due to $b \to s + \gamma$ constraint

First, as we will see, most of the allowed part of parameter space is in co-annihilation region. Here m_0 is essentially determined by $m_{1/2}$ (for a fixed A_0) and is not a free parameter:

Fig. Corridors in $m_0 - m_{1/2}$ plane allowed by relic density constraint for $\tan\beta = 40$, $m_h > 114$ GeV, $\mu > 0$, $A_0 = 0, -2m_{1/2}, 4m_{1/2}$ (bottom to top).

Thus:

(1) Lower bound on a_μ^{SUGRA} determines upper bound on $m_{1/2}$.

(2) But also, m_h increases with $m_{1/2}$ and $\tan\beta$. Thus lower bound on m_h (with an upper bound $m_{1/2}$ already determined) implies a lower bound on $\tan\beta$.

We find at 95% C.L.:

$m_h > 114$ GeV

$\tan\beta > 7$; $A_0 = 0$

$\tan\beta > 5$; $A_0 = -4m_{1/2}$

$m_h > 120$ GeV

$\tan\beta > 15$; $A_0 = 0$

$\tan\beta > 10$; $A_0 = -4m_{1/2}$

$\mu > 0$, $m_h > 114$ GeV, $\tan\beta = 40$

FIGURES

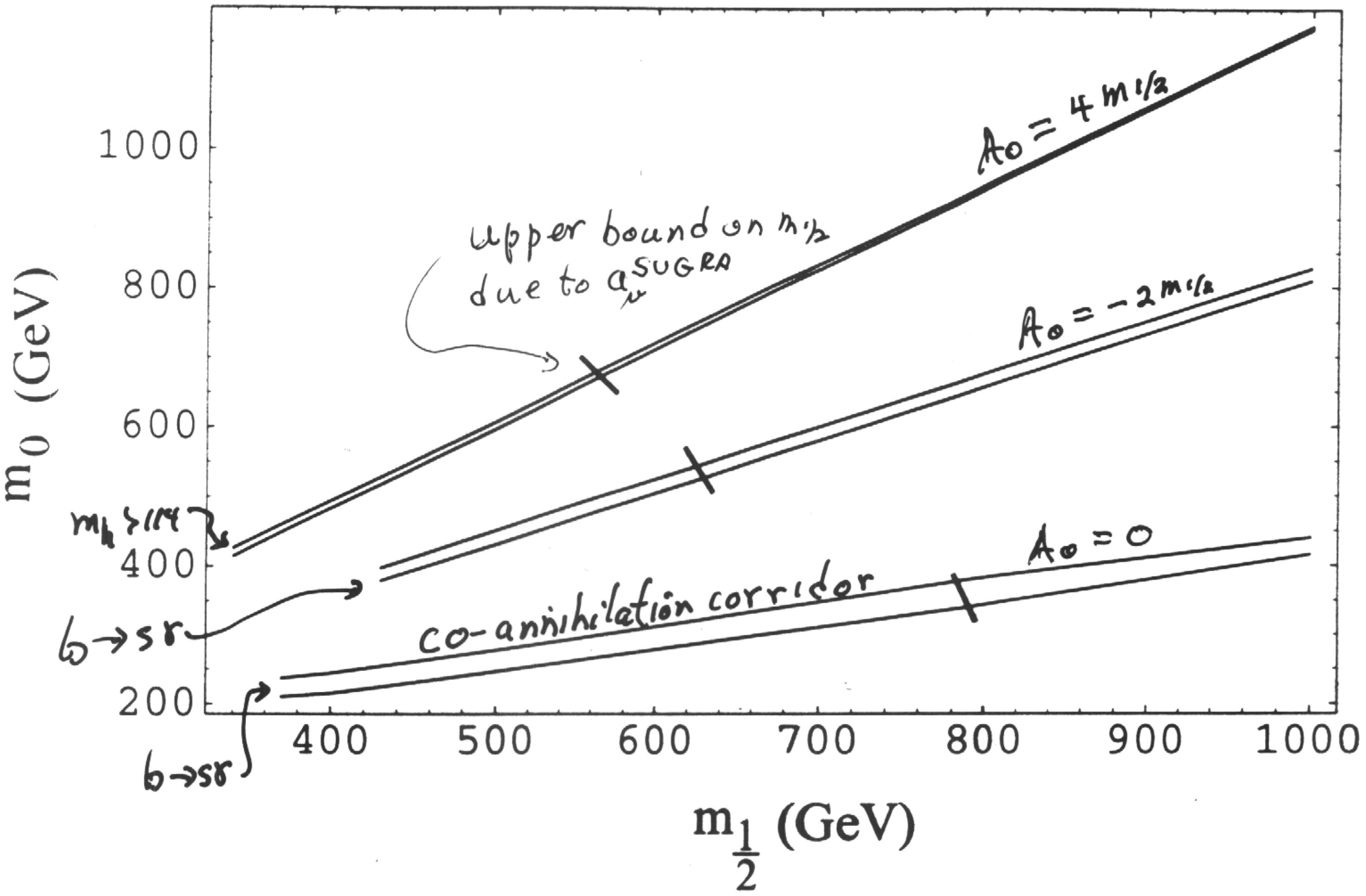


FIG. 1. Corridors in the $m_0 - m_{1/2}$ plane allowed by the relic density constraint for $\tan\beta = 40$, $m_h > 111$ GeV, $\mu > 0$ for $A_0 = 0, -2m_{1/2}, 4m_{1/2}$ from bottom to top. The curves terminate at low $m_{1/2}$ due to the $b \to s\gamma$ constraint except for the $A_0 = 4m_{1/2}$ which terminates due to the m_h constraint. The short lines through the allowed corridors represent the high $m_{1/2}$ termination due to the lower bound on a_μ of Eq. (1).

Note: The $b \to s\gamma$ constraint and m_h constraint forces parameter space into coannihilation domain ($m_{1/2} \gtrsim (350-400)$ GeV)

Thus the combined constraints

m_h, a_μ^{SUGRA}, $b \to s\gamma$, relic density

have begun to <u>strongly</u> <u>limit</u> the parameter space, and thus affect predictions of model:

(1) <u>a_μ^{SUGRA}</u>

<u>Fig.</u> a_μ^{SUGRA} for $A_0 = 0$, $\mu > 0$ for $\tan\beta = 10, 30, 40$ (bottom to top).

One sees that the mSUGRA model can no longer accommodate large values of a_μ. If final E821 result were significantly higher than $\approx 40 \times 10^{-10}$, this would be a signal for non-universal soft breaking.

(2) <u>Accelerator Physics</u>

To see effect of reduction of parameter space on predictions for SUSY particles at accelerators, consider current 90% C.L. on a_μ i.e. $a_\mu > 21 \times 10^{-10}$:

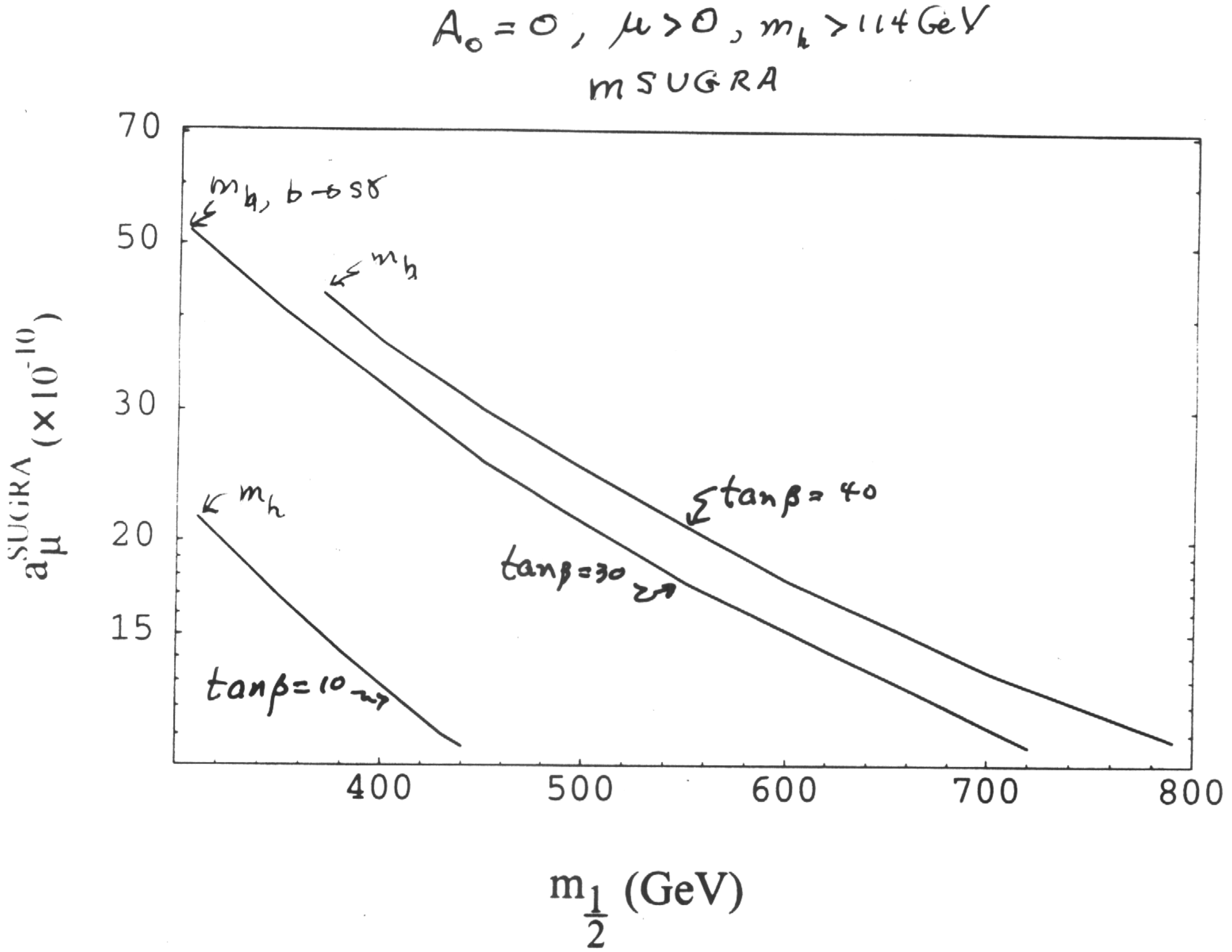


FIG. 6. mSUGRA contribution to a_μ as a function of $m_{1/2}$ for $A_0 = 0$, $\mu > 0$, for $\tan\beta = 10$, 30 and 40 (bottom to top).

For $A_0 = 0$:

$\tan\beta > 10$

and m_0 and $m_{1/2}$ are constrained:

$m_{1/2} = (290 - 550)$ GeV; $m_0 = (70 - 300)$ GeV; $\tan\beta \leq 40$

SUSY particle ranges are:

Table mSUGRA SUSY masses for 90% C.L. for a_μ and $A_0 = 0$.

Tri-lepton SUSY signal would be out of reach of Tevatron RUN II ($\tan\beta$, $m_{1/2}$ too large).

Reaches of various accelerators would be:

Tevatron RUN II: light Higgs if $m_h \lesssim 130$ GeV

500 GeV NLC: h; $\tilde{\tau}_1$; $\tilde{e}_1$ (partially); $\tilde{\chi}_1^{\pm}$ (marginally)

LHC: All

mSUGRA, $A_0 = 0$; 90% C.L. on a_μ

Table 1. Allowed ranges for SUSY masses in GeV for mSUGRA assuming 90% C. L. for a_μ for $A_0 = 0$. The lower value of $m_{\tilde{t}_1}$ can be reduced to 240 GeV by changing A_0 to $-4m_{1/2}$. The other masses are not sensitive to A_0.

$\tilde{\chi}_1^0$	$\tilde{\chi}_1^\pm$	$\tilde{g}$	$\tilde{\tau}_1$	$\tilde{e}_1$	$\tilde{u}_1$	$\tilde{t}_1$
123-237	230-451	740-1350	134-264	145-366	660-1220	500-940

For $A_0 = -4m_{1/2}$ lower bound on $m_{\tilde{t}_1}$ is

$m_{\tilde{t}_1} > 240$ GeV

(Other masses insensitive to A_0.)

(3) Dark Matter ($\tilde{\chi}_1^0$) Detection

The $\tilde{\chi}_1^0$-p cross section, $\sigma_{\tilde{\chi}_1^0-p}$, governs the dark matter detection rate for halo neutralinos. In general $\sigma_{\tilde{\chi}_1^0-p}$ decreases with increasing m_0, $m_{1/2}$. Since the a_μ minimum value has reduced the upper bounds on $m_{1/2}$, m_0, it raises possible lower bounds on $\sigma_{\tilde{\chi}_1^0-p}$.

Fig. $\sigma_{\tilde{\chi}_1^0-p}$ for $\tan\beta = 40$, $\mu > 0$, $A_0 = -2m_{1/2}, 4m_{1/2}, 0$ (bottom to top) for $m_h > 114$ GeV.

We see there is significant dependence on A_0; and the parameter space is quite restricted. Over the full range one finds

$$\sigma_{\tilde{\chi}_1^0-p} > 6\times 10^{-10}\ \text{pb} \qquad \tan\beta = 40$$

If we reduce $\tan\beta$ one expects $\sigma_{\tilde{\chi}_1^0-p}$ to fall. However, for lower $\tan\beta$, the a_μ bound becomes more constraining, eliminating more of the high $m_{1/2}$, m_0 region mSUGRA ; $\mu > 0$; $\tan\beta = 40$

$m_h > 114$ GeV

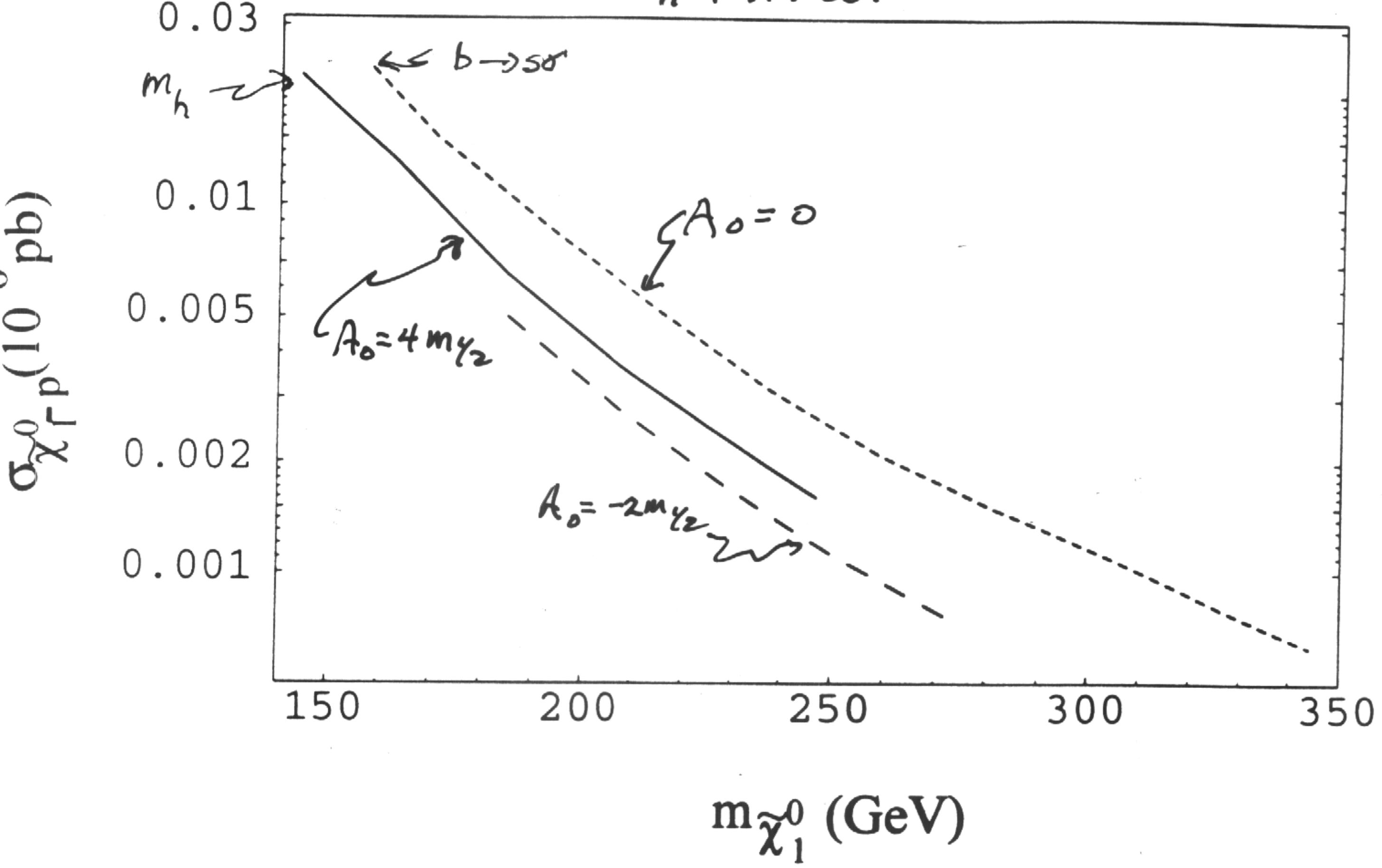


FIG. 3. $\sigma_{\tilde{\chi}_1^0 - p}$ as a function of the neutralino mass $m_{\tilde{\chi}_1^0}$ for $\tan\beta = 40$, $\mu > 0$ for $A_0 = -2m_{1/2}, 4m_{1/2}, 0$ from bottom to top. The curves terminate at small $m_{\tilde{\chi}_1^0}$ due to the $b \to s\gamma$ constraint for $A_0 = 0$ and $-2m_{1/2}$ and due to the Higgs mass bound ($m_h > 111$ GeV) for $A_0 = 4m_{1/2}$. The curves terminate at large $m_{\tilde{\chi}_1^0}$ due to the lower bound on a_μ of Eq. (1).

For $m_h > 120$ GeV, lower bounds on $m_{1/2}$ increase:

$A_0 = -2m_{1/2}$: 200 GeV

$A_0 = 0$: 215 GeV

$A_0 = 4m_{1/2}$: 246 GeV

Fig. $\sigma_{\tilde{\chi}_1^0 - p}$ for $\tan\beta = 10$, $\mu > 0$, $A_0 = 0$ (upper), $A_0 = -4\, m_{1/2}$ (lower), $m_h > 114$ GeV

We have here

$$\sigma_{\tilde{\chi}_1^0 - p} > 4 \times 10^{-10}\ \text{pb} \qquad \tan\beta = 10$$

Almost all the SUSY parameter space should now be accessible to planned future dark matter detectors.

mSUGRA, $\mu > 0$, $\tan\beta = 10$

$m_h > 114$ GeV

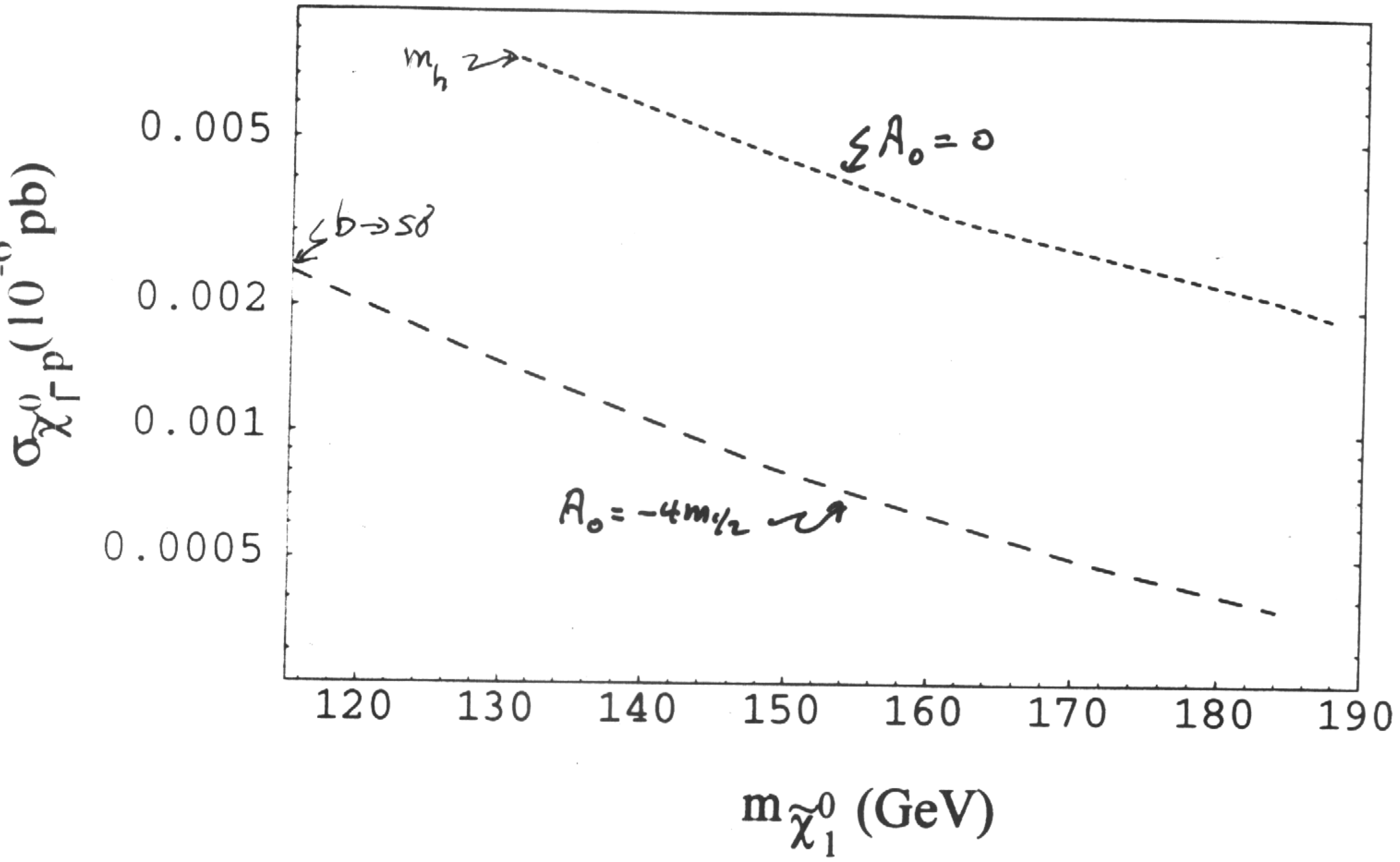


FIG. 2. $\sigma_{\tilde{\chi}_1^0-p}$ as a function of $m_{\tilde{\chi}_1^0}$ for $\tan\beta = 10$, $\mu > 0$, $m_h > 111$ GeV for $A_0 = 0$ (upper curve), $A_0 = -4m_{1/2}$ (lower curve). The termination at low $m_{\tilde{\chi}_1^0}$ is due to the m_h bound for $A_0 = 0$, and the $b \to s\gamma$ bound for $A_0 = -4m_{1/2}$. The termination at high $m_{\tilde{\chi}_1^0}$ is due to the lower bound on a_μ of Eq. (1).

For $m_h > 120$ GeV entire $\tan\beta = 10$ parameter space eliminated.

(1) $\delta_2 = 1$; $\delta_i = 0$, $i \neq 2$

Fig $\delta_2 = 1$, $\tan\beta = 40$, $A_0 = m_{1/2}$, $\mu > 0$; allowed m_0-$m_{1/2}$ region $m_h > 114$ GeV

An extra channel at high m_0 opens up in m_0-$m_{1/2}$ plane satisfying relic density constraints.

Fig. $\sigma_{\tilde{\chi}_1^0 - p}$ for $\delta_2 = 1$, $\tan\beta = 40$, $A_0 = m_{1/2}$, $\mu > 0$; $m_h > 114$ GeV

We see that $\sigma_{\tilde{\chi}_1^0 - p}$ due to new Z-channel annihilation is fairly large, and will be tested when CDMS moves to SOUDAN mine.

(2) $\delta_{10} (= \delta_3 = \delta_4 = \delta_5) = -0.7$

Fig. $\delta_{10} = -0.7$, $\tan\beta = 40$, $A_0 = m_{1/2}$, $\mu > 0$; allowed region in m_0- $m_{1/2}$ plane; $m_h > 114$ GeV.

The $\tilde{\tau}_1 - \tilde{\chi}_1^0$ co-annihilation mSUGRA corridor is moved up in m_0, and the Z s-channel annihilation channel opens.

Fig. $\sigma_{\tilde{\chi}_1^0 - p}$ for $\delta_{10} = -0.7$, $\tan\beta = 40$, $A_0 = m_{1/2}$, $\mu > 0$; $m_h > 114$ GeV.

Again Z-s channel region gives large cross sections, and $\sigma_{\tilde{\chi}_1^0 p} \gtrsim 5 \times 10^{-9}$ pb for $\tilde{\tau}_1 - \tilde{\chi}_1^0$ co-annihilation corridor.

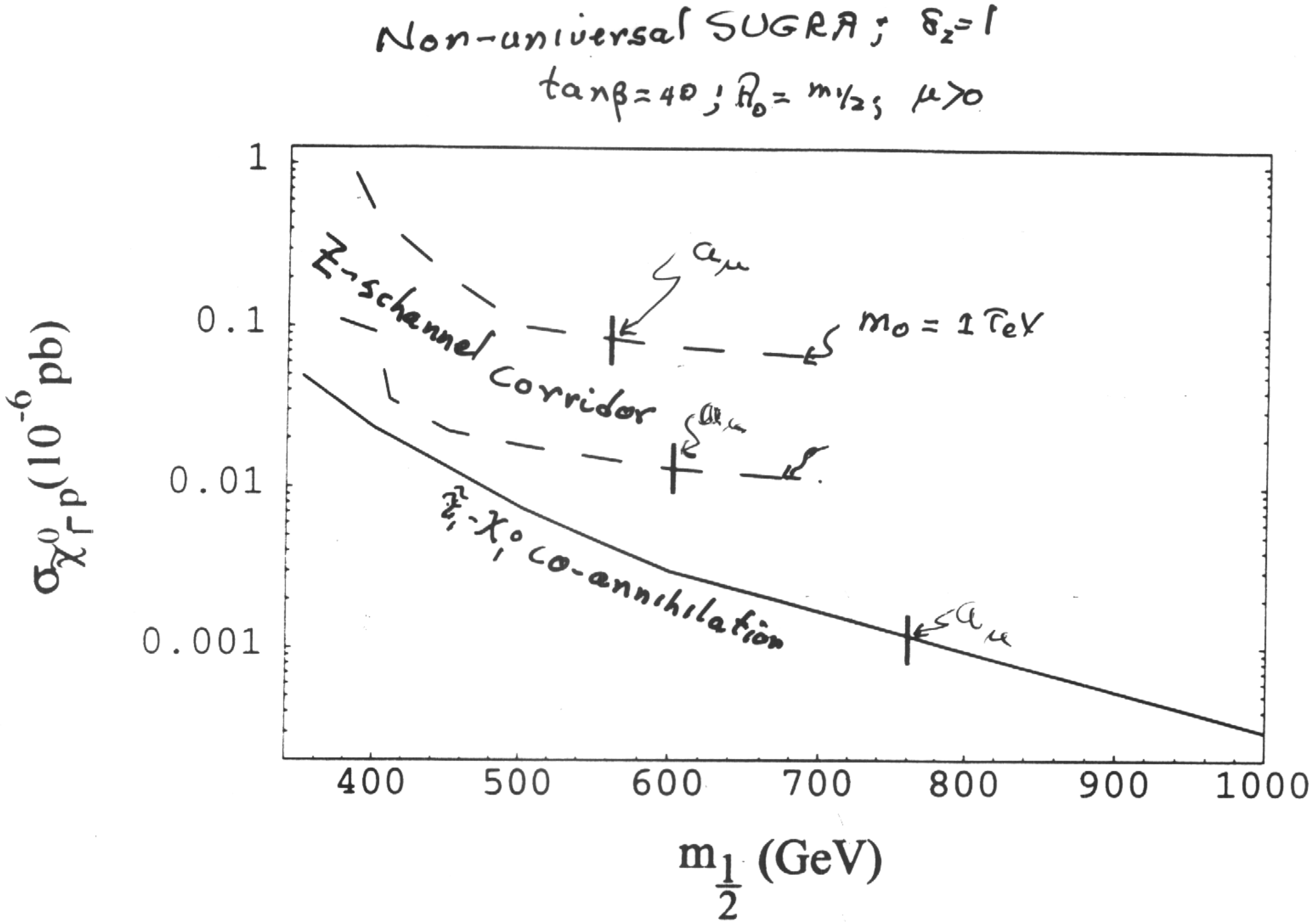


FIG. 4. $\sigma_{\tilde{\chi}_1^0-p}$ as a function of $m_{1/2}$ ($m_{\tilde{\chi}_1^0} \cong 0.4 m_{1/2}$) for $\tan\beta = 40$, $\mu > 0$, $m_h > 111$ GeV, $A_0 = m_{1/2}$ for $\delta_2 = 1$. The lower curve is for the $\tilde{\tau}_1 - \tilde{\chi}_1^0$ co-annihilation channel, and the dashed band is for the Z s-channel annihilation allowed by non-universal soft breaking. The curves terminate at low $m_{1/2}$ due to the $b \to s\gamma$ constraint. The vertical lines show the termination at high $m_{1/2}$ due to the lower bound on a_μ of Eq. (1).

4. Non-Universal Models

We parameterize non-universal Higgs and squark/sleptons at M_G by

$$m_{H_1}^2 = m_0^2(1+\delta_1)\,; \quad m_{H_2}^2 = m_0^2(1+\delta_2)$$

$$m_{q_L}^2 = m_0^2(1+\delta_3)\,; \quad m_{t_R}^2 = m_0^2(1+\delta_4)\,; \quad m_{\tilde{\tau}_R}^2 = m_0^2(1+\delta_5)$$

$$m_{b_R}^2 = m_0^2(1+\delta_6)\,; \quad m_{l_L}^2 = m_0^2(1+\delta_7)\,; \quad -1 \leq \delta_i \leq +1$$

μ^2 governs much of the physics:

$$\mu^2 = \frac{t^2}{t^2-1}\left[\left(\frac{1-3D_0}{2} + \frac{1}{t^2}\right) + \frac{1-D_0}{2}(\delta_3+\delta_4) - \frac{1+D_0}{2}\delta_2 + \frac{\delta_1}{t^2}\right] m_0^2$$

$$+ \text{universal parts} + \text{loop corrections}\,; \qquad t \equiv \tan\beta$$

If one lowers μ^2, one can open $\tilde{\chi}_1^0$ annihilation through s-channel Z-poles in relic density analysis. Lowering μ^2 also increases $\sigma_{\tilde{\chi}_1^0-p}$. To illustrate, consider two examples:

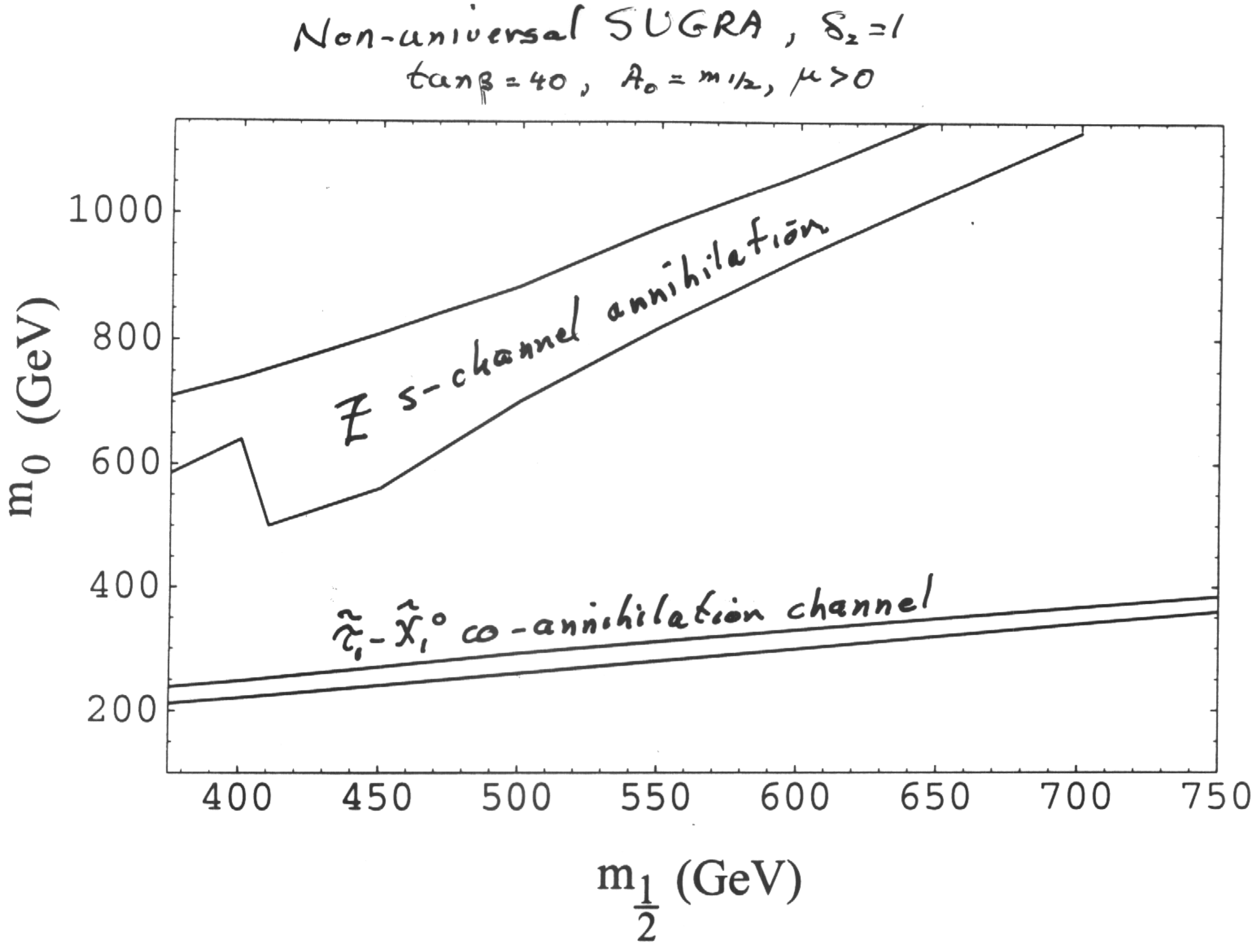


FIG. 7. Effect of a nonuniversal Higgs soft breaking mass enhancing the Z^0 s-channel pole contribution in the early universe annihilation, for the case of $\delta_2 = 1$, $\tan\beta = 40$, $A_0 = m_{1/2}$, $\mu > 0$. The lower band is the usual $\tilde{\tau}_1$ coannihilation region. The upper band is an additional region satisfying the relic density constraint arising from increased annihilation via the Z^0 pole due to the decrease in μ^2 increasing the higgsino content of the neutralino.

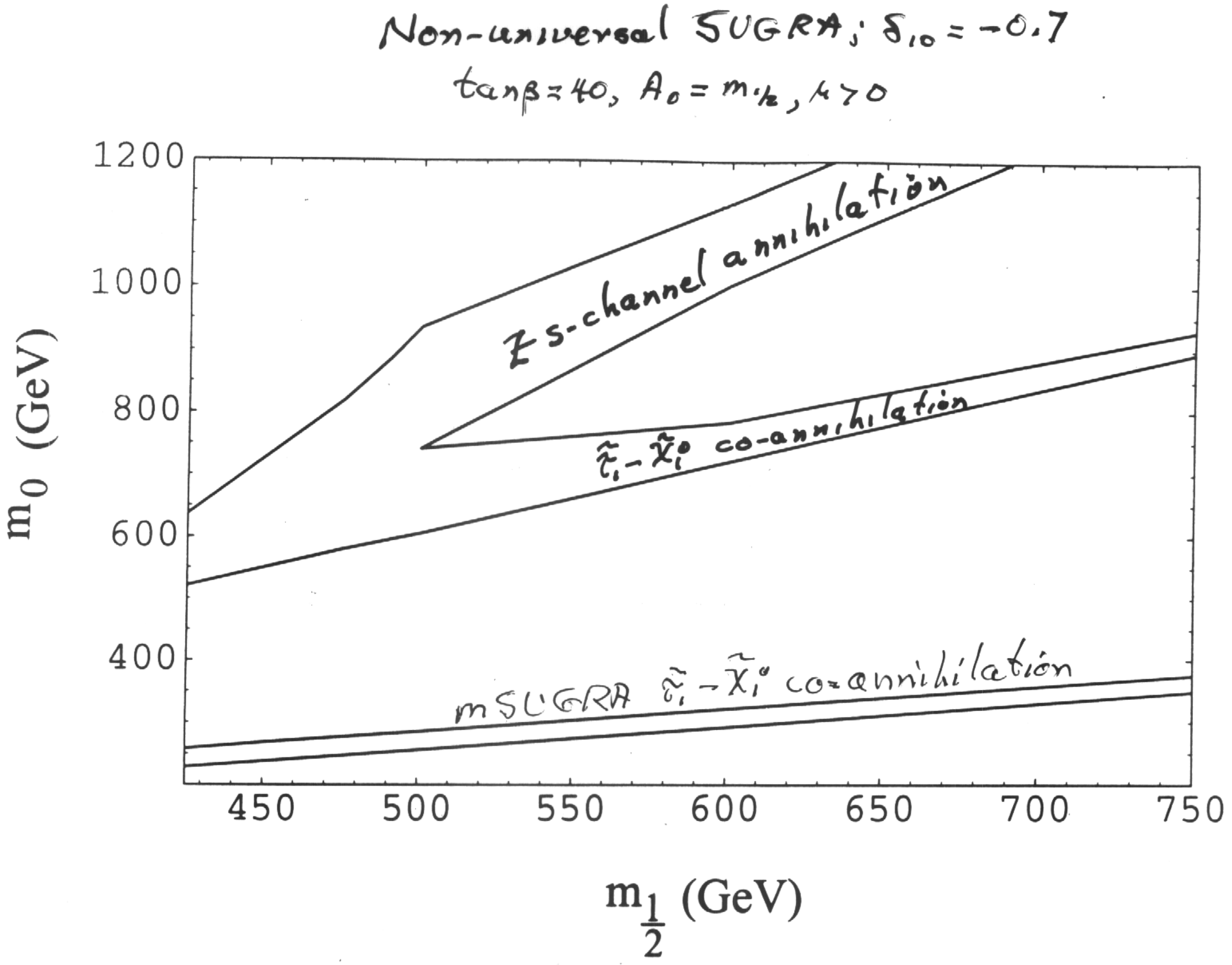


FIG. 8. Allowed regions in the $m_0 - m_{1/2}$ plane for the case $\tan\beta = 40$, $A_0 = m_{1/2}$, $\mu > 0$. The bottom curve is the mSUGRA $\tilde{\tau}_1$ coannihilation band. The middle band is the actual $\tilde{\tau}_1$ coannihilation band when $\delta_{10} = -0.7$. The top band is an additional allowed region due to the enhancement of the Z^0 s-channel annihilation arising from the nonuniversality lowering the value of μ^2 and hence raising the higgsino content of the neutralino. For $m_{1/2} \lesssim 500$ GeV, the two bands overlap.

Non-universal SUGRA; $\delta_{10} = -0.7$
$\tan\beta = 40$, $A_0 = m_{1/2}$, $\mu > 0$; $m_h > 114$ GeV

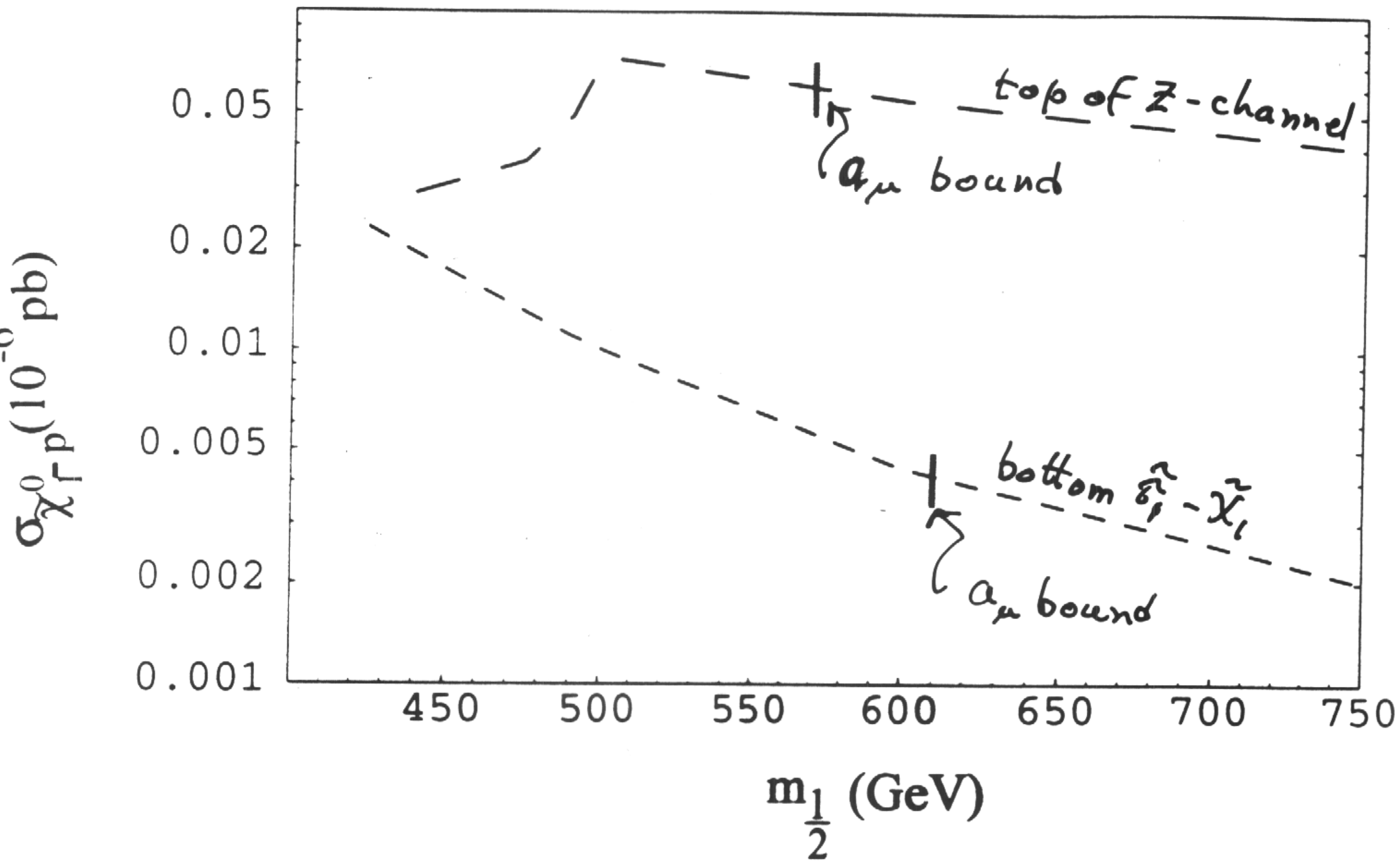


FIG. 5. $\sigma_{\tilde{\chi}_1^0-p}$ as a function of $m_{1/2}$ for $\tan\beta = 40$, $\mu > 0$, $A_0 = m_{1/2}$ and $m_h > 111$ GeV. The lower curve is for the bottom of the $\tilde{\tau}_1 - \tilde{\chi}_1^0$ co-annihilation corridor, and the upper curve is for the top of the Z channel band. The termination at low $m_{1/2}$ is due to the $b \to s\gamma$ constraint, and the vertical lines are the upper bound on $m_{1/2}$ due to the lower bound of a_μ of Eq. (1).

Conclusions

We've examined here the 2.6σ deviation of $a_\mu = \frac{1}{2}(g_\mu - 2)$ from the Standard Model under the assumption that the effect is real and can be understood in terms of supergravity GUT models with R-parity invariance.

For mSUGRA models the combined constraints from a_μ, m_h, $b \to s\gamma$ and relic density of $\tilde{\chi}_1^0$ dark matter greatly limits the SUSY parameter space greatly tightening predictions. The lower bound on a_μ produces an upper bound on $m_{1/2}$, and then the m_h, $b \to s\gamma$ constraints produces lower bounds on $m_{1/2}$ and $\tan\beta$. Thus

* $m_h > 114$ GeV : $\tan\beta > 7(5)$ for $A_0 = 0\ (-4m_{1/2})$
* $m_h > 120$ GeV : $\tan\beta > 15(10)$ for $A_0 = 0(-4m_{1/2})$

The lower bound on $m_{1/2}$ push most of the remaining parameter space in the co-annihilation domain effectively determining m_0 in terms of $m_{1/2}$. However, there is still significant dependence on $\tan\beta$, A_0.

* Predictions for reach of accelerators asuming 90% C.L. range for a_μ are:

Tevatron Run II : h (for $m_h < 130$ GeV)

500 GeV NLC : $\tilde{\tilde{\tau}}_1$, h, $\tilde{e}_1$ (part of parameter space)

LHC : All SUSY particles

* Future planned dark matter detectors should be able to sample almost all of SUSY parameter space.

* Non-universal SUGRA models allow new regions in parameter space to open (due to annihilation in early universe through s-channel Z-poles) with $\sigma_{\tilde{\chi}_1^0-p}$ accessible to be tested by current detectors.

The current a_μ anomaly is a 2.6σ effect. However, E821 has four times additional data, which should allow the determination of whether the effect is real.