Università di Pisa - Dipartimento di Ingegneria Civile e Industriale

Corso di Laurea in Ingegneria Aerospaziale

Fisica Generale II e Elettronica

Appello 5 - 08/01/2016

Soluzioni

PROBLEMA 1

1) $Q = \int_0^r \int_0^{2\pi} \sigma_0 \cos^2(\phi) \rho d\phi d\rho = \frac{\pi}{2}\sigma_0 r^2$

2) $E_z(z) = \frac{1}{4\pi\epsilon_0}\sigma_0 z \int_0^r \frac{xdx}{(x^2+z^2)^{\frac{3}{2}}} \int_0^{2\pi} \cos^2(\phi) d\phi = \frac{\sigma_0 z}{4\epsilon_0}(\frac{1}{|z|} - \frac{1}{(r^2+z^2)^{\frac{1}{2}}})$

per $z > 0$, $E_z(z) = \frac{\sigma_0}{4\epsilon_0}(-\frac{z}{(r^2+z^2)^{\frac{1}{2}}} + 1)$

per $z < 0$, $E_z(z) = \frac{\sigma_0}{4\epsilon_0}(-\frac{z}{(r^2+z^2)^{\frac{1}{2}}} - 1)$

3) $V(z) = \frac{1}{4\pi\epsilon_0}\sigma_0 \int_0^r \frac{xdx}{(x^2+z^2)}^{\frac{1}{2}} \int_0^{2\pi} \cos^2(\phi) d\phi = \frac{\sigma_0}{4\epsilon_0}((r^2+z^2)^{\frac{1}{2}} - \mid z \mid)$

4) $F_z(z) = p\frac{d}{dz}[\frac{\sigma_0}{4\epsilon_0}(1 - \frac{z}{(r^2+z^2)^{\frac{1}{2}}})] = -\frac{p\sigma_0}{4\epsilon_0}\frac{r^2}{(r^2+z^2)^{\frac{3}{2}}}$

5) $\frac{1}{2}mv^2 = \int_h^0 F_z(z)dz = -\frac{p\sigma_0 r^2}{4\epsilon_0}\int_h^0 \frac{dz}{(r^2+z^2)^{\frac{3}{2}}}$

$\frac{1}{2}mv^2 = \frac{p\sigma_0}{4\epsilon_0}\frac{h}{(r^2+h^2)^{\frac{1}{2}}}$

PROBLEMA 2

1) $I(t) = k(1-e^{-t/t_0})\int_0^{r_0}(r_0 - x)xdx \int_0^{2\pi} d\phi$

$I(t) = k(1-e^{-t/t_0})\frac{\pi}{3}r_0^3$

2) $2\pi r B_\phi(r) = \mu_0 k(1-e^{-t/t_0})\int_0^r (r_0 - x)xdx \int_0^{2\pi} d\phi$

per $r \leq r_0$, $B_\phi(r) = \mu_0 k(1-e^{-t/t_0})(\frac{r_0 r}{2} - \frac{r^2}{3})$

per $r \geq r_0$, $B_\phi(r) = \mu_0 k(1-e^{-t/t_0})\frac{r_0^3}{6}\frac{1}{r}$

3) $r_{max} = \frac{3r_0}{4}$, $B_\phi(r) = \mu_0 k(1-e^{-t/t_0})\frac{3r_0^2}{16}$

4) $I(t) = \mu_0 k\frac{l_0 r_0^3}{6R}\ln(2)\frac{1}{t_0}e^{-\frac{t}{t_0}}$

5) $E = RI_0^2\frac{t_0}{2}$, con $I_0 = \mu_0 k\frac{l_0 r_0^3}{6R}\ln(2)\frac{1}{t_0}$

1)

$$\hat{V}_1 = i\omega L \hat{I}_1 + \hat{V}_2 + 2R\hat{I}_1 + \frac{1}{i\omega C}\hat{I}_1$$

$$\hat{V}_1 - \hat{V}_2 = 2V - V = V$$

$$i\omega L + \frac{1}{i\omega C} = i\sqrt{\frac{L}{C}} + \frac{1}{i}\sqrt{\frac{L}{C}} = 0$$

$$V = 2R\hat{I}_1 \qquad \hat{I}_1 = \frac{V}{2R} \qquad I_1 = \frac{V}{2R}\cos\omega t$$

$$W_J = 2R I_1^2 = \frac{V^2}{2R}\cos^2\omega t$$

2)

$$V_1 I_1 = 2V\cos\omega t \, \frac{V}{2R}\cos\omega t = \frac{V^2}{R}\cos^2\omega t \quad \text{erogata da 1}$$

$$-V_2 I_1 = -\frac{V^2}{2R}\cos^2\omega t \quad \text{assorbita da 2}$$

3)

Su C V_c continua e sfasata di 90° rispetto a I_1 e V_1

$$V_c(0) = 0$$

4)

$$I_A = I_1 + I_2$$

$$I_2 = \frac{V_2}{R} = \frac{V}{R}$$

$$I_1 = \frac{V}{2R}$$

$$I_A = \frac{3V}{2R}$$

5)

$$E_i = \frac{1}{2}L I_1^2 + \frac{1}{2}C V_c^2 = \frac{1}{2}L\frac{V^2}{4R^2}$$

$$E_f = \frac{1}{2}L\frac{4V^2}{R^2}$$

$$\Delta E = E_f - E_i = \frac{15}{8}L\frac{V^2}{R^2}$$